

# Joint Synchronization and Channel Estimation with Optimum Training Sequences for OFDM-Based Systems Using EM Algorithm

Navid Daryasafar 1† and Omid Borazjani 2††,

Faculty of Electrical Engineering, Dashtestan Branch, Islamic Azad University, Borazjan, Iran.

## Summary

In this paper, we propose a joint carrier frequency synchronization and channel estimation algorithm for multiple-input and multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. A major challenge to MIMO-OFDM systems is how to obtain the channel state information accurately and promptly for coherent detection of information symbols and channel synchronization. MIMO-OFDM systems are very sensitive to synchronization errors, especially the carrier frequency offset (CFO). The carrier frequency synchronization and channel estimation are carried out repetitively through the Expectation–Maximization (EM) algorithm using an OFDM preamble symbol. It is revealed by the simulation results that the proposed scheme achieves ideal performance.

## Key words:

*MIMO-OFDM, Frequency Synchronization, CFO, Channel Estimation, EM Algorithm*

## 1. Introduction

The combination of orthogonal frequency division multiplexing (OFDM) with space-time coding has received much attention recently to combat multipath delay spread and increase system capacity [1]-[5]. OFDM is sensitive to timing and frequency errors [6]. To guarantee the fast and accurate data transmission, the Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI) caused in the transmission should be reduced as much as possible. In OFDM system, ISI can be minimized by inserting cyclic prefix with length greater than the channel impulse response, and the ICI can be reduced by maintaining the orthogonality of carriers under the condition that the transmitter and the receiver have the exact same carrier frequency. However, in the real world, frequency offsets will be arising from the frequency mismatch of the transmitter and the receiver oscillators and the existence of Doppler shift in the channel. Channel parameters are needed in order to coherently decode the transmitted signal. Roger Pierre Fabris Hoefel [7] explored the execution of Least Square (LS), Time Delay Truncation (TDT) and Model- Based

(MB) channel estimation conspires uncommonly intended to work in an OFDM numerous info various yield (MIMO) IEEE

802.11n framework. Recently MIMO-OFDM systems have gained considerable attentions from the leading industry companies and the active academic community [8], [9].

A collection of problems including channel measurements and modeling, channel estimation, synchronization, IQ (in phase-quadrature) imbalance and PAPR (peak-to-average power ratio) have been widely studied by researchers [10]-[12]. Clearly all the performance improvement and capacity increase are based on accurate channel state information. Channel estimation plays a significant role for MIMO-OFDM systems. The maturing of MIMO-OFDM technology will lead it to a much wider variety of applications. WMAN (wireless metropolitan area network) has adopted this technology. Similar to current network-based wireless location technique [13], we consider the wireless location problem on the WiMax network, which is based on MIMO-OFDM technology.

A joint carrier frequency synchronization and channel estimation scheme is proposed here for OFDM systems that is based on the EM algorithm. Using an OFDM preamble symbol, Expectation and maximization steps in the proposed scheme, provide both channel and carrier frequency offset estimate iteratively compared to other equalization methods, the stimulation results show that the proposed algorithm achieves satisfying performance.

## 2. System Model and Channel Estimation

### 2.1 OFDM System

The structure of the OFDM system is shown in Fig.1.

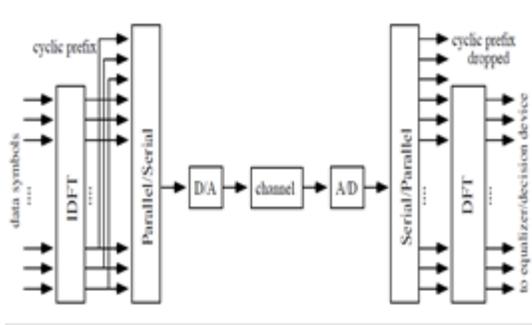


Fig. 1 The structure of the OFDM system

## 2.2 Synchronization Methods

### Before FFT algorithms

The above-mentioned algorithms are divided to two groups of input based algorithms and non-input based algorithms as follows:

Non input based algorithms: this group of algorithms estimates the synchronization parameters using the special structure of OFDM symbols. This group is also called cyclic prefix based methods [14]. Input based algorithms: this group of algorithms uses the educational symbols sent in information frames to estimate synchronization parameters [15]-[18].

