Fuzzy Reasoning in Description Logic

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Summary

Fuzzy description logics are intended to represent and reason not only crisp notions but also fuzzy ones. They have a key role in various domains, and in particular in the semantic web domain. They are now a promising research orientation, on which we positioned our works, assuming that the reasoning over vague concept is complex, and the integration of new expansion rules is essential. This is the subject of this paper. The approach which we propose for this purpose is to extend Description Logics by concrete domains which allows us to describe the precise parts of the concepts, the thing that allows us at the reasoning phase by applying expansion rules proposed to extract the vague part of the concept by assigning a degree of certainty, and also execute an inference according to these degrees.

Key words:

Semantic Web, Ontologies, Description Logics, Fuzzy logic, Reasoning, Concrete domain.

1. Introduction

The consideration of semantics is also essential in the research for information and the evaluation of Web queries. Many works from the Semantic Web community were realized to describe semantic of applications by building ontologies. Indeed, Semantic Web is a Web which concerns many Internet users and researchers hopes whether in the field of information research, e-business or Competitive Intelligence, etc.. It will have the mission of giving a meaning to the data and allowing the machines to analyze and to understand the circulating information.

To have a meaning, we have first to create the description of this information (meta-data), and then trying to link them together through inference and deducing rules to construct ontologies are so central to the Semantic Web. Which on the one hand, it seeks to rely on modeling of Web resources from conceptual representations of the domain concerned, and on the other hand, it aims to enable programs to make inferences above.

`Toleration of inconsistency can only be done by fuzzy systems. We need a semantic web, which will provide guarantees and about which one can reason with logic [1]. Such are the words of Tim Berners-Lee, founder and President of the World Wide Web Consortium, where he tried to show us that all these meta-data are created by humans, and so they should contain many uncertainties and imprecision, which will affect the construction of ontologies. Since fuzzy logic was conceived to find solutions to the problems of inaccuracies and uncertainties in a flexible way, researchers have had the idea to integrate this logic in the field of the Semantic Web in general, and to use it in the construction of ontologies by the Description logic in particular.

Description logics are a good model to describe the semantics of the data from the Web by restrictions, which are necessary to obtain reasoning algorithms that pass in the scale to detect inconsistencies or logical correlations between data or data sources, and to compute the set of answers to conjunctive queries, but they are very weak when we want to model a domain whose knowledge and information is vague and imprecise. For this reason, there were many proposals to extend description logics by mathematical theories that deal with uncertainty and imprecise information.

There are several works in the literature after the first essay of J.YEN in 1991[2] which handle the problem of the imperfection[3][4][5][6], where they built their works on the idea to give a membership (certainty degree) to the fuzzy concepts of the ontology, however little works trying to study the reasoning optimization techniques [7] [8][9], but neither of them considers reasoning without modified the TBox, the thing that has an impact on the representation structuring of the knowledge and hence we shall have two modifications during the ontology management, the first one on the representation, and the other one on the reasoning, modifying both of the representation and the reasoning will have a big effect on the ontology itself.

The objective of our work is to create the ontology in a very normal way based on the concrete domain and datatype, which will allow us to introduce numeric information to the concepts, by this way we can just represent the true part of concept, by other words we just take the precise party of the information, for example if we talk about age, we find the child age is less than 12 years and a young have age between 16 and 35 years, in our approach although we have a gap between child age and young age but just we represent the precise age of the both. This representation allows us to treat the vagueness in the reasoning phase by calculate the degree of certainty of the assertion in its vague concept part instead of affecting it to concepts in the phase of representation, and thus arriving

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at a more precise and more coherent result with the initial data. Expansion rules were proposed to reason with representation KB.

To illumine well our idea we thought to apply it to a real case, for this reason we have done study with cardiologist on diseases that can be diagnosed only with ECG (electro cardio gram) like cardiac dysrhythmias, so we have choose four parameters from the ECG (AmplitudeP, QRS width, Rhythm RR, Heart rate) that can drive us to three disease Slow fibrillation"SF", Continuous arrhythmia atrial fibrillation "CAAF", Continuous tachycardia atrial fibrillation "CTAF".

To reflect our objectives, the paper is organized as follows. Section 2 and 3 give basic idea about description logic and concrete domain. Section 4 presents preliminary fuzzy membership function. Section 5 details the method of fuzzy reasoning; and in section 6 we interpret the result obtained; and finally the paper ends with a discussion and conclusion.

2. Description Logic

Description logics (DLs)[10][11][12] are a family of knowledge representation languages which can be used to represent the terminological knowledge of an application domain in a formal and structured manner [11]. There exist several description logics which are different in their expressive power and naturally by the complexity of the algorithms of satisfiability associated. In the following, we will introduce the syntax and semantics of the description logic.

Definition 1: *(Syntax).Let NC and NR be two disjoint countable sets indicating concept names and role names. We use A,B for the atomic concepts, R for role names and C,D for the complex concepts. The symbols • and ⊥* indicate the universal and empty concepts respectively. The basic ALC DL language is defined as follows:

$$C;D \rightarrow A \mid \top \mid \bot \mid \neg C \mid (C \sqcap D) \mid (C \sqcup D) \mid \forall R.C \mid \exists R.C$$

Definition 2: (Semantic). *An interpretation $I = (\Delta^I, .^I)$ consists of a set Δ^I, the domain of I, and function $.^I$ that associates every concept C to a subset C^I of Δ^I and to every role R a subset R^I of $\Delta^I x \Delta^I$, such as for all C,D concepts and R roles, the following properties are satisfied:*

$$\top^I = \Delta^I \ / \ \bot^I = \varnothing$$
$$\neg C = \Delta^I - C^I$$
$$(C \sqcap D)^I = C^I \bigcap D^I$$
$$(C \sqcup D)^I = C^I \bigcup D^I$$
$$(\forall r.C)^I = \{x \in \Delta^I / \forall y : (x,y) \in r^I \rightarrow y \in C^I\}$$
$$(\exists r.C)^I = \{x \in \Delta^I / \exists y : (x,y) \in r^I \wedge y \in C^I\}$$

