# Efficient Window Approach of FIR Filter Design (MSK$^2$)

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**Summary**
There are several windows used to truncate the impulse response in order to fix filter size. Kaiser and Tukey windows are the most important types; from which we can generate other types of windows depending on the variation of ripple parameter. The proposed new window approach is called MSK$^2$ that was implemented to compare its parameters with these two main windows. The characteristics of MSK$^2$ are tested for the 100 window size. The proposed new window is generated via mixing Kaiser and Tukey windows. After implementing the filter with the proposed new window it is obvious that the performance is good and stable.

***Key words:***
*Adaptive Filter Design, Kaiser Window, Tukey Window, FIR and MSK$^2$.*

## 1. Introduction

Window is a mathematical function that is zero-valued outside of some chosen interval. A more general definition of window functions does not require them to be identically zero outside an interval, as long as the product of the window multiplied by its argument is square integrable, that is, that the function goes sufficiently rapidly toward zero. In typical applications, the window functions used are non-negative smooth curves, though rectangle and triangle functions and other functions are sometimes used [1, 2]

Windowing of a simple waveform, like cos(ωt) causes its Fourier transform to have non-zero values (commonly called spectral leakage) at frequencies other than ω. It tends to be worst (highest) near ω and least at frequencies farthest from ω. If there are two sinusoids, with different frequencies, leakage can interfere with the ability to distinguish them spectrally. If their frequencies are dissimilar, then the leakage interferes when one sinusoid is much smaller in amplitude than the other. That is, its spectral component can be hidden by the leakage from the larger component. But when the frequencies are near each other, the leakage can be sufficient to interfere even when the sinusoids are equal strength; that is, they become unresolvable [3,4].

Windows are used in the design of digital filters to convert the impulse response of infinite duration, to a finite impulse response (FIR) filter design. Windows are very effective for digital filter design, and there are many types of windows; Modified Bartlett-Hann window, Bartlett window, Blackman window, Blackman-Harris window, Bohman window, Chebyshev window, Flat Top weighted window, Gaussian window, Hamming window, Hann (Hanning) window, Nuttall-defined minimum 4-term Blackman-Harris window, Parzen (de la Valle-Poussin) window, Rectangular window, Taylor window, Triangular window, Tukey window, in addition you can generate other windows depending on the coefficients generation [5,6].

## 2. Literature Review

Many works are published related this field; some of them are listed below:

Yuan-Pei Lin and P. P. Vaidyanathan [7] proposed limiting the search of the prototype filters to the class of filters obtained using Kaiser windows. The design process is reduced to the optimization of a single parameter. An example will be given to show that very good designs can be obtained in spite of the limit of search. The design of the prototype filter is formulated as a problem of optimizing the cutoff frequency in Kaiser window design.

Paraskevas Kaliva, et al. [8] proposed a scheme based on the structure of the carry save array multiplier where each cell implements the computation of an FIR filter at the bit level. This structure leads to latency independent of the number of the filter taps. The proposed scheme is pipelined at the bit-level, is systolic at the cell-level and requires less hardware than other schemes based on discrete multipliers.

Hakan Johansson [9] introduced two classes of frequency-response masking linear phase finite impulse response filters for interpolation and decimation by arbitrary integer factors. The proposed filters are low complexity sharp transition linear phase FIR interpolation and decimation filters. The new ones offer lower complexity and more freedom in selecting the locations of the passband and stopband edges.

Hai Huyen Dam et al. [10] investigated the design of VDFs with discrete coefficients as a means of achieving low complexity and efficient hardware implementation. The filter coefficients are expressed as the sum of signed power-of-two terms with a restriction on the total number

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of power-of-two for the filter coefficients. An efficient design procedure is proposed that includes an improved method for handling the quantization of the VDF coefficients for both the min–max and the least-square criteria leading to an optimum quantized solution.