### After FFT algorithms

The algorithms of this group are also categorized in two groups of pilot based algorithms and direct decision algorithms. In comparing the two algorithms, before FFT algorithms are faster than after FFT algorithms, but after FFT algorithms have a higher throughput spectral.

## 3. Maximum Likelihood Estimation (MLE) Algorithm

One of the principal disadvantages of OFDM is sensitivity to frequency offset in the channel. For example, the coded OFDM system developed by CCETT (Centre Common. d'Etudes de Telediffusion et Telecommunications) for digital sound broadcasting to mobile receivers incorporates an AFC (automatic frequency control) loop in the receiver to reduce frequency offset caused by tuning oscillator inaccuracies and Doppler shift.

There are two deleterious effects caused by frequency offset; one is the reduction of signal amplitude in the output of the filters matched to each of the carriers and the second is introduction of ICI from the other carriers

which are now no longer orthogonal to the filter. Because, in OFDM, the carriers are inherently closely spaced in frequency compared to the channel bandwidth, the tolerable frequency offset becomes a very small fraction of the channel bandwidth. Maintaining sufficient open loop frequency accuracy can become difficult in links, such as satellite links with multiple frequency translations or, as mentioned previously, in mobile digital radio links that may also introduce significant Doppler shift.

The algorithm generates extremely accurate estimates even when the offset is great to demodulate the data values. The estimation error is insensitive to channel spreading and frequency selective fading.

The transmitter sends  $X(K)$  data for  $K=0, \dots, N-1$ :

$$S(n) = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X(k) e^{j \frac{2\pi}{N} kn}; n = 0, \dots, N-1 \tag{1}$$

Adding the cyclic prefix (cp) and considering carrier frequency offset estimation, the  $r(n)$  vector is formed as following:

$$r^{cp}(n) = e^{j2\pi\Delta f n} \times s^{cp}(n) h(n) + Z(n); n = \dots, N_s - \tag{2}$$

Finally, we remove cp:

$$r(n) = e^{j2\pi\Delta f n} s^{cp}(n) * h(n) + Z(n); n = N_{cp}, \dots, N_s - \tag{3}$$

To estimate the carrier frequency offset of  $r(n)$  vector,  $r_1$  and  $r_2$  vectors are formed as follow:

$$r_1(n) = e^{j2\pi\Delta f N_{cp}} e^{j2\pi\Delta f n} (s(n) * h(n)); n = 0, \dots, \frac{N}{2} - 1 \tag{4}$$

And

$$r_2(n') = e^{j2\pi\Delta f N_{cp}} e^{j2\pi\Delta f n'} (s(n') * h(n')); n' = \frac{N}{2}, \dots, N-1 \tag{5}$$

$$r_2^* \cdot r_1 = e^{j2\pi\Delta f (\frac{N}{2})} |d(n)|^2 \tag{6}$$

Assuming  $d(n)=s(n)*h(n)$  and performing some calculations we have:

$$\Gamma = \frac{r_2^* \cdot r_1}{|r_2^* \cdot r_1|} = e^{j2\pi\Delta f (\frac{N}{2})} = e^{-j2\pi\Delta f N} = \cos \pi N \Delta f - j \sin \pi N \Delta f \tag{7}$$

Finally the carrier frequency offset estimation is performed as follow:

$$-\tan(\pi N \Delta f T) = \frac{\text{Im}(\Gamma)}{\text{Re}(\Gamma)} \Rightarrow \hat{\Delta f} = \frac{1}{\pi} \tan^{-1} \left( -\frac{\text{Im}(\Gamma)}{\text{Re}(\Gamma)} \right) \tag{8}$$