Definition 3: (Satisfiability, subsumption and equivalence of concepts)

- *A concept C is staisfiable if there exist en interpretation*
- *I such as $C^I \neq \emptyset$.*

- *A concept C subsumes a concept D, noted D ⊑ C, if $D^I \subseteq C^I$ for any interpretation I.*

- *Two concepts C and D are called equivalent, denoted D ≡ C if $D^I = C^I$ for any interpretation I.*

Definition 4: (Knowledge Base). *A knowledge base K associated with DL, also called ontology, contains two parts: a terminological part (TBox) and assertionnelle part (ABox). The TBox T describes the terminology by listing the concepts, roles and relationships. It is a finite set of terminals axioms D⊑C; (General Concepts Inclusion (GCI)), D≡C (C is equal to D, ie C⊑D and D⊑C). The ABox contains information about individuals. An Abox A is a finite set of assertions. (a: C (belonging to a concept), (a, b) R :(role).*

Definition 5: (Satisfiability assertion and bases model).

- *An assertion (a:C) is satisfiable if there is an interpretation I such that $a^I \in C^I$, an assertion (a,b):R is satisfiable if there is an interpretation I such* as *$(a^I;b^I)$> R^I.*

- *An interpretation I is a model of an axiom of a DL (TBox or ABox) if it satisfies the axiom, and is a model of the knowledge base K if it satisfies all axioms of K.*

- *A formula Ø (assertion in the form a:C; (a, b) : R or terminological axiom (A≡D) and (A⊑D) is logically implied by a base K, noted K Ø, if Ø is satisfied by any model of K.*

Table 1. Description Logic : Syntax and Semantic

Constructor	Syntax	Semantics	Example
Atomic concept	A	$A^I \sqsubseteq \Delta^I$	Human
Individual	A	$a^I \sqsubseteq \Delta^I$	Mohamed
Top	$\top$	Δ^I	Thing
Bottom	$\bot$	Φ	Nothing
Atomic role	R	$R^I \sqsubseteq \Delta^I x\ \Delta^I$	hasSpeed
Conjunction	$C \sqcap D$	$(C \cap D)^I$	Human ⊓ Male
Disjunction	$C \sqcup D$	$(CUD)^I$	Male ⊔ Female
Negation	$\neg C$	Δ^I / C^I	¬ Human
universal restriction	$\forall R.C$	$x \in \Delta_I / \forall y \in \Delta_I : (x,y) \in R^I => y \in C^I$	∀hasChild .human
existential restriction	$\exists R.C$	$x \in \Delta_I / \exists y \in \Delta_I : (x,y) \in R^I \wedge y \in C^I$	∃hasChild.Girl
Values restriction	R:{A}	$x \in \Delta_I / \exists y \in \Delta_I : (x,y) \in R^I => y = a^I$	has-child.{Mohamed}
Numbers restriction	$(\geq n .R)$	$x \in \Delta_I / \vert\{y \vert (x,y) \in R^I\}\vert \geq n$	(≥3 .hasChild
	$(\leq n .R)$	$x \in \Delta_I / \vert\{y \vert (x,y) \in R^I\}\vert \leq n$	(≤1 .hasFather)
Subsomption	$C \sqsubseteq D$	$C^I \subseteq D^I$	Man⊑ Human
Concept definition	$C \equiv D$	$C^I = D^I$	Father=Man⊓∃hasChild.Human
Concept assertion	$a : C$	$a^I \in C^I$	Mohamed :Man
Role assertion	$(a,b) : R$	$(a^I, b^I) \in R^I$	(Mustapha, Mohamed):hasChild

3. Concrete Domain

Concrete domains of description logics introduced in [13] [14] allow the introduction of numeric or textual information about concepts. More generally, they are used to represent concrete properties of real objects such as size, visual appearance or their spatial organization. For example by respecting the formalism of table1, the concept (Person A Age B 20) represent all persons whose age is less than or equal to 20. In this example B 20 is a predicate on the concrete domain of natural numbers N. Formally concrete domains are defined as follows:

Definition 6: (Concrete Domains). A concrete domain D is a pair (ΔD, ΦD) where ΔD is a non-empty set and ΦD is a non-empty set of predicate names defined on ΔD. Each predicate name P∈ΦD associated an arity n and an n-ary predicate PD⊆ ΔDn. A concrete field is said admissible if (i) all of its predicate names is closed by negation, ie for all P∈ΦD, there exists $\bar{P}$ ∈ΦD Interpreted as $\bar{P}$D= ΔD\PD, (ii) contains the predicate TD interpreted as ΔD, and (iii) the problem of satisfiability of a conjunction of predicates $\wedge_{i=i}^{n} P_i(u_i)$ is decidable (there exists an application δ / $\delta(u_i) \epsilon P_i^D$, for all $1 \leq i \leq n$).

Definition 7: Let NC, NR and NcF be non-empty and pair-wise disjoint sets of concept names, role names and concrete features, moreover NaF a countable subset of NcF. The elements of NaF are called abstract attributes. A chain u is a composition f1;..; fn g of n abstracts attributes f1;..; fn(n<0) and concrete attributes g. For a given concrete domain D, all ALC(D) concepts are smallest set such that:

1- every concept name is a concept,

2- if C and D are concept names, R role names, $u_1..u_n$ chains and $P \in \Phi^D$ a predicate name with an arity n, then the ALC(D) language is defined as flows :

$$C;D \to A \mid \top \mid \bot \mid \neg C \mid (C \sqcap D) \mid (C \sqcup D) \mid \forall R.C \mid \exists R.C \mid \exists u_1 \ldots\ldots\ldots u_n.P$$

The Abox A associated with this language becomes a finite set of assertions (a:C(Membership in a concept) (a, b): R(rôle), (a, x): f(attribute) and (x1;:::; xn):P (Membership in a predicate of concrete domain)).

