An Investigation and Comparison of the Performance of Different Techniques of Reducing PAPR in OFDM Systems

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Summary
Higher Peak-to-Average Power Ratio (PAPR) in output signal of transmitters is considered one of the drawbacks of multicarrier modulations such as OFDM. This will require amplifiers with wider linear performance range that is not cost-effective. However, if amplifiers with lower linear performance range are utilized, inter-band distortion and emission of beyond the sending signal's band will be seen. In this research, SLM-based techniques are investigated to reduce PAPR.

Key words:
OFDM, Peak-to-Average Power Ratio (PAPR), SLM technique

1. Introduction

In each sending frame, when subcarriers integrate coherently, OFDM causes Peak-to-Average Power Ratio, abbreviated as PAPR, to be increased. This quantity increases further with the increase in subcarriers. On the other hand, the linear performance range of amplifiers is limited and it is not matched to PAPR of OFDM's signal. Moreover, amplifiers usually have limited linear performance range.

Sending OFDM's signal is given by:

$$y(t) = m_I(t)\cos 2\pi f_c t - m_Q(t)\sin 2\pi f_c t \tag{1}$$

where $m_I(t)$ and $m_Q(t)$ can be written as follows:

$$m_I(t) = \sum_{m=0}^{N-1}(a_m \cos 2\pi f_m t - b_m \sin 2\pi f_m t)$$
$$m_Q(t) = \sum_{m=0}^{N-1}(a_m \sin 2\pi f_m t + b_m \cos 2\pi f_m t) \tag{2}$$

Here, modulated complex signal is:

$$x(t) = m_I(t) + jm_Q(t) = \sum_{m=0}^{N-1} X_m e^{j2\pi \frac{m}{T_S}t} \tag{3}$$

In addition, transmitted signal y(t) in (1) can also be written as:

$$y(t) = \sqrt{m_I^2(t) + m_Q^2(t)} \times \cos[2\pi f_c t + \tan^{-1}(\frac{m_Q(t)}{m_I(t)})] \tag{4}$$

Where

$$Envelop\ of\ y(t) = \tilde{y}(t) = \sqrt{m_I^2(t) + m_Q^2(t)} \tag{5}$$

is equal to transmitted signal posh. PAPR of transmitted signal is defined by [1]

$$PAPR = \frac{\max\{[\tilde{y}(t)]^2\}}{E\{[\tilde{y}(t)]^2\}} = \frac{\max\{|x(t)|^2\}}{E\{|x(t)|^2\}} \tag{6}$$

Here, x(t) is modulating complex signal and E[.] is statistical mean. If instantaneous power of the posh transmitted by OFDM's signal is given by

$$P(t) = [\tilde{y}(t)]^2 = |x(t)|^2 \tag{7}$$

Thus, instead of PAPR calculation of bandpass signal y(t), one can perform this on the base band in signal x(t) and get the similar results. In addition, calculating PAPR of signal x(t) can be approximated by time samples of signal x(t), (xn, n=0,1,2,…,N-1) [1].

2. Selected Mapping (SLM) technique

In SLM technique, the transmitter generates a number of data blocks from main data block that all of them transfer similar data. Then, among these sets of blocks, a block with the least PAPR will be sent. In this technique, each block which is the symbol of main data $X = [X_0, X_1, ..., X_{N-1}]$ is multiplied by U number of different phase sequence, each with a length of N and in the form of $B^{(u)} = [b_{u,0}, b_{u,1}, ..., b_{u,N-1}], u = 1,2,...,U$ in order to generate a new block with equal data as

$$X^{(u)} = [X_0 b_{u,0}, X_1 b_{u,1}, ..., X_{N-1} b_{u,N-1}]^T, u = 1,2,...,U \tag{8}$$

Then, based on them, U frames of OFDM are built as:

$$x^u(t) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} X_n b_{u,n}.e^{j2\pi.n\Delta ft} \quad 0 \le t < NT, u = 1,2,...,U \tag{9}$$