2.3 Signal Model with Frequency Offset

Signal model is as follows:

$$x^{cp}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi}{N} k(n-Ncp)} \tag{9}$$

With eliminating cp and doing a series of operations, we have:

$$r(n) = e^{j 2\pi \Delta f (n+Ncp)} \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) H(k) e^{j \frac{2\pi}{N} kn} + Z(n) \tag{10}$$

Where Z (n) is White Gaussian Noise with an average of 0. The output of the receiver is as follows:

$$y(k) = e^{j \frac{2\pi}{N} k n} \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r(n) e^{-j \frac{2\pi}{N} k n} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \sum_{i=0}^{N-1} X(i) H(i) e^{j \frac{2\pi}{N} i n} e^{-j \frac{2\pi}{N} k n} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} Z(n) e^{-j \frac{2\pi}{N} k n} \tag{11}$$

$$y(k) = x(k) H(k) + Z(k) \quad k = 0, \dots, N-1 \tag{12}$$

2.3 EM Channel Estimation

By studying the equation (11) again, we can observe that this relation has two unknown parameters of w and h(i). Since v is supposed the white Gaussian noise with a zero average, estimating the maximum similarities of w and hi unknown parameters, equals estimating their minimum squares. Estimating the maximum similarities of w and hi can be obtained from the following cost function [19]:

$$\min_{w, h_i} \left\{ \sum_{i=1}^M |r_i - \Gamma(w) A h_i|^2 \right\} \tag{13}$$

$$\Gamma(w) = \text{diag}(1, e^{jw}, \dots, e^{jw(k-1)})$$

$$A(n) = [e^{jwN_s}, A_1(n), \dots, e^{jwN_s} \cdot A_N(n)]_{k \times N \cdot L}$$

Step 1 (estimating the carrier frequency offset):

the question of estimating maximum similarity of carrier frequency offset can be presented as follows:

$$\hat{\omega} = \arg_w \max \sum_{i=1}^M \log \int p(r_i | w, h_i) p(h_i) dh_i \tag{14}$$

The EM algorithm estimates carrier frequency offset in two following steps:

Step E: Expectation:

$$Q(w | w^{(k)}) = E \left\{ \left[ \sum_{i=1}^M \log p(r_i | w, h_i) \right] \middle| r_i, w^{(k)} \right\} \tag{15}$$

Step M: Maximization:

$$w^{(k+1)} = \arg_w \max Q(w | w^{(k)}) \tag{16}$$

In above relations, w(k) show the estimated carrier frequency offset in the k<sup>th</sup> repetition of EM.

At the end“ Equation 16” after a series of calculations becomes like the following:

$$w^{(k+1)} = \arg_w \max \left( \sum_{i=1}^M \sum_{p=0}^{k-1} \text{Re} \left[ r_i^* (\rho) V_i^{(k)} (\rho) e^{jwp} \right] \right) \tag{17}$$

With the assumption of  $S_i^{(k)}(p) = r_i^* (p) v_i^{(k)}(p)$ , relation number 17 is rewritten like the following:

$$w^{(k+1)} = \arg_w \max \text{Re} \left( \sum_{i=1}^M R_i^{(k)} \right) \tag{18}$$

The above relation can be calculated using FFT. Thus the proposed method has a suitable calculation speed.

Step 2 (estimating the coefficients of fading channel): after the estimation of rate of carrier frequency offset, it’s time to estimate the maximum similarities of coefficients of fading channel hi (i =1,...,M ). Thus the estimation of maximum similarities of channel’s coefficients can be obtained from the following relation:

$$\hat{h}_i = (A^H A)^{-1} A^H \Gamma^H(\hat{\omega}) r_i, i = 1, \dots, M \tag{19}$$

4. Performance Evaluation

In this chapter, a MIMO-OFDM system with 2 transmitter antennas and 2 or 1 receiver ones is used for the simulation. The assumed system has a QPSK or BPSK modulation. The total number of subcarriers, N, is 64 or 32 and L is the tap of channel.