4. Fuzzy Membership Function

Fuzzy Sub-Sets were introduced to model human knowledge representation, and thus to improve the performances of the decisional systems using modeling. In a set of reference E, a fuzzy subset A of this reference is characterized by a function of membership m of A, which associates with each element X of E, the degree μ(X), ranging between 0 and 1, for which X belongs to A. This function is the extension of the function characteristic of a classical subset [15] [16]. It can be represented in the form of triangular or trapezoidal or parabolic function [17]. In a preoccupation with clearness and in order to facilitate calculations, we will use the trapezoidal and triangular form.

The purpose of the fuzzy subset is to allow gradations in the membership of an element X to a class A, to authorize an element to more or less strongly belong to this class. A fuzzy subset A on the domain of variation E of x is defined by the triplet (A,a,μA), where:

- A: is a subset of S;
- a: a linguistic term, qualitatively characterizing part of the values of x;
- and μA, the function that gives the membership degree of an observation of x in fuzzy subset A. This μA function is called "membership function" of A. It associates each element x of E, the degree μA(x) in the range [0,1].

For example, if we take the family of cardiac arrhythmias; According to cardiologists the detection of these abnormalities is primarily determined by four parameters: heart rate, irregular rhythm, amplitude of the P wave and the QRS width.
Each input parameter is represented by linguistic values[21]. The predominance intervals of these values are defined by trapezoidal membership functions. Figure 1, Figure 2, Figure 3 shows these membership functions.

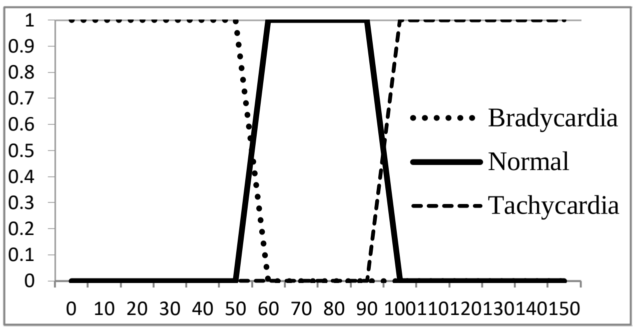


Figure 1: Heart Rate

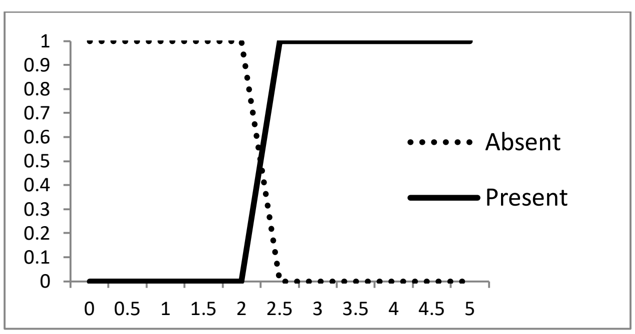


Figure 2: Amplitude of P

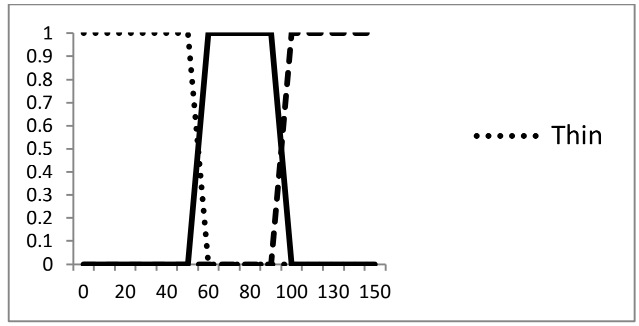


Figure 3: QRS width

In this example, if we take the linguistic variable “Heart rate”, we found that could be associated with one of the following terms {tachycardia, Normal, Bradycardia}.Note that the heart rate can be considered as belonging to several different terms such as: Normal, tachycardia, and if all values term permit it, we can group all the membership functions for the same function.
The linguistic term characterized the support of the membership function; it can be described by a set of pairs (x, μA(x)). For a heart rate described as "Normal"; the following pairs {(50|0),(«55|0.5),(60|1),(90|1),(100|0)} can describe the membership function shown in Figure 1[Heart rate].

5. Tableau Reasoning and Architecture

Descriptions Logics (DLs) are decidable fragments of first-order logic enable us to reason about axioms expressing logical constraints on unary and binary predicates. They cover a wide range of logic-based languages classes for which the reasoning problems are decidable with complexity that depends on all constructors and axioms allowed in the language.
A knowledge base in DL consists of an intentional part (the Tbox) which can be seen as ontology and assertion part (the Abox): the Tbox defines the conceptual data model of the Abox. Generally it defines the ontology as an explicit formal specification of a shared conceptualization of a given domain. An ontology thus will consist of at least a conceptual vocabulary, i.e. a set of linguistic variable, equipped sometimes by a set of terms (variable =heart rate, terms = {Tachycardia, Normal cardia, Bradycardia}). We find in literature a lot of reasoners that supported the vagueness like FUZZYDL system [18] and DELOREAN system [19].
Our approach is founded on a description logic enriched by fuzzy representation in «concrete domains ». In this formalism we separate the purely conceptual layer from the linguistic representation of its encoding in the concrete domain which is the set of the fuzzy subset definite on the linguistic variable. This allows us, via fuzzy logic

operators and mathematical morphology, to build fuzzy reasoning system that specifies zones of research in a vague and imprecise context.

If we go back to our example, we find that the heart rate of Bradycardia is less than 50 and the Normal cardia is between 60 and 90, so in this case how can we know to which linguistic term, a Heart rate between 50 and 60 belongs.

The objective of our work is to modify the reasoning algorithm without changing the syntax and semantic of DL to solve this problem.

Our fuzzy reasoning allows, as we have seen, to guide the treatment of uncertain and imprecise information. However, the reasoning services used are those offered by the description logic language for knowledge representation (e.g. satisfiability research, production, etc.). These tools were not originally designed to fuzzy calculation. They do not, therefore, offer any expected tools of a fuzzy reasoning formalism. So, we try here to enrich the description logic with new reasoning services dedicated to the fuzzy inference, the finality being the construction of fuzzy description logic decidable. Following the example of our approach in the construction of vague ontology, we privilege the construction of operators being able to act at the same time in the conceptual layer (terminological) and in vague calculation, so that they are operational for the interpretation of vague information. We thus enrich the concrete domain by fuzzy mathematical morphology operators, where the predicates represent spaces of fuzzy sets that support vague information.

