Levenberg-Marquardt and Moving Horizon Estimation for the Synthesis of Nonlinear Observers

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Summary
This paper presents a method of synthesizing an observer for a class of nonlinear systems. The method is based on the moving horizon estimation technique. It can transpose the observation problem into an optimization problem. It consists on the minimization of the difference between the system measurement and its prediction on a predetermined moving time horizon. The optimization algorithm used is "Levenberg-Marquardt". This methodology is applied to an example of CSTR chemical reactor.
Key words:
Observer, Moving horizon, Optimization, Nonlinear systems, CSTR reactor.

1. Introduction

At the outset, an observer is a "computer" measurement tool that allows finding the states of a system having a minimum of information [1, 2]. In this, different methods of nonlinear observer synthesis have been suggested such as Kalman extended filter, Luenberger extended observer, the high gain observer.... [3, 4]. Yet, most of these techniques require linear approximations [5, 6, 7].
However, other vital approaches are introduced. i.e., the specific methods based on the optimization of a criterion. They are a set of the model structure independent observers inspired by the foundation of predictive control and observability [8, 9, 10].
Actually, this paper is a proposal of an observer-synthesis method; namely, the moving horizon estimation method (MHSE). In other words, it is a strategy that reformulates the estimation problem as a minimization of a criterion. It is to minimize the difference between the system measurement and its prediction on a predetermined time horizon [11, 12].
The paper is organized as follows: section 2 is devoted to the presentation of the moving horizon state estimation method (MHSE) as well as "Levenberg-Marquardt" optimization technique. An example of the synthesis of an observer and the simulation results are given in section 3. The paper is ended up by a conclusion in Section 4.

2. Theory and synthesis

In this part, the observer synthesis method MHSE is presented. The synthesis involves the use of the criterion to be minimized and the optimization algorithm.

2.1 Principle of the method MHSE

Let the non-linear system be in the form:

$$\begin{cases} \dot{x} = f\left(x,u\right) \\ y = g\left(x\right) \end{cases}, \tag{1}$$

Where $x \in \Re n$ is the system state, $u \in \Re m$ is the controls and $y \in \Re p$ is the output. f and g are two nonlinear functions. Let us consider h bounded observations of the output vector made at regular time intervals Te, such as lh0 of the horizon is lh0 = (h− 1) Te [12, 13, 14].
The moving horizon method can be stated as a nonlinear optimization problem with the following structure:

$$J(x) = \frac{1}{2}\sum_{k=t_k}^{t_k+lh_0} \varepsilon_k{}^2 = \frac{1}{2}\sum_{k=t_k}^{t_k+lh_0} (y_k - y_{mk})^2, \tag{2}$$

Where y is the measured output, ym is the estimated output and x is the state vector to be identified. tk and lh0 are respectively the beginning and the length of the horizon.
The principle of the moving horizon estimator, shown in Fig. 1, is to estimate on a given horizon [tk, tk+lh0] the sole initial state $\hat{x}_0$ which minimizes the criterion J(x). Then, the process dynamic model is used (1) in order to estimate the current state from the previously estimated initial state (Fig.1). In the next sampling period, the estimation horizon shifts a period. Therefore, we resume the estimation procedure on the new interval $t \in \left[t_k, t_k + lh0\right]$ and so on [13, 14].

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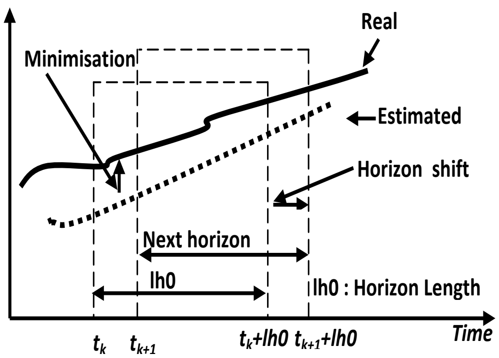

Fig. 1 MHSE method

2.2 Levenberg-Marquardt optimization technique

To solve the above optimization problem, we use the iterative algorithm "Levenberg Marquardt" [3, 15].