Then, among OFDM blocks (all of them have similar data), $X^{(u)}$, $u = 1,2,...,U$, a block with the lowest PAPR value is selected for transmission. In this case, U number of activities of IDFT is required and $\log_2 U$ number of additional bits should be transmitted to phase sequence

Manuscript received November 5, 2016
Manuscript revised November 20, 2016

(phase sequence number multiplied by transmitted signal) along with the data block in order to reveal the main data block. Evidently, the efficacy of this technique depends upon reliability of additional bits. So, additional bits are encoded prior to the transmittance into the channel [2]. In addition, in [3-6], a method is proposed which obviates, by adapting more complexities in receiver, there is no need for sending additional bits to receiver to revise main data bits. Thus, transmitted data rate will not be dropped in the channel.

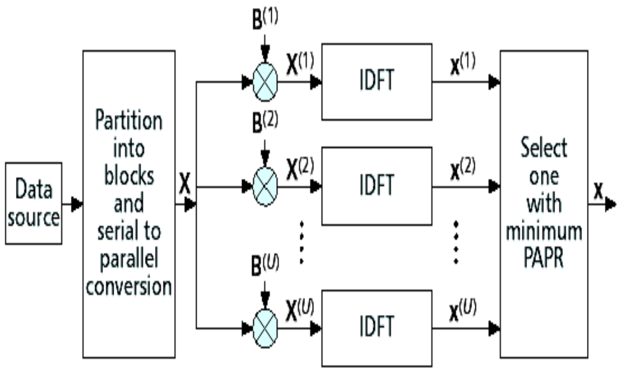


Fig.1 SLM technique

SLM technique is one of the most effective ways to decrease PAPR. One of the main advantages for SLM technique is that it produces no distortions in transmitted signal range. In addition, in conventional SLM technique, a part of the channel capacity should be always assigned to the additional data transfer.

Cumulative distribution function or CDF may be used to evaluate how to examine statistical performance of various techniques in reducing PAPR. In most of the applications, complementary CDF that is CCDF is calculated and used instead of CDF. CCDF expresses the possibility of PAPR of a OFDM's data block to violate a threshold value. When the number of subcarriers is high, CCDF of PAPR quantity of a block with a length of N and threshold of z (when no techniques are used to reduce PAPR) is given analytically by

$$P(PAPR > z) \approx 1-(1-\exp(-z))^{N} \qquad (10)$$

Equation above is established by considering independency and uncorrelation of samples. When sampling rate is more than Nyquist rate, Eq. (10) changes as Eq. (11):

$$P_{clip0} = P(PAPR > z) \approx 1-(1-\exp(-z))^{\alpha N} \qquad (11)$$

Where P_{clip0} is the possibility of clipping signal amplitude when no technique is used to decrease PAPR and α is a number higher than one and dependent on sampling rate, and N is the previous number of subcarriers and z is the threshold level. In other word, having applied SLM technique, the possibility of clipping signal $P_{clip-SLM}$ is

$$P_{clip-SLM} = (P_{clip0})^{U} \qquad (12)$$

Where U is the number of branches. Moreover, when SLM technique is used in transmitter, and if M number of independent and uncorrelated phase sequences is utilized for transmission, then we have [7]:

$$P(PAPR > z) \approx (1-(1-\exp(-z))^{\alpha N})^{M} \qquad (13)$$

Eq. (13) is depicted in terms of various values of M in Fig.2.