The inference rules deducting the properties of the morphological predicates allowed to guide the fuzzy reasoning, in a quantitative way, in the field of the vague information. Applying this reasoning to the terminological level, thus giving it a qualitative nature, which requires the definition of a satisfiability procedure, dedicated to imprecise calculations. We enriched, to this end, the approach by semantic tableau with new expansion rules.

Let us call back briefly the principle of the semantic tableau. A complete review can be found in [10]. Remember also that all modes of reasoning can be rewritten as a test of consistency of Abox. Either A0 is the initial Abox whose consistency is to be tested, the algorithm iteratively applies expansion rules, each one corresponding to a logic description construction, thus transforming the Abox input into a set of output Aboxes (create a tree of Aboxes or a forest if the test relates to a set of individuals connected by a role). The algorithm stops when the tree is complete. It means that there are no more rules to be applied or a contradiction is detected in a pathing tree (we also talk about "clash").

In our approach there are 2 types of expansion rules that enrich the semantic tableau, where in the first type we add rules to simplify and classify the vague information represented by the concrete domain, on the other hand the expansion rule used on the second phase has the role of calculating the degree of certainty of vague concepts and roles in order to validate the satisfiability

5.1 First Phase

In this phase, some rules are applied to the ontology axioms to simplify the representation of fuzzy concepts and roles and so that to reach an effective reasoning in the second phase. The expansion rules proposed in this phase are presented in Table 2:

The first expansion rule of the preprocessing phase allows to simplify the representation of a fuzzy concept associated with the concrete domain.

The second rule differs from the classical conjunction rule by the fact that it imposes the constraints in the concrete domain.

Table 2. Preprocessing Expansion rules

Rule	Premises	Condition	Conclusion
R1	$\{\forall R.P \sqsubseteq C\} \in \mathrm{T}$		$\{\mathrm{T} \cup (R_c[P])\} / \{\mathrm{A}\}$
R2	$\{R_C[P_1 \sqcap P_2 \sqcap ..... \sqcap P_N] \sqsubseteq C\} \in \mathrm{T}$	$N \geq 1$	$\{\mathrm{T} \cup (R_c[P_1], R_c[P_2], ...R_c[P_N])\} / \{\mathrm{A}\}$
R3	$\{R_C[\sim \mathrm{d}] \sqsubseteq C\} \in \mathrm{T}$	$\sim \in \{\leq, \geq\}$	$\{\mathrm{T} \cup (R_c[d])\} / \{\mathrm{A}\}$
R4	$\left\{R_{C_i}\,[\mathrm{d}_i]_{i=1}^{n} \sqsubseteq C\right\} \in \mathrm{T}$	$C_j \neq C_{J+1}, d_j < d_{j+1}, 1 < j < n$	$\{\mathrm{T} \cup (R[C_j(d_j), C_{j+1}(d_{j+1})])\} / \{\mathrm{A}\}$

In the third rule we eliminate the datatype construction operator to obtain the vocabulary, the term and the number of limitation and finally the 4th rule serves for organizing the linguistic terms according to the restriction number.

Our objective is to extract the ambiguity presented in the concrete domain to give a simpler and more organized representation, which will allow us in a second stage to apply an effective fuzzy reasoning.

Illustration

Now we take the example discussed in Section 4 to illustrate the progressive approach for interpreting the expansions rules. For that purpose, we present below the

TBox related to our example associated to help diagnostic rules allowing from electrocardiogram (ECG) to detect three family of atrial fibrillation {Slow fibrillation "SF", Continuous arrhythmia atrial fibrillation "CAAF", Continuous tachycardia atrial fibrillation "CTAF"}.

Table 3. Atrial Fibrillation Diagnostic Rules

Heart rate	AmplitudeP	Rhythm RR	QRS	DIAGNOSTIC
BradyCardia	Absent	Irregular	Thin	**SF**
NormalCardia	Absent	Irregular	Thin	**CAAF**
TachyCardia	Absent	Irregular	Thin	**CTAF**
NormalCardia	Present	Regular	Medium	**NORMAL**

The TBox of our example will be as follow :

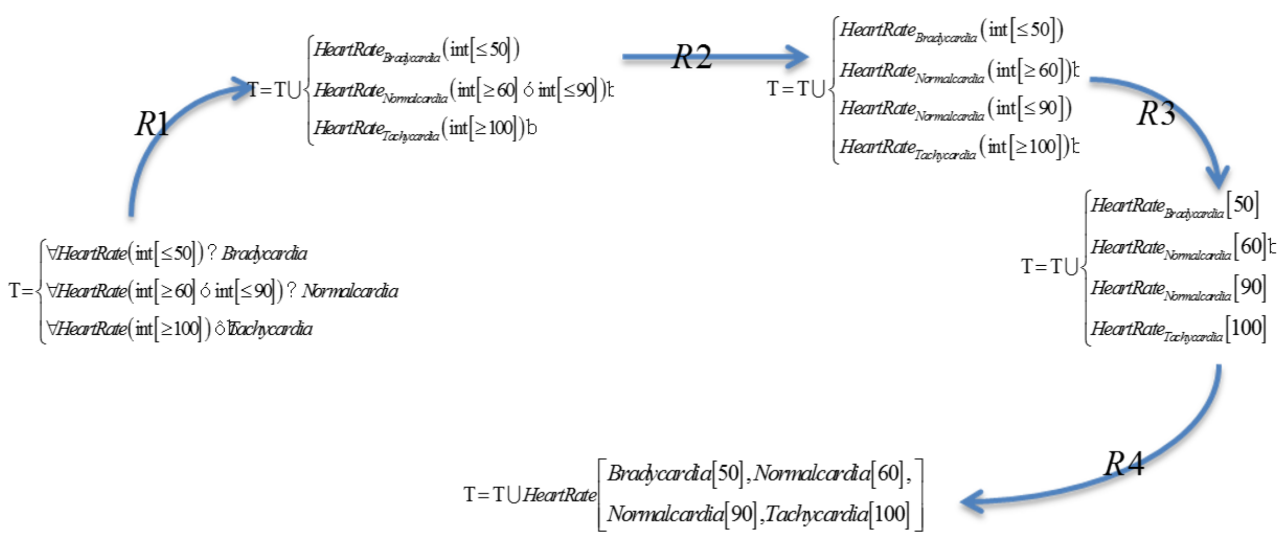
$$\begin{cases} \forall HeartRate(\text{int}[\leq 50]) \sqsubseteq Bradycardia \\ \forall HeartRate(\text{int}[\geq 60] \sqcap \text{int}[\leq 90]) \sqsubseteq Normalcardia \\ \forall HeartRate(\text{int}[\geq 100]) \sqsubseteq Tachycardia \\ \forall AmplitudeP(\text{int}[\leq 2]) \sqsubseteq P.Absent \\ \forall AmplitudeP(\text{int}[\geq 2,5]) \sqsubseteq P.Present \\ \forall QRSWidth(\text{int}[\leq 50]) \sqsubseteq QRS.Thin \\ \forall QRSWidth(\text{int}[\geq 60] \sqcap \text{int}[\leq 90]) \sqsubseteq QRS.Medium \\ \forall QRSWidth(\text{int}[\geq 100]) \sqsubseteq QRS.Large \end{cases}$$