Equation (2) has an optimum if the optimality condition ($\frac{\Delta J}{\Delta xo}=0$) is verified. The limited development of the criterion (2) gives

$$J(x_{i+1}) = J(x_i+\Delta x_i) = J(x_i) + \left[\frac{\partial J}{\partial x}\right]^T_{x=x_i} \Delta x_i + \frac{1}{2}\Delta x^T \left[\frac{\partial^2 J}{\partial x.\partial x_i}\right]_{x=x_i} \Delta x_i + 0\|\Delta x\|^2, \quad (3)$$

The variation ΔJ of the criterion is given by

$$\Delta J = Grad(x_i).\Delta x_i + \frac{1}{2}\Delta x^T Hess(i,j).\Delta x_i + 0\|\Delta x\|^2, \quad (4)$$

With Grad is the gradient and Hess is the hessian and are given by

$$Grad(x_i) = \left[\frac{\partial J}{\partial x}\right]^T_{x=x_i} = -2\sum_{k=1}^{N}(y_k - y_{m_k}(x))\frac{\partial y_{m_k}(x_i)}{\partial x_i}, \quad (5)$$

$$Hess(i,j) = \left[\frac{\partial^2 J}{\partial x_j.\partial x_i}\right] = 2\sum_{k=1}^{N}\left(\frac{\partial y_{m_k}(x_i)}{\partial x_j}\right)^2 - 2\sum_{k=1}^{N}(y_k - y_{m_k}(x))\frac{\partial y_{m_k}(x_i)}{\partial x_i.\partial x_j}, \quad (6)$$

By applying the optimality condition $\frac{\Delta J}{\Delta x}=0$ we get:

$$Grad(x_i) + Hess(i,j).\Delta x_i = 0, \quad (7)$$

Whence the state variation is:

$$\Delta x_i = x_{i+1} - x_i = -Hess(i,j)^{-1}Grad(x_i), \quad (8)$$

The principle of the "Levenberg-Marquardt" method is to neglect the term which can make the Hessian matrix negative and add a diagonal matrix to adjust the values of the descent matrix [16, 17, 18].

The new value of the state in iteration i+1 is given by the following equation:

$$x_{i+1} = x_i - \left[Hess(i,j) + \lambda_i I\right]^{-1}\nabla J, \quad (9)$$

With

- Hess is the hessian matrix given by:

$$Hess(i,j) = 2\sum_{k=1}^{N}\left(\frac{\partial y_{m_k}(x_i)}{\partial x_j}\right)^2, \quad (10)$$

- λi is the relaxation coefficient. It adjusts the eigen values of the Hessian matrix by dividing or multiplying one or more times until the convergence of the method [16].

Levenberg-Marquardt algorithm is described by the Fig. 2:

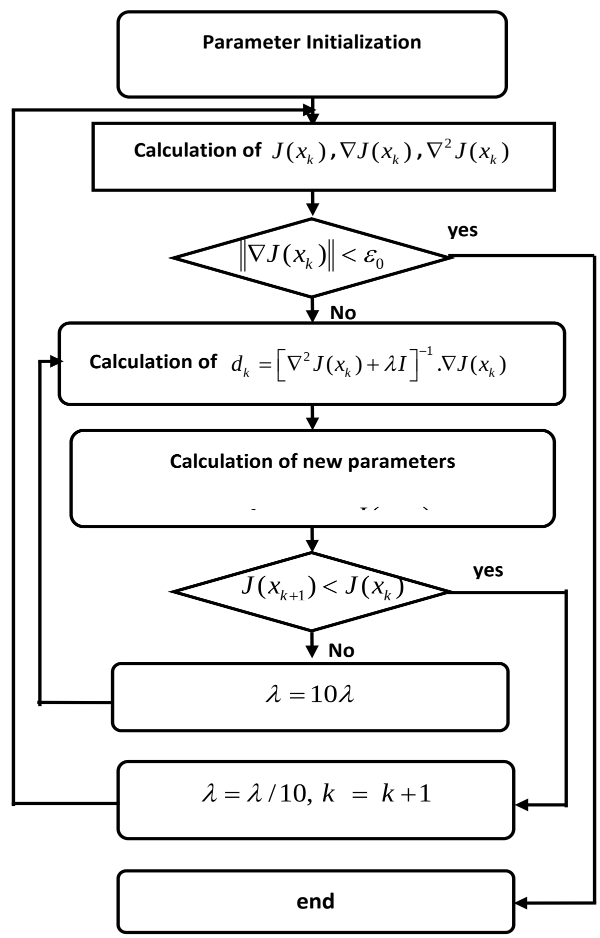

Fig. 2. Levenberg-Marquardt technique

2.3 Observer synthesis

The following iterative algorithm gives us a state estimate of the dynamic system (1) at the end of the chosen horizon, i.e. at the time $l = t_k + lh0$ .This algorithm uses the following terminology: T_{max} is the maximum estimate time (duration of the experiment), t_k is the beginning of the horizon, $lh0$ is the length of the horizon, $\sum(.)$ is the dynamic system model, u is the control over the preset horizon, y is the measurement of the system output on the preset horizon, $\hat{x}_0$ is the limited estimated state at the beginning of the horizon (the solution of the optimization problem (2)), $\hat{x}_l$ is the estimated state at the current moment $l = t_k + lh0$.

<u>Observation Algorithm</u>

Input : Tmax, tk, lh0, u, y, ∑(.)

Output: $\hat{x}_0$, $\hat{x}_l$

Initiation: tk=1, l= tk+lh0

While l≤ Tmax

1. Optimization problem resolution (2) (Levenberg-Marquardt)

$$\hat{x}_0 := Optimization\left(J\left(x\right), u, y, lh0\right)$$

2. Calculation of the sequence of states until the end of the horizon $\hat{x}_l := Trajectory(x_0, \sum(.), t_k, lh0, u)$

3. Move the horizon of a step to calculate the next estimate: $\hat{x}_{l+1}$

tk= tk +1 (Horizon shift)

End

The observer synthesis technique is described by the following flow chart

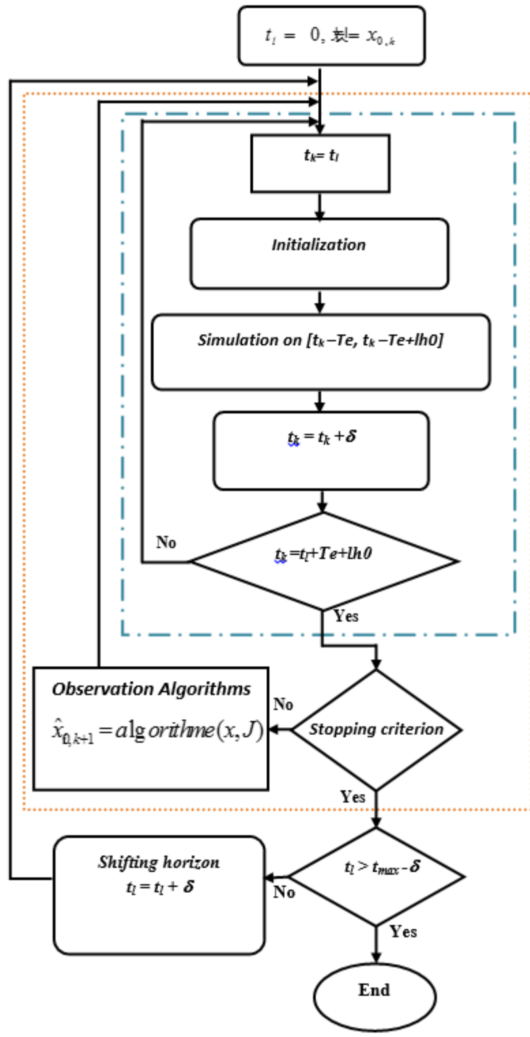

Fig. 3 Observer synthesis technique

The principle of the method can be given by the block diagram of fig. 4:

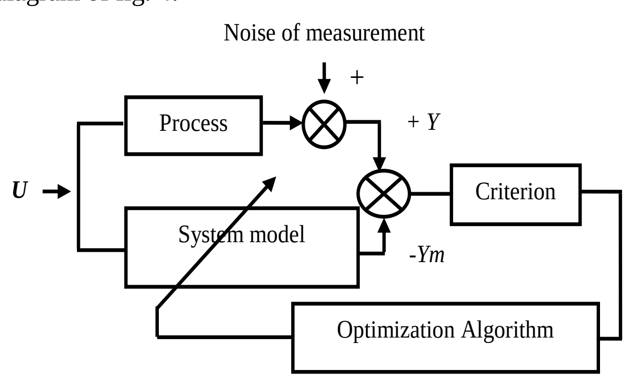

Fig. 4 Model Method

3. Example: CSTR Chemical Reactor

As an illustration of the previously presented methodology, we consider the problem of estimating the concentration of a component A of a continuous reactor as shown by Fig. 5.

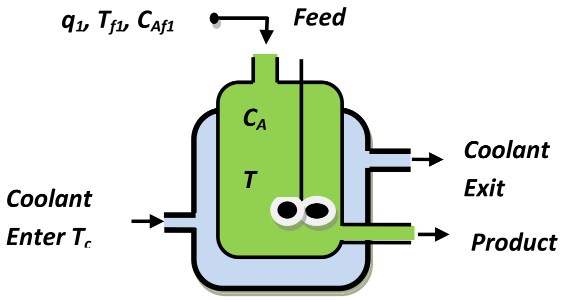

Fig. 5 CSTR reactor

The studied reactor is modeled by the following differential system [19]:

$$\begin{aligned} \frac{dC_A}{dt} &= \frac{Q}{V}(C_{Af} - C_A) - k_0 \exp\left(-\frac{E}{RT}\right) C_A \\ \frac{dT}{dt} &= \frac{Q}{V}(T_f - T) - \frac{H_r}{\rho c_p} k_0 \exp\left(-\frac{E}{RT}\right) C_A + \frac{UA}{\rho c_p V}(T_c - T), \end{aligned} \quad (11)$$

C_A is the concentration of the component A and T is the reaction temperature [6].

Let us consider the following representation: $\begin{bmatrix} x_1 & x_2 \end{bmatrix}^T = \begin{bmatrix} C_A & T \end{bmatrix}^T$ is the state vector, $y = T$ is the output and $u = T_c$ is the control.

Numerical parameters:

The reactor system is characterized by the numerical parameters given by Table 1.

Table 1: The numerical parameters of the CSTR reactor

Notation	Description	Numerical value
Q	Flow rate	50 *L/min*
V	Volume of the reactor	100*L*
K_0	Reaction rate constant	72.10^{10} min^{-1}
E0/R	Activation energy them	8750 *K*
C_p	Heat capacity of fluid	0.239 *J/gK*
ΔH	Enthalpy of the reaction	-5. 104 *J/mol*
C_{Af}	Concentration	1.5*mol/L*
T_f	Temperature	350*K*
ρ	Density oh the fluid	100 *L*
U_A	Heat transfer constant	5.104 *J/minK*

The model is simulated according to the following initial conditions:

- Initial state: X_0= [1000, 308].
- Control: U= 20 *L/min*
- Initial state of the estimator: X_{m0}= [1100, 308].

The estimation results of the concentration of the component A are represented by fig. 6.