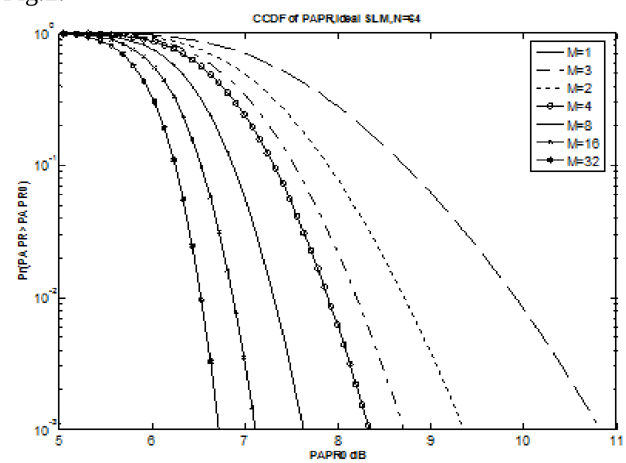


Fig.2 CCDF, OFDM in SLM technique

In Fig. 3 CCDF, PAPR in SLM technique is shown, obtained from simulation (using Monte Carlo method versus more than 1 million binary bits) in terms of randomized phase sequences (with uniform distribution in (0, 2π)) for N=64 and QAM modulation.

As figures illustrate, the theoretical relationships are completely matched with the simulation results.

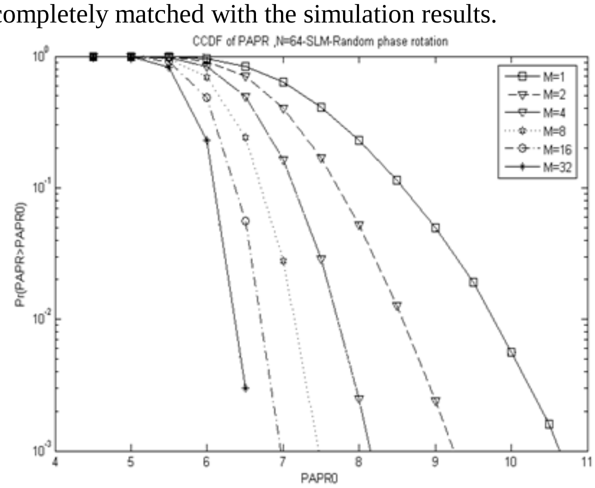


Fig.3 CCDF, OFDM in SLM technique

3. Performance of perpendicular matrices along with BSLM technique on the reduction of PAPR in OFDM system

In conventional SLM technique, part of channel capacity should be assigned to the transmission of additional data in order to enable receiver to restore the main data. In addition, in channels with feeding, this additional data should be secured against destroying effects of channel. To this goal, we ought to use channel coding, increasing system complexities and decreasing channel effective capacity [7]. As it will be shown further, using these matrices will reduce significantly PAPR effect in OFDM system [8]. In this technique, by applying changes in the receiver structure, it will be seen that there is no need to transmit such additional data and in some cases, error rate also decreases. The use of Hadamard matrix along with BPSK modulation is very effective in decreasing PAPR value.

4. BSLM using Perpendicular Matrices

In this method, instead of multiplying transmitted sequence of data by M different phase sequences, it is multiplied by M perpendicular (N*N) matrices; then, a block with the lowest PAPR is transmitted. In Fig.4, Matrix H1 may be one of the perpendicular (N*N) matrices such as Hadmard or Holmert matrix. Matrices H_2 and H_3 to H_M are the same H_1 matrix which their rows are exchanged randomly (H_1 and H_2 to H_M still remain perpendicular).