Hime A. Oliveira Jr. et al. [11] presented an adopted numerical optimization algorithm based on the well-known simulated annealing paradigm and its implementation. The design minimized globally the particular cost function at optimal filter. This method is suitable to application in other problems of digital filter design, where the matter under study can be stated as finding the global minimum of a numerical function of filter parameters.

Anton Blad and Oscar Gustafsson [12] implemented FIR filter with partial product generation for the multiplications, and carry save adders to merge the partial products. This approach focused on the efficient pipelined reduction of the partial products, which is done using a bit level optimization algorithm for the tree design. However, the method is not limited only to filter design, but may also be used in other applications where high speed reduction of partial products is required.

Tanveet Kaur [13] designed of FIR filter using Kaiser window. By using Kaiser window attempt is made to reduce the sidelobe level & control by modifying nth order Bessel function I0(x) & controlling the shape factor by adjustable parameter α. Few dB sidelobe level responses are also reduced. It can achieve direct control over the stopband attenuation thereby sustaining optimality and flexibility.

## 3. Filter Design

Filters are divided into two types; IIR and FIR. The main aspect of FIR filter design is to generate an adequate number of filter coefficients that reach an optimal design. The main aspect of filter design is concentrated on the design of analog Low Pass Filter (LPF), and then the next step is to generate the digital LPF depending on the analog design. On the other hand, you can follow any procedure to generate the other types of digital filters such as High Pass Filter (HPF), Band Pass Filter (BPF) and Band Stop Filter (BSF).

FIR filter design may be implemented via the many steps: Filter specifications, Filter type selection, Windows method design, Impulse response calculation, Filter structure, Filter analysis, Filter implementation.

The design of a FIR filter starts with its specifications in either time domain or frequency domain, or both domains. In time domain, the design aims to generate a specified impulse response. In the frequency domain, the requirement is concentrated on magnitude response; passband ripple and stopband ripple as shown in Figure (1).

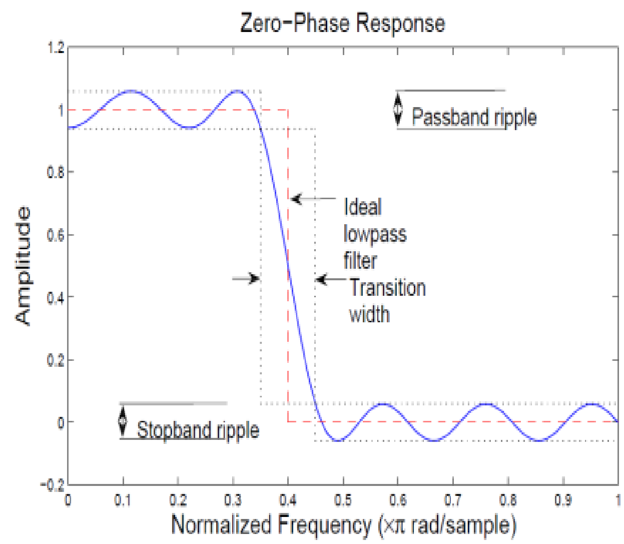


Fig. 1 Typical normalized low pass filter.

The impulse response of this ideal low pass filter is a sinc function.

$$\begin{aligned} h(x) &= 2ft\ sinc\ (2\pi ftx) \\ &= 2ft\ \frac{\sin(2\pi ftx)}{2\pi ftx} \\ &= \frac{\sin(2\pi ftx)}{\pi x} \end{aligned} \qquad (1)$$

## 4. Kaiser Window

Kaiser window is an important window for its effective characteristics. In addition, there are two important factors that can be used in as a main part of the filter design; these factors are ripple parameter δ, and the transition width. This window is defined in the interval –N ≤ n ≤ N, and otherwise it is zero. The Kaiser Window algorithm steps are mentioned below:

Step 1: calculate the attenuation value via the adjustment of the ripple (δ).

$$\propto = -20\ \log_{10}\delta \qquad (2)$$

Step 2: calculate the value of $\beta$ according to the attenuation value.