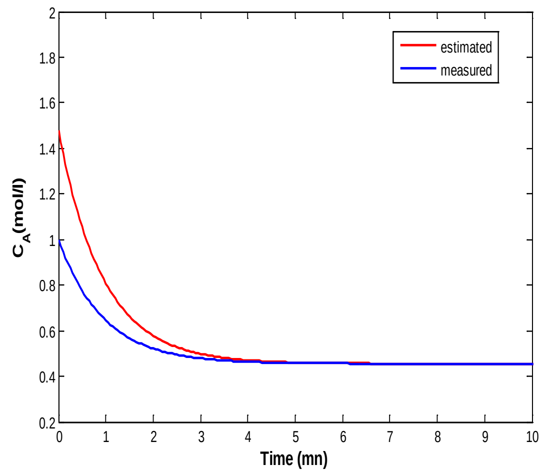

Fig. 6 Estimation of the concentration CA.

The MHSE algorithm tends to minimize the error between the simulated value of the concentration and the estimated one that is null from a minimum time t_m (fig. 6). It is the time required by the estimator to remove all the errors between these two values.

The estimation results give us good estimates of the state variables. Moreover, the process model is treated as a whole without using any transformation or linearization.

Adjustment of parameters of the observer

Some tests are carried out so as to study the effect of the length of the horizon on the quality of the observer.

In the least squares sense, a number of samples higher than *n* has to be taken (*n is the number of the system states, Te is the period of Sample Rate*). Hence, the minimum value of the length of the horizon is *lh0=nTe* [13]. Let the relative errors be:

$$Err_{C_A} = \frac{C_A - \hat{C}_A}{C_A} \quad \text{and} \quad Err_T = \frac{T - \hat{T}}{T}$$

Three values of the length of the horizon are presented: *lh0=nTe, lh0=2nTe and lh0=6 nTe.*

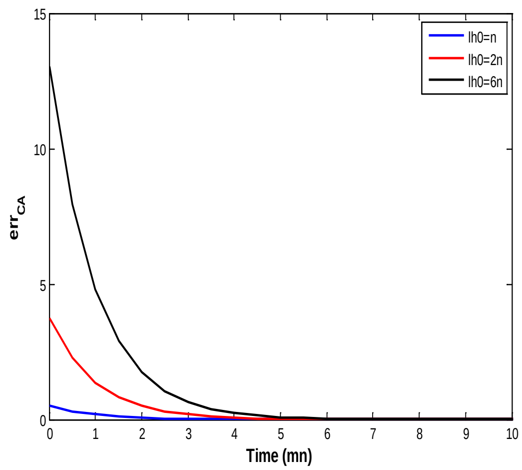

Fig. 7 The concentration error (test on lh0).

Fig. 7 shows that the values of the relative error obtained for *lh0=nTe* and *lh0 =2nTe* are acceptable. However, for an excessive length, *lh0 =6nTe*, the error increases strongly, In fact, if we study the dynamics of the system, the latter evolves and naturally responds in a certain time t_m called response time. In the sense of controllability, one must take a length *lh0* lower than the system response time to excite the system and see all these moved states. Thus, the value t_m can only be the system response time and the length of the horizon is bounded by $nTe \leq lh0 \leq t_m$.

Robustness of the observer

To test the robustness of the observer in the presence of a measurement noise, a Gaussian noise is added to the measurements.

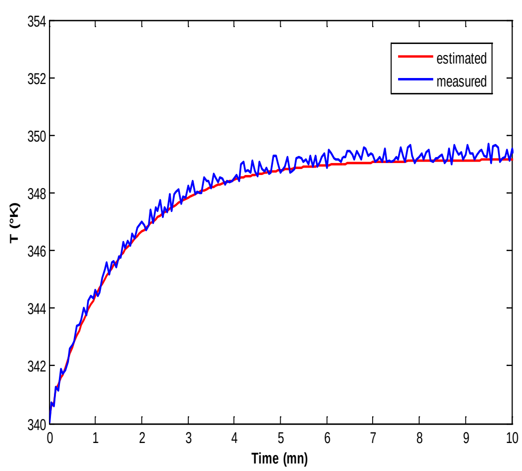

Fig. 8 Estimation of the temperature T in the presence of a measurement noise

In the presence of noise on the outputs and for some amplitude, the estimates show an excellent compatibility between the simulated values and the results of the estimates of the states. In fact, we see, in fig. 8, the MHSE method characterizes the trajectory of the state without problem and compensates and filters these disturbances; that is how the robustness of the estimator is recognized.