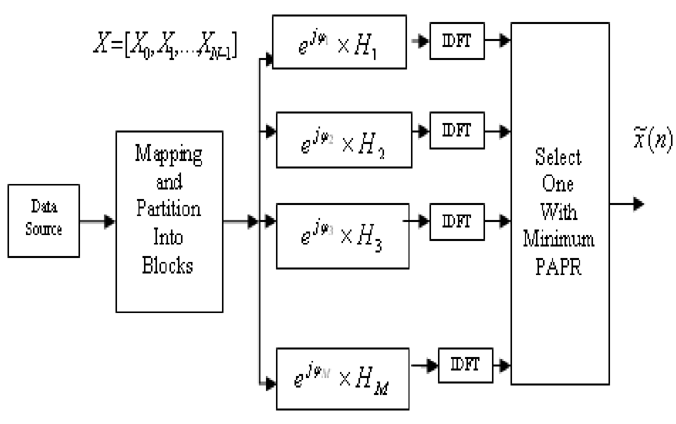


Fig.4 BSLM transmitter's block diagram

In this blind technique, since we do not want to transmit additional data of branches' number to the receiver, Matrices H_1 and H_2 to H_M are therefore multiplied by phases $e^{j\varphi_1}$ and $e^{j\phi_2}$, respectively. That is, system diagram related to each branch is rotated. Based on the phase rotation occurring on transmitted symbols, a receiver perceives transmitted branch's number and restores data signals (it should be noted that multiplication of matrices by phases has no effect on PAPR of transmitted symbols). Finally, IFFT is derived from the output of M branches, and a branch with the lowest PAPR is transmitted to the receiver. Fig. 4 shows the block diagram of transmitters BLSM. As seen, a data block with a length of N (N is the number of subcarriers) is multiplied by Hadmard matrix in the first branch. In the second branch, the same data block is multiplied by Hadmard matrix except that its rows are interleaved in quasi-random fashion. This action continues until *M-1*th branch. It is noteworthy that interleaving rows for each branch is distinct from the above branch.

$$\tilde{x}(n) = \text{select i with } \min \ PAPR \begin{Bmatrix} IDFT(X \times e^{j\varphi_i} \times H_i) \\ i = 1,2,...,M \end{Bmatrix} \quad (14)$$

In this way, as we saw in simulations, passing data symbols through Hadmard matrices causes system performance improvement in terms of error in Gaussian and multipass channels.

Once the transmitted signal is passed through the channel, it is added to the noise. Thus, a signal arrived in receiver is given by

$$y(n) = \tilde{x}(n) + w(n) \quad (15)$$

Where w(n) is white Gaussian noise with a zero mean and variance of δ^2. In the receiver, FFT at first derived from received signal to obtain vector Y. Next, vector Y is multiplied by the inverse of transmitter matrices and the resulting scalar matrix (the inverse of perpendicular matrix is equal to its transposed matrix). In this case, M numbers of vectors $\hat{Y}_1, \hat{Y}_2, ..., \hat{Y}_M$ are generated.

$$\begin{aligned} \hat{Y}_i &= FFT\{y(n)\} \times e^{-j\varphi_i} \times (H_i)^{-1} \\ &= FFT\{y(n)\} \times e^{-j\varphi_i} \times (H_i)^T \quad i = 1,2,...,M \end{aligned} \quad (16)$$

Then, based on ML criterion, a vector with the highest similarity to the system space used in transmitter is selected and named $\hat{X}$.

Depending on the type of system used in transmitter, Vector $\hat{X}$ will be demodulated to get the binary bits.

$$\hat{X} = Select\ \hat{Y}_i\ that\ has\ Minimum\ Distance\ to\ Constellation \quad i = 1,2,...,M \quad (17)$$

