Magnetic Resonance(MR) Image Reconstruction using Compressive Sensing

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Summary
MRI is done using two processes. First step, phase and frequency information are measured in K-Space, secondly mathematical computations are carried on the k-space data to reconstruct MRI image. The conventional methods with full k-space are time consuming. The mathematical theory, Compressed Sensing has been used vastly in Magnetic Resonance Imaging for acceleration of imaging process. As it highly exploits data redundancy, i.e. with significant fewer measurements, it produces accurate reconstruction in very short time. In implementation of CS, various algorithms are developed to solve nonlinear system of equations for having better quality of images and speed of reconstruction. For selection of an optimal CS algorithm, a systematic and comparative analysis of these algorithms is necessary. Four algorithms are given in our thesis; Conjugate Gradient, RecPF, FCSM and WaTM, which are studied and analyzed on different part of body. Results show that FCSM has less computation time and WaTM has high SNR ratio.

Key words:
Magnetic Resonance Imaging, Compressed Sensing, compressive sensing models

1. Introduction

A new mathematical theory has recently been published, termed as compressive sensing (CS) or compressive sampling. Compressive sensing is a signal processing technique for efficiently acquiring and reconstructing a signal by finding solutions to underdetermined linear system. According to the mathematical results found in compressive sensing techniques if the underlying image exhibits transform sparsity and if the under sampling of K-space results is incoherent artifacts in the transform domain, then the image can be recovered from randomly under sampled frequency domain data, provided an appropriate nonlinear recovery scheme is used.

The sparsity which is implicit in MR Images is exploited to significantly under sample K-space. Some MR Images like angiogram are already sparse in the pixel representation in some transform domain.

Image reconstruction using CS

In this section a formal framework of reconstruction using CS is presented. Let the reconstructed image (MR image) be represented by a complex vector x, have dimensions nx1, the under sampled Fourier transform (measurement matrix) is denoted by the term R, having dimensions mxn. In k –space, 'b' having dimensions mx1, is the sampling measurement of x, given as in equation (1)

$$b = Rx \qquad (1)$$

The CS problem is to reconstruct x given b and R. Let ψ be a linear operator used to transform pixel representation into the selected representation which result in producing sparsity in x. reconstructed image is obtained by solving the constrained optimization problem given in equation (2);

$$\text{minimize} \, ||\psi x||_1 \quad \text{s.t.} \, ||Rx - b|| < \epsilon \qquad (2)$$

Here b is k-space data obtained from MRI scanner. ϵ is threshold parameter that controls the fidelity of reconstruction to the measured data. It can roughly represent the expected noise. The l_1 norm is given as in equation (3)

$$||x||_1 = \sum_i |x_i| \qquad (3)$$

And l_1 norm, minimizing of function $||\psi m||_1$ produces sparsity. Data consistency is enforced by $||Rx\text{-}b||<\epsilon$. It means many solutions exist which are consistent with measured data but we want to have that solution which is compressible by transform operator ψ. If we chose the finite differential as sparsifying transform then the objective will become a well known problem called the total variation (TV), given as in eq (4)

$$||x||_{TV} = \sum_i \sum_j \left(\left(\nabla_1 x_{ij}\right)^2 + (\nabla_2 x_{ij})^2 \right) \qquad (4)$$

Where ▽1 and ▽2 are the forward finite difference operators on the first coordinate i and on second coordinate j

2. Compressive sensing Models:

The linear combination of total variation minimization and sparse wavelet regularization is very popular in existing CS models. Classical method, conjugate gradient [3] was first to solve this problem. RecPF [4] use a variable splitting method to solve the respective problem. FCSM [9] decomposes this problem in two simple sub-problems and solves each of them separately. This is done with the combination of phenomenon of variable and operator splitting. WaTM[5] utilizes tree sparsity in combination

Manuscript received January 5, 2015
Manuscript revised January 20, 2015

with TV and wavelet regularization which enhances performance in terms of SNR.

2.1 Conjugate Gradient:

In this model, CS problem is solved through a specific interior-point method, search step is computed by pre-conditioned conjugate gradient method (CG). This formulation can be effectively used for solving large CS domain problems. It makes use of quick algorithms for sparsifying transform and their inverse transforms.