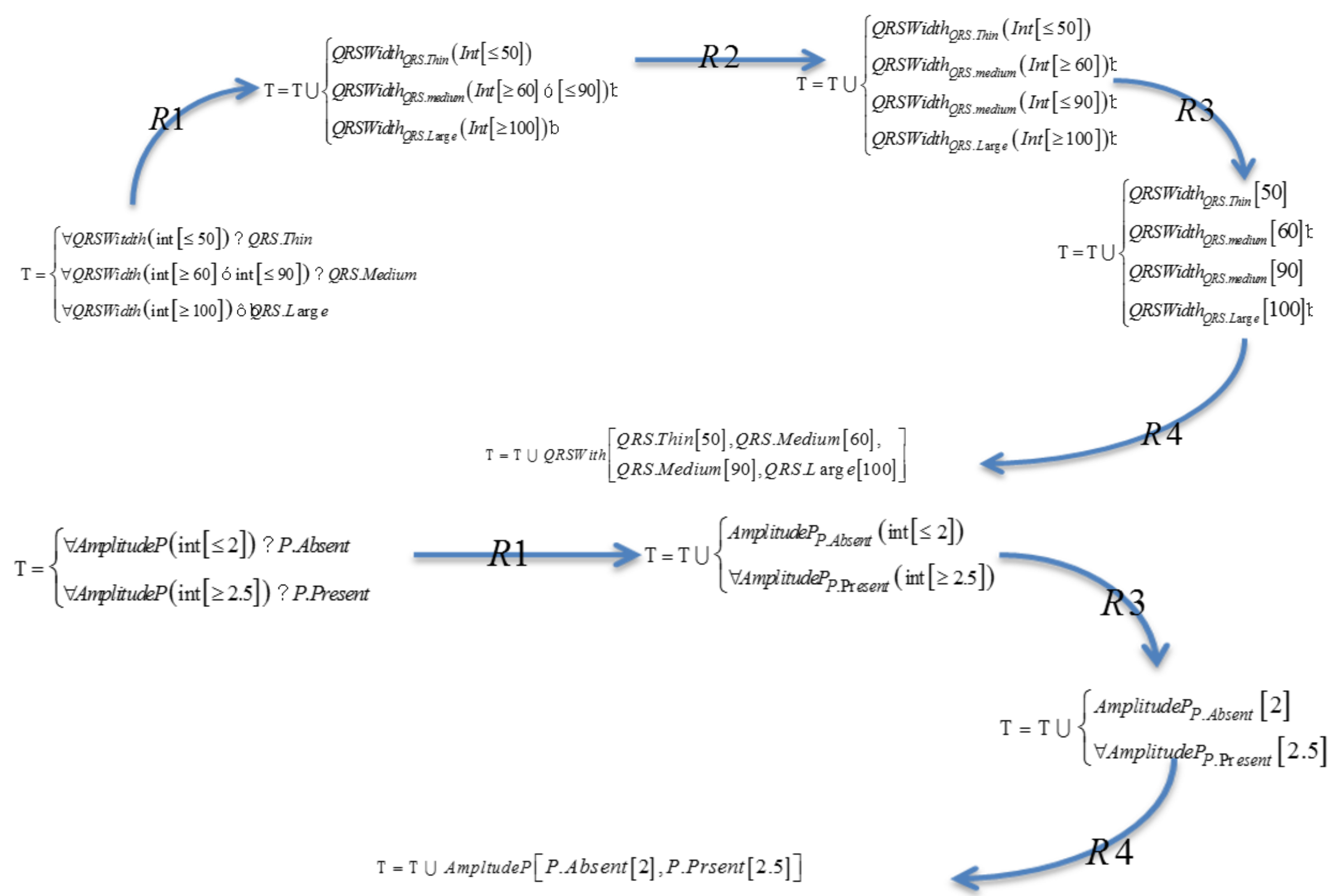
In this TBox we find concepts « Bradycardia, Normalcardia, Tachycardia, P.Absent, P.Present, QRS.Thin, QRS.Medium, QRS.Large, RR.Irregular, RR.Regular, SF, CAAF, CTAF, NORMAL » and concrete role (Feature role) « HeartRate, AmplitudeP, QRSWidh ».