$$\beta = \begin{bmatrix} 0.1102(\propto -8.7) & \propto > 50 \\ 0.5842(\propto -21)^{0.4} + 0.07886(\propto -21) & 50 \geq \propto \geq 21 \\ 0 & \propto < 21 \end{bmatrix} \qquad (3)$$

Step 3: calculate the transition width TW according to the number of coefficients, which is the difference between stop band ws and pass band wp.

$$TW = w_s - w_p \tag{4}$$

Step 4: calculate the cutoff frequency

$$w_c = (w_s + w_p)/2 \tag{5}$$

Step 5: calculate the window size (N).

$$N \geq \frac{(\propto -7.95)}{28.72 * TW} \tag{6}$$

Step 6: calculate the zero order Bessel function.

$$I_0(x) = \sum_{n=0}^{\infty} \left[\frac{\left(\frac{x}{2}\right)^n}{n!}\right]^2 \tag{7}$$

Step 7: calculate Kaiser window depending on the above information.

$$w(n) = \frac{I_0\left(\pi\alpha\sqrt{1-\left(\frac{2n}{N-1}-1\right)^2}\right)}{I_0(\pi\alpha)} \qquad -N \leq n \leq N \tag{8}$$

## 5. Tukey Window

Tukey window is another important window for its effective characteristics. Tukey window is a rectangular window with the first and last r/2 percent of the samples equal to parts of a cosine. We can use all functions of Kaiser window accept the window generation that can be implemented the following equation:

$$w(n) = \begin{cases} \frac{1}{2}\left\{1+\cos\left(\frac{2\pi}{r}[n-r/2]\right)\right\} & 0 \leq n < r/2 \\ 1 & r/2 < n < 1-r/2 \\ \frac{1}{2}\left\{1+\cos\left(\frac{2\pi}{r}[n-1+r/2]\right)\right\} & 1-r/2 \leq n \leq 1 \end{cases} \tag{9}$$

The parameter r is the ratio of cosine-tapered section length to the entire window length with $0 \leq r \leq 1$.

## 6. Window Measurements

Many factors can be measured to improve the efficiency of the window filter; we will explain some of the important factors.

Coherent Gain (CG): Once a window is applied to the signal, however, the magnitudes of the signal computed with the Fourier transform will decrease. The coherent gain of the window is a measure of this decrease. The Coherent Gain (CG) of a window w(k) of length N is given by:

$$CG = \frac{1}{N}\sum_{n=0}^{N-1} w(n) \tag{10}$$

Equivalent Noise Bandwidth (ENBW): ENBW measures the noise power that is accumulated in the magnitude response of the filter. ENBW increases as the cutoff frequency of the low pass filter increases. Consider a window w(n) defined in discrete time and applied to a finite impulse response filter. If we treat this window as a filter, then its equivalent noise bandwidth is given by:

$$ENBW = \frac{N\sum_{n=0}^{N-1}|w(n)|^2}{\left(\sum_{n=0}^{N-1} w(n)\right)^2} \tag{11}$$

Processing Gain (PG): The processing gain of a filter or a window or a system is given by:

$$PG = \frac{1}{ENBW} \tag{12}$$

The processing gain can be expressed in decibels:

$$PG = 10\log_{10}\frac{1}{ENBW} \tag{13}$$

Leakage Factor (LF): the ratio of power in the sidelobes to the total window power.
Relative SideLobe Attenuation (RSLA): the difference in height from the mainlobe peak to the highest sidelobe peak.
MainLobe Width (MLW) (-3dB): the width of the mainlobe at 3 dB below the mainlobe peak.