The l_1 regularized CS (equation 5) problem is transformed in to a convex quadratic program given in equation (6)

$$\text{minimize} \parallel Rx - b \parallel_2^2 + \lambda \parallel x \parallel_1 \qquad (5)$$

where the variable $x \epsilon R^n$

$$\parallel Rx - b \parallel^2 + \sum_{i-1}^{n} \lambda u_i$$

$$-u_i \leq x_i \leq u_i \quad i = 1, \ldots \ldots, n, \qquad (6)$$

where the variables are $x \epsilon R^n$ and $u \in R^n$

In this multiplying of a vector by A and multiplying a vector by A^T **(A transpose)** are computed fast because of using fast algorithms for sparsifying transform and their inverse transforms. This make this procedure lead to large scale problems, search direction is computed by application of PCG method to the Newton System Method. A symmetric optimistic definite pre-conditioner is used that somewhat equals to Hessian of $\emptyset_t(x, t)$, given as equation (7)

$$\nabla^2 \emptyset_t(x, t) = t\nabla^2 \parallel Rx - b \parallel^2 + \nabla^2 \phi(x, t) \in R^{2n*2n} \qquad (7)$$

$$P = diag(t\nabla^2 \parallel Rx - b \parallel^2) + \nabla^2 \phi(x, t) \in R^{2n*2n} \qquad (8)$$

The second term in equation (8) is same for pre-conditioner but the first term is retained with its diagonal entries. Here diagonal entries have to be calculated only once for the whole interior point iterations.

2.2 RecPF

In RecPF model, Yang solve CS recover problems that may be solve through either L1 or TV, with special condition, Φ should be a subsample of the Fourier transform. In particular, the following problem is solved through RecPF

$$\min_{x \in R^N} \parallel x \parallel_{tV} + \frac{\lambda}{2} \parallel Rx - b \parallel_{t_2}^{t} \qquad (9)$$

Here λ is Lagrangian parameter and it can be difficult to tune its value.

The basic idea of this algorithm is to recast above (Eq 9) into the following problem given by equation 10:

$$\min_{(x,w)} \sum_{i=0}^{n-1} (w_i \parallel_{t_2} + \frac{\beta}{2} w_i \parallel -D_i x \parallel_{t_2}^{t} + \frac{\lambda}{2} \parallel Rx - b \parallel_{t_2}^{t}$$

$$\text{s.t.} \quad x \in \mathbb{R}^N \text{ and } \forall_i \; w \in R^2 \qquad (10)$$

Where each Di is the 2xN Matrix which is used to compute two spatial derivatives of x at pixel 'i'. Whenβ→∞, then both equations (9 and 10) will become equivalent. Then the algorithm, for a fix value of x, minimize above equation in w. then it uses previous optimize value of w in x, and so on.

- as equation 10 is separable in w, minimization in w can be done in O(N)
- Quadratic problem is involved in minimization in *x, so this can be obtained in* O(*n* log(*n*)), this is because of specific properties of the Fourier transform in calculating the convolution product.

In totality, thus O (n log (n)) is an algorithmic complexity in each iteration of RecPF procedure.

2.3 FCSM

In, an efficient model is proposed for minimization of linear combination of least square data fitting, L1 norm regularization and TV minimization as given in equation (11). First the main problem is divided into two sub-problems and each solve through existing techniques. Then weighted average of solutions from sub-problems is calculated through iterative framework for obtaining reconstructed image.

$$\hat{x} = arg\min_{x} \left\{ \frac{1}{2} \parallel Rx - b \parallel^2 + \alpha \parallel x \parallel_{TV} + \beta \parallel \Phi x \parallel_1 \right\} \qquad (11)$$

This algorithm is based on splitting techniques of operator and variable. Variable x is split into two variables x1 and x2. Then perform operator splitting to minimize L1 and TV minimization sub-problems over the two variables. The results are then combined through re-weighted average to have solution x. This scheme is called Composite splitting technique. This is further accelerated by scheme given in Fast Iterative Shrinkage Thresholding Algorithm (FISTA)