The TBox which proposes yet another definition of the concepts SF, CAAF, CTAF, NORMAL- consists of the flowing axioms:

$$\begin{cases} SF \equiv Bradycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\ CAAF \equiv Normalcardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\ CTAF \equiv Tachycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\ NORMAL \equiv Normalcardia \sqcap P.Present \sqcap QRS.Medium \sqcap RR.Regular \end{cases}$$

In this preprocessing phase the reasoner tries to simplify and classify the TBox by applying the expansions rules cited above. In this phase the reasoner takes the axioms defined by datatype and tries the simplified it until we get a list of a linguistic variable that contains these terms assigned to values





NB: we note that in the simplification of the language variable "AmplitudeP" we have not used the Rule "R2" becaufe it is not necessary to use

After the application of expansion rules we manage to simplify it in a list that will be useful in the second phase of reasoning.

5.2. Second Phase

The reasoning in DL serves to check the validity of satisfaction of ontology. In our work we try to extend this satisfaction to answer the fuzzy needs for ontology. In this case the satisfiability of such ontology will be acceptable if we return a degree of certainty associated with the concept related to the request.

Fuzzy reasoning provides effective treatment to define the behavior of systems that are overelaborate or too badly defined to allow the use of a precise mathematical analysis. The result of this reasoning is not any more a simple answer but a result with a rate of satisfiability; there are thus more chances than it is coherent with objectives and starting constraints. To do so, we add three new rules that we can use according to the results of the previous phase:

Table 4. Reasoning Expansion rules

Rule	Premises	Condition	Conclusion
R-R1	$\{C(a),R(a,d)\in A\}$	$R\{P_i[d_i]_{i=1}^{n}\}\in T\wedge\{(d<d_i)\wedge(P_i\neq P_{i+1})\wedge(i=1)\}$ $\vee\{(d>d_n)\wedge(P_i\neq P_{i-1})\wedge(i=n)\}$	$P_i(a)=1$
R-R2	$\{C(a),R(a,d)\in A\}$	$\{R\{P_i[d_i]_{i=1}^{n}\}\in T\wedge\{d_i<d<d_{i+1}\}\wedge\{P_i\neq P_{i+1}\}\}$	$\left(P_i(a)=\left\lvert\frac{d-d_{i+1}}{d_{i+1}-d_i}\right\rvert\right)$ $\left(P_{i+1}(a)=\left\lvert\frac{d-d_i}{d_i-d_{i+1}}\right\rvert\right)$
R-R3	$\{C(a),R(a,d)\in A\}$	$\{R\{P_i[d_i]_{i=1}^{n}\}\in T\wedge\{d_i<d<d_{i+1}\}\wedge\{P_i=P_{i+1}\}\}$	$P_i(a)=1$

In the reasoning phase we apply the rules presented above, according to the results of the previous phase, where the first rule is applied when the concept that represents linguistic term belongs to the first or the last element of result list of the previous phase, also in the case when two successive elements are similar. The result of the certainty degree of this rule is always equal to 1. In the second and the third rules, the certainty degree is also calculated, but the difference will be in the likeness of the concept of motion relative to the elements of the result of the previous phase list.

After calculating the degree of certainty we move to the implementation of the tableau algorithm in a usual way, where we try to respond to the un-satisfiability of the negation statement. The difference is that in the last sheet of the branch of the reasoning tree, when we arrive at a contradiction □ in one branch, we said to be closed and shall cease to expand. When all the branches of a table are closed and there is no linguistic term with a degree of certainty, we say that the table is closed while the initial statement is considered unsatisfiable, otherwise you take the lower degree in the disjunction case, or the highest in the conjunction case, knowing that all concepts have a degree of certainty equal to 1. If one or more branches have a certainty degree and an existing clash branches, we ignore it. The final result will be the satisfiability of the statement with a certainty degree equal to 1 minus the obtained degree.

If one tree contains at least an open branch and there is no branch with a degree of certainty, we conclude that the initial statement is not satisfiable

Illustration

Let us return to the previous example and apply the rules of the second step to execute reasoning. For doing that, we add the following assertions:

$$Patient(x),\ HeartRate(x,97.5),\ AmplitudeP(x,1.9),$$
$$RR.Irregular(x), QRS(x,55)$$

By putting the request: CTFA(x)?

To answer this request we put the description ofthe ontology (TBox & ABox) as follows:

$$T = \begin{cases} \forall HeartRate(\text{int}[\le 50]) \sqsubseteq Bradycardia \\ \forall HeartRate(\text{int}[\ge 60] \sqcap \text{int}[\le 90]) \sqsubseteq Normalcardia \\ \forall HeartRate(\text{int}[\ge 100]) \sqsubseteq Tachycardia \\ \forall AmplitudeP(\text{int}[\le 2]) \sqsubseteq P.Absent \\ \forall AmplitudeP(\text{int}[\ge 2,5]) \sqsubseteq P.Present \\ \forall QRSWidth(\text{int}[\le 50]) \sqsubseteq QRS.Thin \\ \forall QRSWidth(\text{int}[\ge 60] \sqcap \text{int}[\le 90]) \sqsubseteq QRS.Medium \\ \forall QRSWidth(\text{int}[\ge 100]) \sqsubseteq QRS.Large \\ SF \equiv Bradycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\ CAAF \equiv Normalcardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\ CTAF \equiv Tachycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\ NORMAL \equiv Normalcardia \sqcap P.Present \sqcap QRS.Medium \sqcap RR.Regular \end{cases}$$

$$A = \begin{cases} Patient(x),\ HeartRate(x,97.5),\ AmplitudeP(x,1.9), \\ RR.Irregular(x), QRS(x,55) \end{cases}$$

as we have seen, the application of the rules R1, R2, R3, R4 according to their necessities allows us to simplify the TBox. The results obtained after the application of these rules is a new TBox that contains new concept defined with lists contains the different linguistic terms related to these concepts. Values are assigned to every terms for quantitative identification and as such value that the terms are stored in these lists. Therefore, the following has these values we can check the consistence of TBox