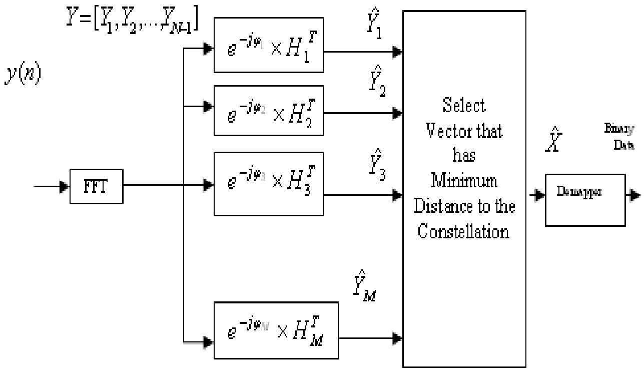
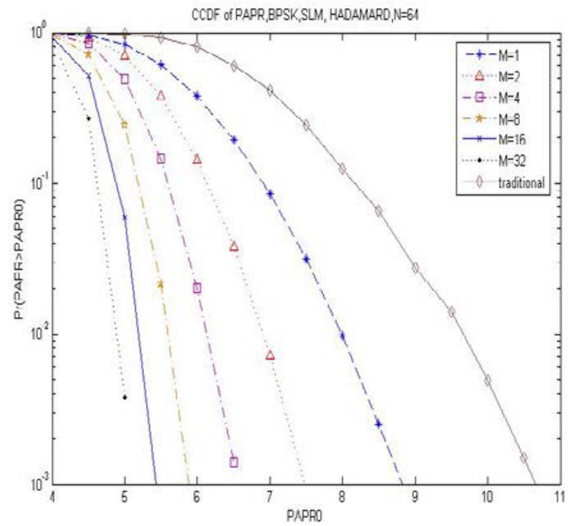


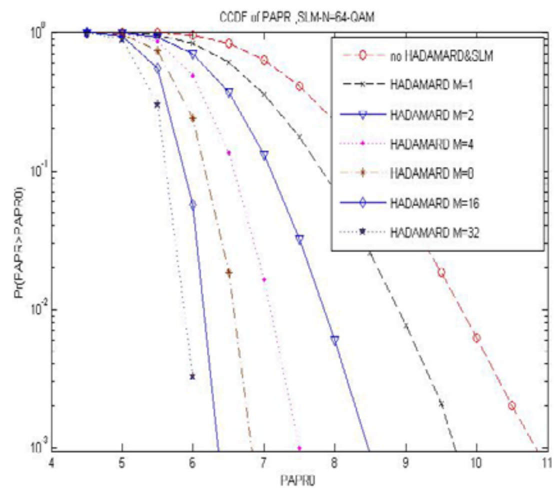
Fig. 5 BSLM receiver's block diagram

As seen, in this method, in contrary to conventional SLM, a receiver does not need to know branch number to restore main data. Simulation in receiver is done using Monte Carlo method for more than 1 million bits. The number of subcarriers is 64, and modulation type is considered separately QPSK, QAM, and BPSK. And the number of branches are M=1,2,4,8,16,32.

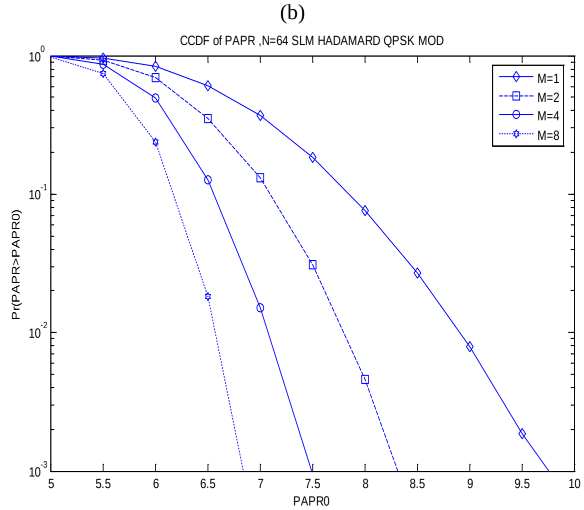
As depicted in Fig. 6, when no technique is used to decrease PAPR, threshold value of PAPR for the probability of 10^{-3} is roughly 10.8. When M=1 and modulation is of QAM type (Fig. 6-b), that is, data are passed only through main Hadmard matrix, threshold value is decreased to 9.6 which is considered as a significant value. If the number M, expressing number of branches, is large the probability of having higher amplitude in channel for transmitted signal will be high. Thus, the possibility of clipping amplitude and transmitted signal distortion in a transmitter will be low. In addition, for constant M, QAM and BPSK have similar performance along with Hadmard matrix in terms of reducing PAPR value. And, using BPSK technique together with Hadmard matrix will decrease PAPR value even for M=1 by 2dB. For M=32, the value of PAPR threshold is approximately reduced by 5.5dB with a probability of 10^{-3}. Comparing Fig. (6-a) ideal SLM, it may be concluded that the performance of this technique in reducing PAPR is better than optimal state performance, that is conventional SLM is superior over the other technique. As we know, increasing M and N will, indeed, increase system calculation and complexities.