- for each iteration computational complexity is O(nlog(n)) only

2.4 WaTM

A new model of CS is proposed, that combines gradient sparsity, wavelet and tree sparsity and is shown in equation (12). In modeling of tree structure, coefficients of each pair of parent-child are assigned to one group, which compel them to be simultaneously zero or non-zero. As this is problem of overlapping group, it becomes difficult to solve directly. To overcome this, a new variable is introduced which divide this problem into three simple sub-problems. Then existing techniques are used to solve each problem efficiently as they now have a closed form solution.

$$\min_x \left\{F(x) = \frac{1}{2}\| Rx - b \|_2^2 + \alpha \| x \|_{TV} + \beta(\| \Phi x \|_1 + \sum_{g\in G} \| \Phi x_g \|_2)\right\} \quad (12)$$

Let $G\Phi_x = z$ to make the non overlapping convex optimization problem given by equation (13)

$$\min_{x,z}\left\{F(x) = \frac{1}{2}\| Rx - b \|_2^2 + \alpha \| x \|_{TV} + \beta(\| \Phi x \|_1 + \sum_{i=1}^{s} \| z_{g_i} \|_2) + \frac{\lambda}{2}\| z - G\Phi_x \|_2^2\right\} \quad (13)$$

Now z is given as in equation (14)

$$z_{g_i} = \arg\min_{x g_i}\left\{\beta \| z_{g_i} \|_2 + \frac{\lambda}{2}\| z_{g_i} - (G\Phi_x)_{g_i} \|_2^2\right\}, i = 1,2,\dots,s \quad (14)$$

Here gi is the ith group and s is total number of groups. And now x is given as in following eq (15)

$$x = \arg\min_x\left\{\frac{1}{2}\| Rx - b \|_2^2 + \alpha \| x \|_{TV} + \beta \| \Phi x \|_1 + \frac{\lambda}{2}\| z - G\Phi_x \|_2^2\right\} \quad (15)$$

- A convex formulation model is present that based on tree structure combined with wavelet sparsity and total variation minimization
- An extremely fast convergence performance. Each iteration cost aboutO(nlogn) only.

3. Experimental Setup

The CS algorithms discussed in section 2, are simulated on MRI images in this study to analyze the performance of these algorithms, namely CG, FCSA, RecPF and WatMRI, for our MR image data set. Partial Fourier transform of MR images consist of m rows and n columns. Sampling ratio is than defined as m/n. As much this ratio will be less, means the less number of samples (measurement) of k-space are required, resulting in lesser scanning time eventually. For the experimental results presented this ratio equals to 20%. The number of Fourier coefficients taken as measurements from higher frequencies and more than the coefficients from lower frequencies. A 0.01 [what this figure represents] Gaussian noise is added on all measurements. Original image is given with reconstructed images for visual comparison. SNR and CPU time are used for numerical evaluation. Experiments are carried on a laptop, having processor intel core i3. Matlab version is matlab R2010 a. The values for various parameters are chosen as α=.0001, β=.035 and λ=0.2 x β. All images are resized to 256 x 256 for fair comparison. These algorithms run for 50 iterations except for CG due to its higher complexity.

3.1 Performance Evaluation

For having appropriate method of MR images reconstruction, image quality and reconstruction speed are critical constraints. SNR and CPU Time are the respective parameters for above mention considerations.

Arteries

For MR images of arteries, RecPF , WaTM and FCSM give acceptable visual results can be seen in fig 1. WaTM give good result than FCSM in SNR, fig 2 (a) but takes more CPU time than FCSM as can be seen in fig 2(b). CG gives very poor SNR and takes lager time among all.

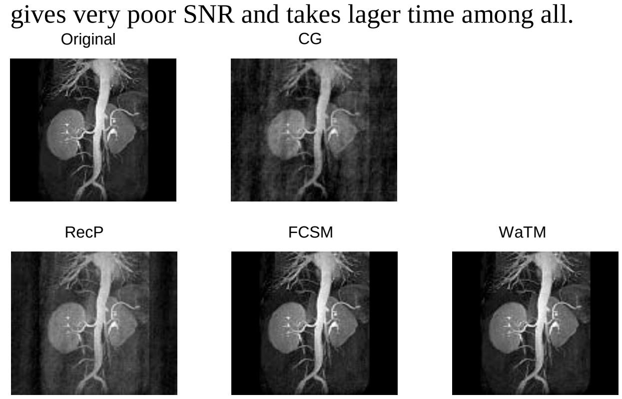

Fig 1: (a) original image of arteries, (b) reconstructed image through CG (c),(d),(e) reconstructed images through RecPF, FCSM and WaTM respectively