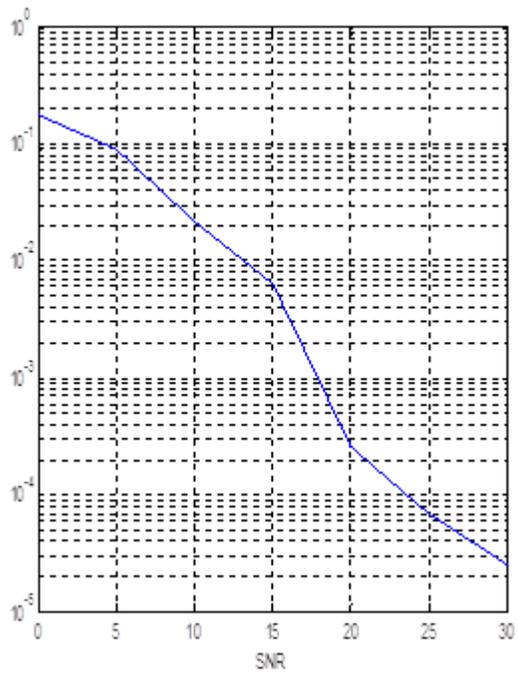


Fig.2 CFO estimation for SISO-OFDM systems with MLE algorithm, L=5

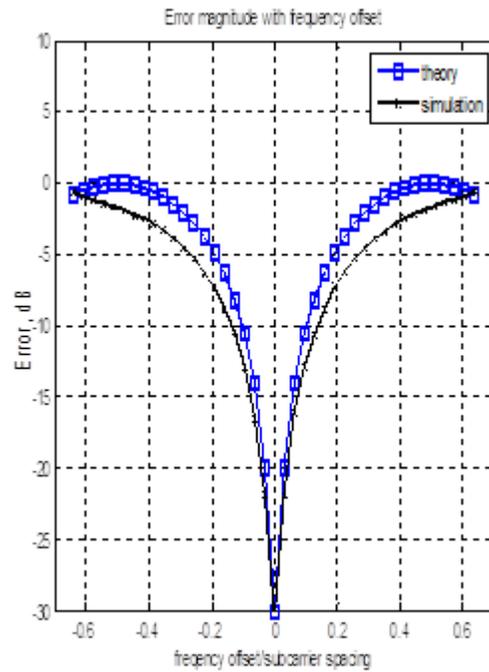


Fig.4 Error Magnitude vs frequency offset for OFDM

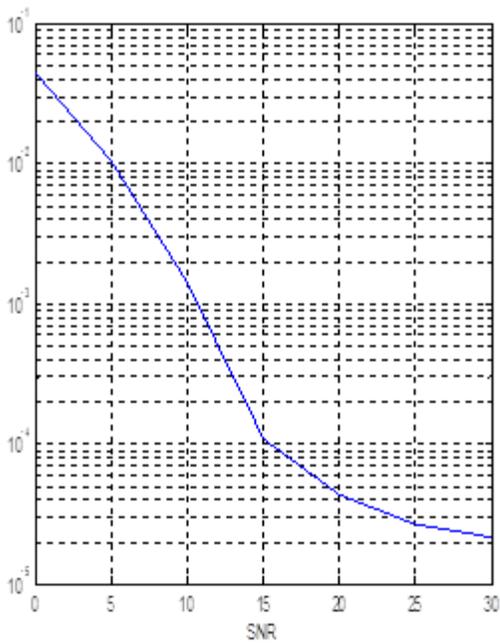


Fig.3 CFO estimation for 2\*2 MIMO-OFDM systems with MLE algorithm, L=4

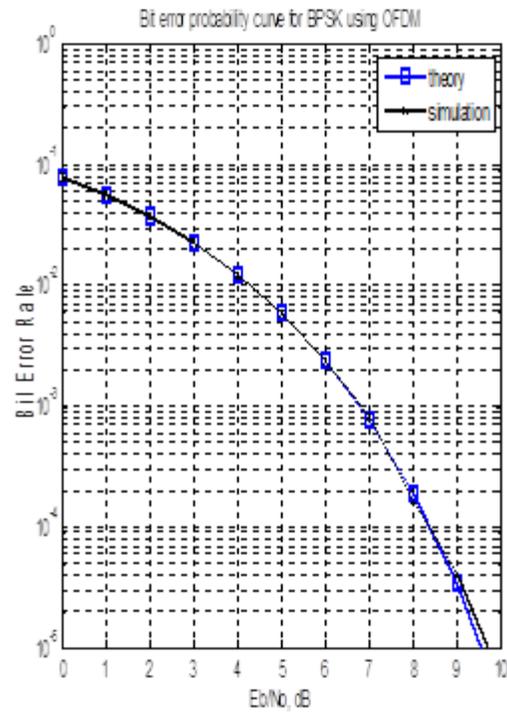