$$
T = \begin{cases}
\forall HeartRate(\text{int}[\leq 50]) \sqsubseteq Bradycardia \\
\forall HeartRate(\text{int}[\geq 60] \sqcap \text{int}[\leq 90]) \sqsubseteq Normalcardia \\
\forall HeartRate(\text{int}[\geq 100]) \sqsubseteq Tachycardia \\
\forall AmplitudeP(\text{int}[\leq 2]) \sqsubseteq P.Absent \\
\forall AmplitudeP(\text{int}[\geq 2,5]) \sqsubseteq P.Present \\
\forall QRSWidth(\text{int}[\leq 50]) \sqsubseteq QRS.Thin \\
\forall QRSWidth(\text{int}[\geq 60] \sqcap \text{int}[\leq 90]) \sqsubseteq QRS.Medium \\
\forall QRSWidth(\text{int}[\geq 100]) \sqsubseteq QRS.Large \\
SF \equiv Bradycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\
CAAF \equiv Normalcardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\
CTAF \equiv Tachycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\
NORMAL \equiv Normalcardia \sqcap P.Present \sqcap QRS.Medium \sqcap RR.Regular \\
HeartRate[Bradycardia[50], Normalcardia[60], Normalcardia[90], Tachycardia[100]] \\
AmplitudeP[P.Absent[2], P.Present[2,5]] \\
QRSWidth[QRS.Thin[50], QRS.Medium[60], QRS.Medium[90], QRS.Large[100]]
\end{cases}
$$

$$
A = \begin{cases}
Patient(x),\ HeartRate(x,97.5),\ AmplitudeP(x,1.9), \\
RR.Irregular(x), QRS(x,55)
\end{cases}
$$

In a second step we apply the rules 2 and 3 of the second phase to find the linguistic term related to the linguistic label and its certainty degree.

For example: we take the assertion role "heartrate(x,97,5)"; if we observe our TBox we find that 97.5 is between 90 and 100, so it is between Normalcardia and Tachycardia. Beyond we 'are opting for the application of R-R2 Rule, when we can calculate two certainty degrees one for Normalcardia and the second for Tachycardia.

$$Tachycardia\left(\left|\frac{97.5-90}{100-90}\right|\right) = 0.75$$

$$Normalcardia\left(\left|\frac{97.5-100}{100-90}\right|\right) = 0.25$$

The same rule will be applied to assertion role QRSWidth, because 55 is between QRSThin and QRSMedium; the result will be as follow:

$$QRSThin = \left(\left|\frac{55-60}{60-50}\right|\right) = 0.5$$

$$QRSMedium = \left(\left|\frac{55-50}{60-50}\right|\right) = 0.5$$

Now if we return to the other assertion role, we find that we must apply R-R1 rule to "AmplitudeP" because "1.9<2.0" and P.Absent not equal to P.present. The result after applying the rule is "P.Absent" without certainty degree (certainty degree=1).

NB: if the value is contain in the defined region, our approach return 1 as certainty degree by applying the R-R1 or R-R2 rules.

After applying the reasoning rules to the basic ABox, we face a new ABox with new assertions associate to degrees of certainty. Our knowledge base will be as follow:

$$
T = \begin{cases}
\forall HeartRate(\mathrm{int}[\le 50]) \sqsubseteq Bradycardia \\
\forall HeartRate(\mathrm{int}[\ge 60] \sqcap \mathrm{int}[\le 90]) \sqsubseteq Normalcardia \\
\forall HeartRate(\mathrm{int}[\ge 100]) \sqsubseteq Tachycardia \\
\forall AmplitudeP(\mathrm{int}[\le 2]) \sqsubseteq P.Absent \\
\forall AmplitudeP(\mathrm{int}[\ge 2,5]) \sqsubseteq P.Present \\
\forall QRSWidth(\mathrm{int}[\le 50]) \sqsubseteq QRS.Thin \\
\forall QRSWidth(\mathrm{int}[\ge 60] \sqcap \mathrm{int}[\le 90]) \sqsubseteq QRS.Medium \\
\forall QRSWidth(\mathrm{int}[\ge 100]) \sqsubseteq QRS.Large \\
SF \equiv Bradycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\
CAAF \equiv Normalcardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\
CTAF \equiv Tachycardia \sqcap P.Absent \sqcap QRS.Thin \sqcap RR.Irregular \\
NORMAL \equiv Normalcardia \sqcap P.Present \sqcap QRS.Medium \sqcap RR.Regular \\
HeartRate[Bradycardia[50], Normalcardia[60], Normalcardia[90], Tachycardia[100]] \\
AmplitudeP[P.Absent[2], P.Present[2,5]] \\
QRSWidth[QRS.Thin[50], QRS.Medium[60], QRS.Medium[90], QRS.Large[100]]
\end{cases}
$$

$$
A = \begin{cases}
Patient(x),\ HeartRate(x,97.5),\ AmplitudeP(x,1.9), RR.Irregular(x), QRS(x,55), \\
Normalcardia(\mathrm{x})_{0.25}, Tachycardia(x)_{0.75}, QRS.Thin(x)_{0.5}, QRS.Medium(x)_{0.5}, P.Absent(x)
\end{cases}
$$

After calculating the certainty degree, we pass now to the reasoning, where we have to check the un-satisfiability of the statement negation of CTFA(X): $\neg CTFA(x)$

$$
\neg \mathrm{CTFA}(\mathrm{x}) \rightarrow \neg \begin{pmatrix} \mathrm{Tachycardia} \\ \sqcap \\ \mathrm{P.Absent} \\ \sqcap \\ \mathrm{QRS.Thin} \\ \sqcap \\ \mathrm{RR.Irregular} \end{pmatrix} (\mathrm{x}) \rightarrow \begin{pmatrix} \neg\mathrm{Tachycardia}(\mathrm{x}) \\ \sqcup \\ \neg\mathrm{P.Absent}(\mathrm{x}) \\ \sqcup \\ \neg\mathrm{QRS.Thin}(\mathrm{x}) \\ \sqcup \\ \neg\mathrm{RR.Irregular}(\mathrm{x}) \end{pmatrix}
\begin{matrix}
\rightarrow \neg\mathrm{Tachycardia}(\mathrm{x}) \rightarrow \neg\mathrm{Tachycardia}(\mathrm{x})_{0.75} \rightarrow \mathrm{Tachycardia}(\mathrm{x})_{0.25} \\
\\
\rightarrow \neg\mathrm{P.Absent}(\mathrm{x}) \rightarrow \neg\mathrm{P.Absent}(\mathrm{x}) \\
\\
\rightarrow \neg\mathrm{QRS.Thin}(\mathrm{x}) \rightarrow \neg\mathrm{QRS.Thin}(\mathrm{x})_{0.5} \rightarrow \mathrm{QRS.Thin}(\mathrm{x})_{0.5} \\
\\
\rightarrow \neg\mathrm{RR.Irregular}(\mathrm{x}) \rightarrow \neg\mathrm{RR.Irregular}(\mathrm{x})
\end{matrix}
$$