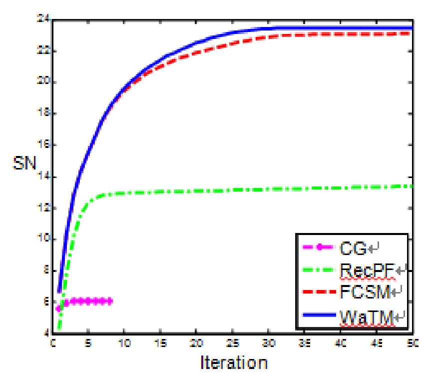

Fig2(a) arteries-Graph of SNR versus Iterations

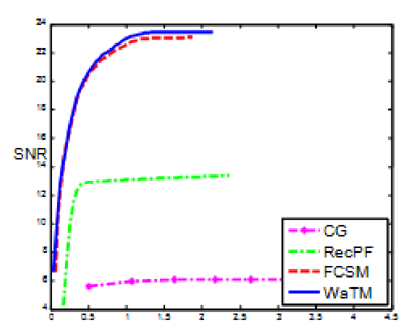

Fig 2(b) arteries- Graph of SNR versus CPU Time

Brain

In Fig 3, we can observe that RecPF, WaTM and FCSM give comparable close result to original image. Fig 4(b) and (a) shows WaTM, FCSM and RecPF are close in terms of CPU time and SNR respectively.CG is with poor SNR and takes lager time among all.

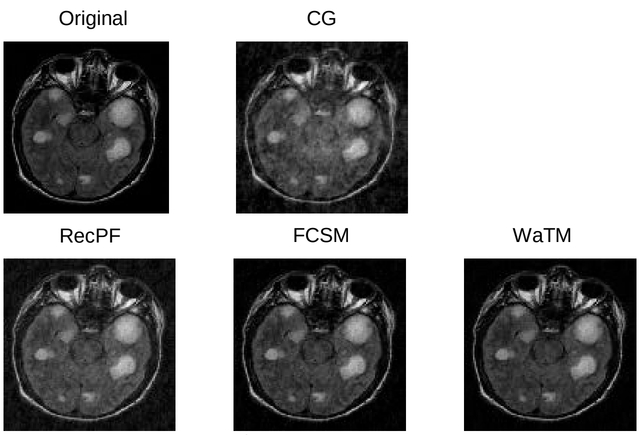

Fig 3:(a) original image of brain, (b) reconstructed image through CG (c),(d),(e) reconstructed images through RecPF, FCSM and WaTM respectively

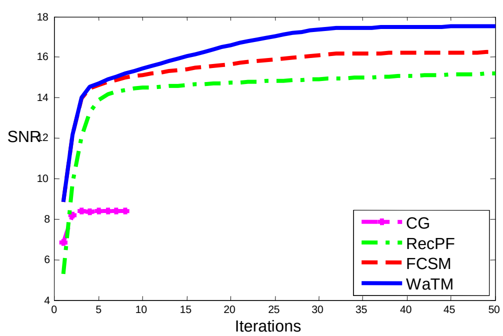

Fig4:brain(a)Graph of SNR versus Iterations

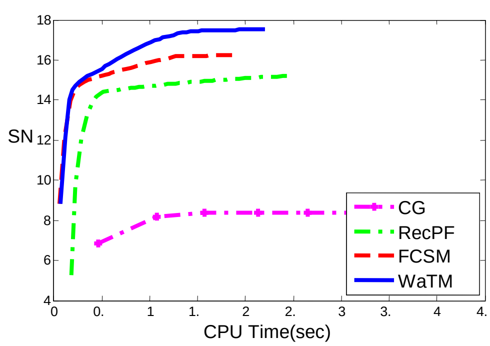

Fig 4: brain (b) Graph of SNR versus CPU Time

Spine

Fig 5 shows FCSM and WaTM are seem to visually close to original image. RecPF and CG are faded in this case and minute details seem to be vanishing. Fig 6(a),(b) shows FCSM gives high SNR and fastest computation time. WaTM also has good SNR and CPU Time. RecPF and CG gives bad result with respect to CPU time and SNR