Fig.5 Bit Error Rate plot for BPSK using OFDM modulation with synchronization

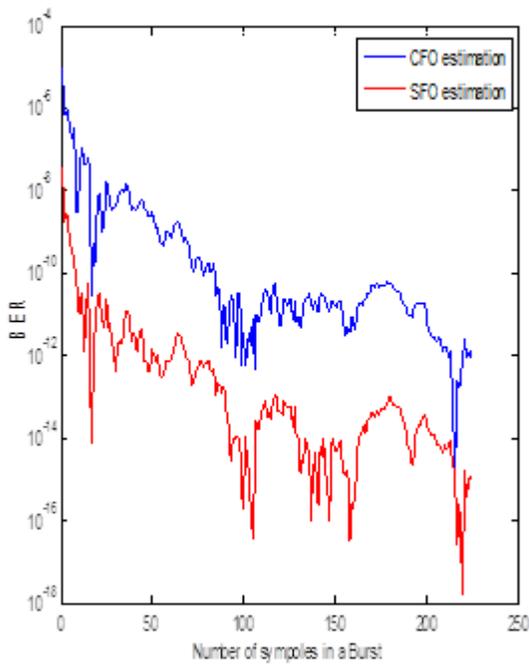


Fig.6 SISO-OFDM, joint channel estimation and synchronization

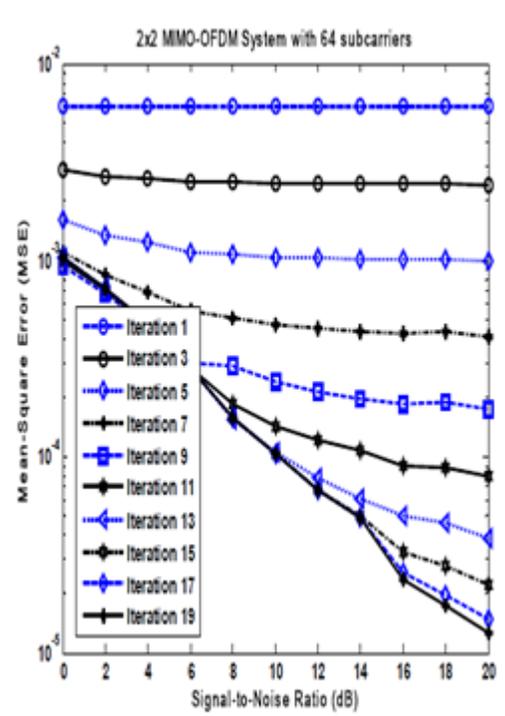


Fig.8 Mean square error according to SNR for MIMO-OFDM system with 64 sub carriers, which uses 2 transmitter antennas and 2 receiver antennas with EM algorithm