We note that the tree is closed but there is a linguistic term with a degree of certainty, so the initial statement is considered unsatisfiable with a degree of certainty which is the lower degree between Tachycardia and QRSThin (0.25). Our result will be the satisfiability of the statement CTFA(x) with certainty degree (1 - 0.25).

We note that the tree is closed but there is a linguistic term with a degree of certainty, so the initial statement is considered unsatisfiable with a degree of certainty which is the lower degree between Tachycardia and QRSThin (0.25). Our result will be the satisfiability of the statement CTFA(x) with certainty degree (1 - 0.25).

So if we interpret the result, we say that the initial statement CTFA (X) is satisfiable with a 0.75 degree (CTFA (X)0.75).

6. Results Interpretation

In our approach the results obtained during the reasoning phase are related to a certainty degree, the latter did not understand and does not give a clearance to the result. To answer this vagueness we decide to interpret the obtained results of the closest ways notion of fuzzy modifiers.

We know that A fuzzy modifier mod is a function fmod: $[0,1] \rightarrow [0,1]$ which applies to a fuzzy set to change its membership function[19][20]. We adopt the same idea to present ours, where our idea will be based on the symbolic similarity notion of fuzzy sets.

To do this, we must give at first the certainty modifiers to fuzzy concepts relaying on the enumeration notion of description logic (Ϭ). Secondly we need to create a subset of fuzzy interval by performing a partition of the interval [0,1] on the number of enumeration.

This produces two sets or two lists:

Certainty Modifiers: $\{x_1, x_2, \ldots, x_n\}$;

Fuzzy interval list: $\{\{y_1,y_2,\ldots,y_n\} = \{]0,1/n],]1/n,2/n]\ldots\ldots,](n-1/n), 1[\}\}$.

Each interval yi is associating t certainty, modifier xi defined in ordered set XN.

Our approach takes the certainty degree of the result obtained during the reasoning concept phase, looking for adequate interval from the list Yi and selects its similar in the list Xi, the similar items in this list correspond to the modifier complies with certainty degree.

NB1: Usual modifiers precede the concept, but in our case the modifier comes after the fuzzy concept.

NB2: Fuzzy concepts with certainty degree “0” or “1” do not fall within the post reasoning treatment (so either true or false result).

The final result will be of the following form: C xi / { C: Result Fuzzy Concept; x_i: the certainty modifier appropriate.

To clarify our idea, we go back returning to the previous example and declare the enumeration of certainty modifiers for the CTFA fuzzy concept for the individual x:

Modi_CTFA={improbable, less probable, probable}.

Note that the number of elements in the list Mod_CTFA is 3, therefore post reasoning operation is to calculate the element interval and generates the similar interval list by dividing 1 on the number of Modi_CTFA elements. The result will be:

Interval_list={]0,0.33],]0 ;33,0.66],]0.66,1[}

In our example the result contains a 0.75 certainty degree that belongs to the interval] 0.66,1 [which is the item number 3 of interval list, then we look into its similar item in Modi_CTFA list, which is “probable”.

The final result of our reasoning will be "CTFA Probable".

7. Discussions

In many areas; the knowledge of the expert is often tainted with fuzzy. The expression of this knowledge in qualitative rather than quantitative way, allows better to consider this fuzziness. In the description logic and for more than a decade, researchers are trying to find a solution to manage this vagueness. In this section we will begin by discussing the reasoning in fuzzy extensions DLs, and then we will show what it has made our approach compared to other.

Many works that extend the Fuzzy description logic are proposed and their main role is to represent and reason with vague information, we also find other work that effort on optimizing the fuzzy reasoning.

But the difference compared to our work is that the other works that treat fuzzy reasoning requires modifications in the ontologies representation, this causes a change in the management of ontologies editors and also affects the reasoned. In contrast, our approach does not affect the presentation part, but devotes all changes on the reasoning algorithm that includes 3 phases: the first one is the pre-treatment phase where the algorithm tries to standardize and normalize the knowledge base, especially fuzzy predicates based on their representation in the concrete domain and datatype; the second is the main phase where the algorithm corresponds to the base query and it is considered more similar to the tableau algorithm. The only difference is the existence of some concepts and roles associated with a degree of certainty; finally the third phase which includes a novelty in relation to other work which treated the quantitative qualitative passage to give more meaning to the result on using certain modifiers to express the degree of truth.

8. Conclusions

This paper discussed fuzzy reasoning technic, where we proposed a new approach for managing the vagueness in description logic; based on concrete domain to represent the fuzziness of a fuzzy concept. Our approach in its representation part counts on the definition of the true part of the fuzzy concept by ignoring that false part, the thing which allows us to represent the vague concepts of an ontology without the need to modify in already existed tools (ex: protege) which isn't the case in another approaches.

On the other hand, the reasoning phase is divided into two parts, The first one serves to normalize and to simplify the knowledge base where we extract the various linguistic terms with their truth intervals which allows us to verify the inconsistency, that drive us next to affect a certainty degree to each linguistic term. In the second phase we based on classic tableau algorithm by adding the principle to deduce the certainty degree of the query in question from degrees assigned to various fuzzy concept in the different axioms of TBox.

The obtained result will be more coherent with the beginning data , that mean that the results takes into account the non-defined regions in the KB representation; and finally we returned a result with a certainty rating which will be interpreted using modifiers to give a more significant result to users.

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