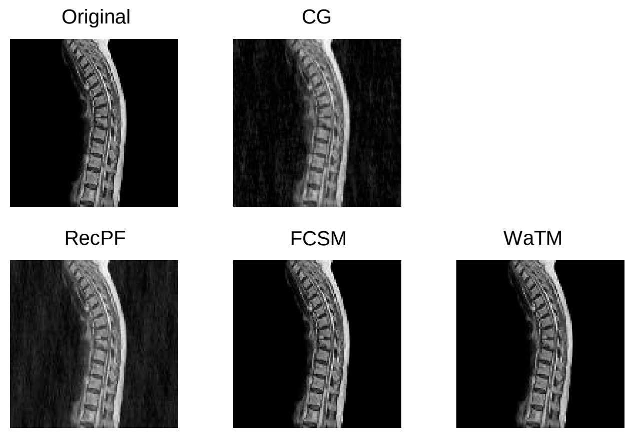

Fig 5: (a) original image of spine, (b) reconstructed image through CG (c),(d),(e) reconstructed images through RecPF, FCSM and WaTM respectively

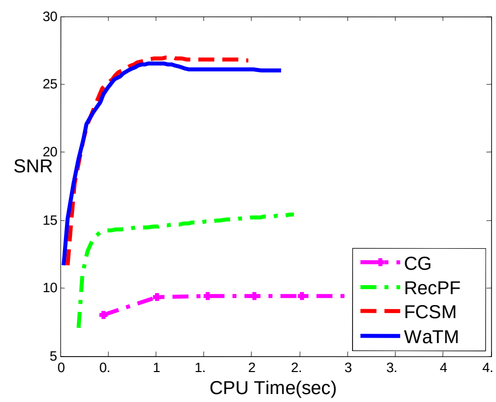

Fig6:spine(a)Graph of SNR versus Iterations

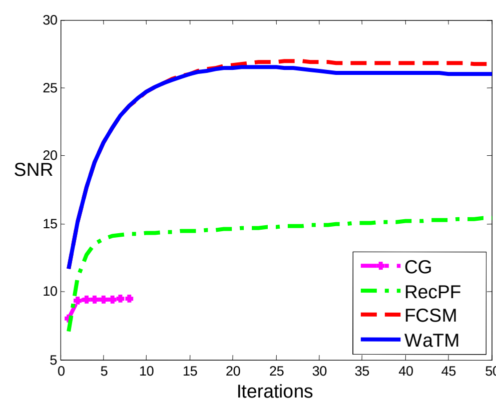

Fig 6: spine (b) Graph of SNR versus CPU Time

Quantitative results of SNR and CPU Time are summarized in below table 1 and 2.

Table 1: SNR

S.No	Body Parts	CG	RecPF	FCSM	WaTM
1	Brain	8.399	15.196	16.246	17.533
2	Renal arteries	6.074	13.379	23.075	23.452
3	Spine	9.473	15.424	26.775	26.020

Table 2:CPU Time

S.No	Body Parts	CG (sec)	RecPF (sec)	FCSM (sec)	WaTM (sec)
1	Brain	4.37	2.40	1.95	2.26
2	Renal arteries	4.56	2.43	1.73	2.00
3	Spine	4.46	2.53	1.81	2.06

4. Conclusion

Through experimental results we have seen compressive sensing is basically a case sensitive. FCSM reconstruct all images in less time among all models. But in signal to noise ratio WaTM gives the best result. WaTM has reconstructed time slightly high as it introduces tree structure. As WaTM is using tree structure, with less sampling measurements O(K +log n), we are able to reconstruct image with increased signal to noise ratio. Whereas in other models, after sparse transforming such as wavlet, O(K + K log n) measuring samples are needed to reconstruct the robust image. In many cases FCSM give reconstruction comparable to WaTM and difference was no so high. CG gives poor results due to its higher complexity, increases cost in each iteration. It is impractical to use it in MRI process. RecPF(variable splitting method) is good in terms of computation time, as its computations are in fourier domain. But its SNR is less in comparison to WaTM and FCSM. So WaTM and FCSM prove to be best in terms of SNR and Time.