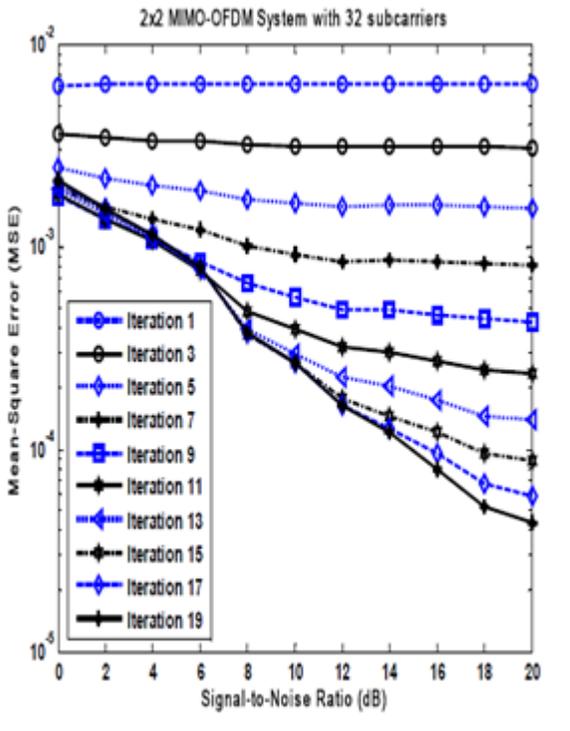


Fig.7 Mean square error according to SNR for MIMO-OFDM system with 32 sub carriers, which uses 2 transmitter antennas and 2 receiver antennas with EM algorithm

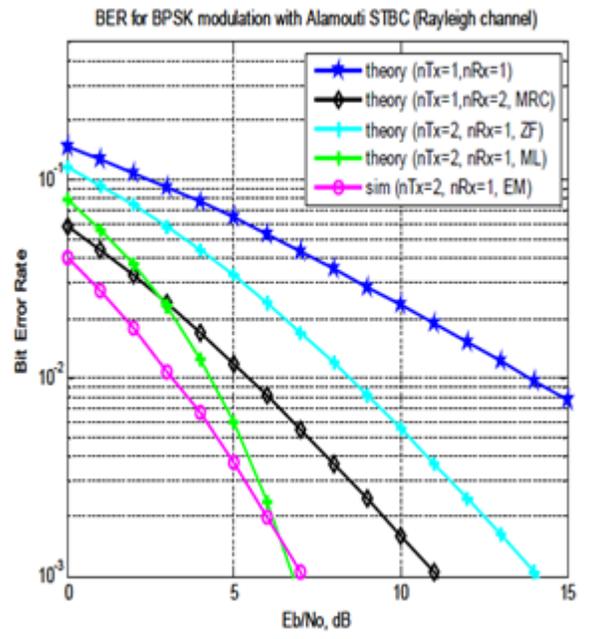


Fig.9 BER for BPSK modulation

## 5. Conclusion

MIMO-OFDM is an attractive technique for high data rate transmission and channel estimation. In this paper we proposed a frequency synchronization technique for OFDM Systems. The synchronization algorithm shows satisfactory performance even at a low SNR and in a selective frequency channel.