## 7. The Proposed FIR Filter Algorithm

The proposed approach for FIR implementation depends on the window selection between Kaiser and Tukey according to the measured parameters.
The steps of the proposed filter approach are listed below while Fig.2 shows these steps.
Step 1: considering pass band ripple (δp) equal to stop band ripple (δs) and equal to the ripple (δ) to be selected.
Step 2: considering pass band frequency (wp), stop band frequency (ws), then these may be selected.
Step 3: calculate all filter parameters.
Step 4: calculate the Kaiser window elements.
Step 5: calculate the Tukey window elements.
Step 6: select the adequate window types according to the factors.
Step 7: implement the filter coefficients according.
Step 8: truncate the filter coefficients with the selected window.

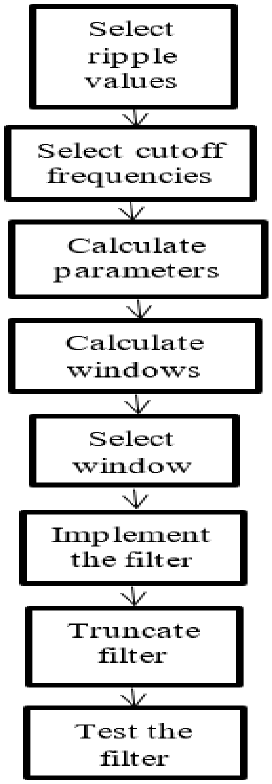


Fig. 2 The proposed FIR filter approach.

## 8. Results and Analysis

The proposed filter algorithm is implemented and tested for different values of attenuation for Kaiser and Tukey Windows. Fig.3 displays the time domain and frequency domain representations of the selected Tukey window for alpha values 0, 0.25, 0.5, 0.75 & 1. Fig.4 displays the time domain and frequency domain representations of the selected Kaiser window for same alpha values multiply by 5. Fig.5 shows the approximated time domain and frequency domain representations of the proposed new window that depends on the Tukey and Kaiser windows.

When selecting an appropriate window function for an application, these comparison graphs are useful to find the filter efficiency Fig.3, Fig.4 and Fig.5. These graphs show the main lobe of the window's frequency response in detail in addition only the envelope of the side lobes is shown to reduce clutter. The frequency axis has units of FFT when the window of length N (101) is applied to data and its transform is computed.

Table 1 shows the window measurements factors of Coherent Gain (CG), Equivalent Noise Bandwidth ( ENBW), Processing Gain (PG), Leakage Factor (LF), Relative SideLobe Attenuation (RSLA) and MainLobe Width (MLW).

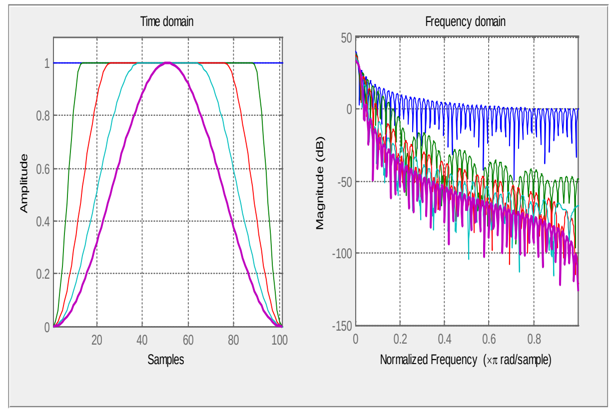


Fig. 3 Tukey window.

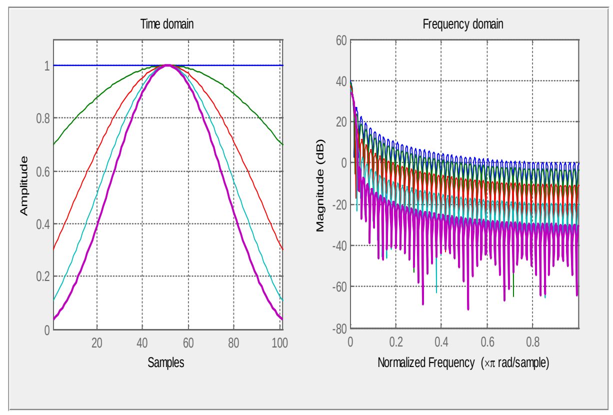


Fig. 4 Kaiser window.