(a)



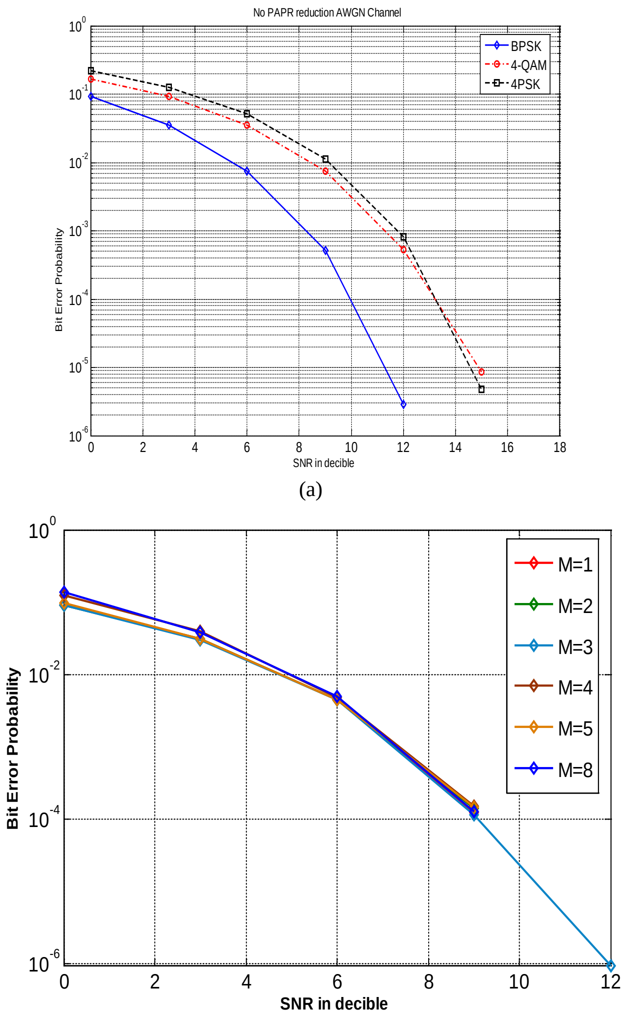
(b)



(c)

Fig. 6 Transmitted signal's distribution function PAPR for BSLM technique (a) BPSK MOD and (b) QAM MOD (c) QPSK MOD

In Fig. 7, error probability curves for Gaussian channel are illustrated. In Fig. (7-a), error curves for conventional OFDM system are shown. In Fig. (7-b), error curves for BSLM technique are depicted. As shown in the figures, this technique could get better error rate compared to conventional method in terms of M=1,2,3,4,8 and BPSK modulation. In other words, this technique has the capability of correcting error. Increasing the number of branches, vectors $\hat{Y}_i$, $i = 1,2,\ldots,M$ get closer for higher M values. This, in turn, leads to increased system complexity and demodulator decision will be more difficult.



(a) (b)

Fig. 7 Error probability curve for Gaussian channel (a) conventional OFDM and modulation of BPSK, QPSK, QAM (b) using BSLM technique along with Hadmard matrix for M=1,2,3,4,5,8 and BPSK modulation

5. Conclusion

In this paper, various techniques are evaluated and studied to decrease PAPR in OFDM system. In techniques based on SLM, part of the channel capacity should be transmitted to send additional data related to branch number, enabling the receiver to restore main information.

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