## REFERENCES

- [1] D. Agrawal, V. Tarokh, A. Naguib, and N. Seshadri, "Space-time coded OFDM for high data-rate wireless communication over wideband channels," in Proc. IEEE VTC'98, pp. 2232-2246, May 1998.
- [2] H. Bolcskei and A. Paulraj, "Space-frequency coded broadband OFDM systems," in Proc. WCNC'02, vol. 1, pp. 1-6, May 2000.
- [3] G. Yi and K. Letaief, "Space-frequency-time coded OFDM for broadband wireless communications," in Proc. GLOBECOM'01, vol. 1, pp. 519-523, 2001.
- [4] Du Yuelin, Zhang Jingxian. The performance of synchronization algorithm in real-time OFDM-PON system. TELKOMNIKA Indonesian Journal of Electrical Engineering; 10(7): 1784-1794, 2012.
- [5] Zhu Yonghong, Feng Qing, Wang Jianhong. Neural network-based adaptive passive output feedback control for MIMO uncertain system. TELKOMNIKA Indonesian Journal of Electrical Engineering; 10(6): 1263-1272, 2012.
- [6] Fan Wu and Mosa Ali Abu-Rgheff, "Time and Frequency Synchronization Techniques for OFDM Systems operating in Gaussian and Fading Channels: A Tutorial", The 8th Annual Postgraduate Symposium on The Convergence of Telecommunications, Networking and Broadcasting (PGNET), Liverpool John Moores University, 28th-29th June.2007.
- [7] Roger Pierre Fabris Hoefel "IEEE 802.11n: On Performance of Channel Estimation Schemes over OFDM MIMO Spatially- Correlated Frequency Selective Fading TGN Channels" 2012.
- [8] Allert van Zelst and Tim C. W. Schenk, "Implementation of a MIMO OFDM- based Wireless LAN system," IEEE Trans. Signal Processing, vol. 52, No. 2, pp. 483-494, Feb. 2004.
- [9] H. Sampath, S. Talwar, J. Tellado, V. Erceg and A. Paulraj, "A fourth-generation MIMO-OFDM broadband wireless system: design, performance and field trial results," IEEE Communications Magazine, No. 9, pp. 143-149, Sep., 2002.
- [10] A. Tarighat, R. Bagheri, and A. H. Sayed, "Compensation schemes and performance analysis of IQ imbalances in OFDM receivers," IEEE Transactions on Signal Processing, vol. 53, no. 8, pp. 3257-3268, Aug. 2005.
- [11] A. Tarighat and A. H. Sayed, "MIMO OFDM receivers for systems with IQ imbalances," IEEE Transactions on Signal Processing, vol. 53, no. 9, pp. 3583- 3596, Sep. 2005.
- [12] Y. Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," IEEE Trans. Wireless Communications, vol. 1, No. 1, pp. 67-75, Jan. 2002.
- [13] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location," IEEE Signal Processing Magazine, vol. 22, no. 4, pp. 24-40, July 2005.
- [14] F. Daffara, and O. Adami, "A novel carrier recovery technique for orthogonal multicarrier systems," Eur. Trans. Telecommun., vol. 8, no. 4, pp. 323-334, July 1996.
- [15] ETSI, "Radio broadcast systems; digital audio broadcasting (DAB) to mobile, portable and fixed receivers: Final draft pr ETS 300 401," European Telecommunications Standards Institute, Tech. Rep., Nov. 1994.
- [16] T. Keller, and L. Hanzo, "Orthogonal frequency division multiplex synchronization techniques for wireless local area networks," in Proc. IEEE Int. Symp. Personal, Indoor, and Mobile Radio Communications, pp. 963-967, 1996.
- [17] U. Lambrette, M. Speth, and H. Meyr, "OFDM burst frequency synchronization based on single carrier training data," IEEE Commun. Lett., vol. 1, pp. 46-48, Mar. 1997.
- [18] T. Schmidl, and D. Cox, "Robust frequency and timing synchronization for OFDM," IEEE Trans. Commun., vol. 45, pp. 1613-1621, Dec. 1997.
- [19] M. O. Pun, S. H. Tsai, and C. C. Jay Kuo, "An EM-Based Joint Maximum Likelihood Estimation of Carrier Frequency Offset and Channel for Uplink OFDMA Systems", IEEE Transactions on communications, Vol. 2, no. 3, pp. 76-84, 2004.