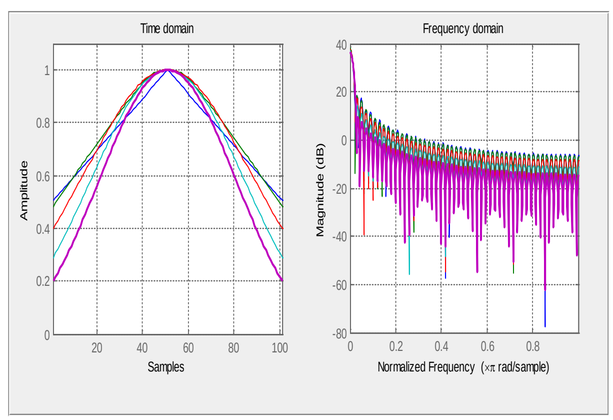


Fig. 5 Hybrid window.

Table 1: Window measurements

| Information | Value of α | CG | ENBW | PG (dB) | LF | RSLA(dB) | MLW |
|---|---|---|---|---|---|---|---|
| **Tukey window** | α1 =0.00 | 1.0000 | 1.0000 | 0.0000 | 9.26 % | -13.3 | 0.015625 |
| | α1 =0.25 | 0.8663 | 1.1131 | -0.4652 | 6.49 % | -13.6 | 0.019531 |
| | α1 =0.50 | 0.7426 | 1.2344 | -0.9147 | 3.61 % | -15.1 | 0.021484 |
| | α1 =0.75 | 0.6188 | 1.3736 | -1.3786 | 1.15 % | -19.4 | 0.025391 |
| | α1 =1.00 | 0.4950 | 1.5150 | -1.8041 | 0.05 % | -31.5 | 0.027344 |
| **Kaiser window** | α2 =5*0.00 | 1.0000 | 1.0000 | 0.0000 | 9.26 % | -13.3 | 0.015625 |
| | α2 =5*0.25 | 0.8939 | 1.0107 | -0.0463 | 5.40 % | -15.5 | 0.017578 |
| | α2 =5*0.50 | 0.7313 | 1.0912 | -0.3791 | 1.28 % | -21.2 | 0.019531 |
| | α2 =5*0.75 | 0.6163 | 1.2268 | -0.8877 | 0.18 % | -28.7 | 0.021484 |
| | α2 =5*1.00 | 0.5398 | 1.3708 | -1.3698 | 0.02 % | -37.0 | 0.025391 |
| **New** MSK$^2$ **window** | α1 & α2 | 0.7500 | 1.0370 | -0.1580 | 3.18 % | -19.0 | 0.017578 |
| | α1 & α2 | 0.7774 | 1.0444 | -0.1886 | 2.70 % | -19.3 | 0.019531 |
| | α1 & α2 | 0.7562 | 1.0654 | -0.2753 | 1.88 % | -20.3 | 0.019531 |
| | α1 & α2 | 0.7064 | 1.1062 | -0.4384 | 1.00 % | -22.9 | 0.019531 |
| | α1 & α2 | 0.6548 | 1.1604 | -0.6460 | 0.46 % | -27.0 | 0.021484 |

## 9. Conclusion

The proposed FIR window filter approach is designed and implemented to merge Tukey and Kaiser window parameters. This approach can be implemented via Tukey window and Kaiser window to generate filter parameters, then these parameters are extended to generate the new window parameters. A comparative study between Tukey window, Kaiser window and the new window parameters is presented in this study.

Coherent Gain, Equivalent Noise Bandwidth, Processing Gain, Leakage Factor, Relative SideLobe Attenuation and MainLobe Width are measured and compared for Tukey window, Kaiser window and the proposed new approach window.

The obtained results showed an improvement of the filter occurs as the attenuation increases. In addition, the obtained results indicate that the implemented filter has good performances and it is stable, and during the implementation of various cases it is possible to reach an optimal design used for final system.

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