4. Conclusion

The main result of this paper is that by reformulating the problem by viewing an optimization problem, we can synthesize observers for nonlinear systems. Synthesis observer is by coupling the sliding horizon technique and the Levenberg-Marquardt optimization algorithm. This synthetic method has been illustrated by an example of the CSTR reactor and the observer has been tested simulation. We expect the future to push the study of moving horizon estimator to the use of global optimization algorithms such as genetic algorithms and simulated annealing.

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References

[1] M. Darouach, “Linear functional observers for systems with delays in the state variables”. IEEE transactions on Automatic Control, vol. 46, no. 7, pp. 491-497, 2001.
[2] J.P. Gauthier and I.A. Kupka, “Observability and observers for nonlinear systems, application to bioreactors”, SIAM. Journal: Control and Optimisation, vol. 32, no. 4, pp.975-994, 1994.
[3] D.G. Luenberger, “An introduction to Observers”, IEEE transactions on Automatic Control, Vol. 16, pp. 596-602, 1971.
[4] N. Kazantzis and C. Kravaris, “Nonlinear observer design using Lyapunov’s auxiliary theorem”, Systems Control Letters, vol. 34, no.5, pp. 241–247, 1998.
[5] K. Zhang, B. Jiang and V. Cocquempot, “Adaptive Observer-based Fast Fault Estimation”, International Journal of Control, Automation, and Systems, vol. 6, pp. 320-326, 2008.
[6] K. Hyun-Sik and L. Doheon, “Intelligent PSR Estimator for Feature Extraction of a Passive Sonar Target”, International Journal of Control, Automation, and Systems, vol. 8, pp. 677-682, 2010.
[7] Kumar and P. Daoutidis, “Nonlinear dynamics and control of process systems with recycle”, Journal of Process Control, vol.12, pp. 475–484, 2002.

[8] M. Themans, N. Zufferey and M. Bierlaire, "Optimisation non linéaire globale avec application aux modèles de choix discret", Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne, 2000.
[9] R. Marino and P. Tomei, "Global adaptative observers and output-feedback stabilization for class nolinear systems", Lecture Note in Control and Information Sciences, vol. 160, pp.455-493, 1991.
[10] K. Holmstrom, "The TOMLAB optimization environment in MATLAB, in: Advanced Modeling and Optimization", pp. 47–69, 1999.
[11] M. Farina, G. Ferrari-Trecate and R. Scattolini, "Distributed moving horizon estimation for nonlinear constrained systems", International Journal of Robust and Nonlinear Control, vol. 22, pp. 123–143, 2012.
[12] V.M. Zavala, C.D. Laird and L.T. Biegler, "A fast moving horizon estimation algorithm based on nonlinear programming sensitivity", Journal of Process Control, vol. 18, pp. 876–884, 2008.
[13] L.P. Russo and R.E.Young, "Moving-Horizon State Estimation Applied to an Industrial Polymerization Process", Proceedings of the American Control Conference. California, vol. 2, 1999.
[14] L. Boillereaux, " Estimation d'Etat de Procédés Non-Linéaires : Méthode à Horizon Glissant avec Indicateur de Qualité", Thèse de Doctorat, Laboratoire d'Automatique de Grenoble, INPG, Grenoble, 1996.
[15] B.M. Wilamowski and H. Yu, "Improved Computation for Levenberg Marquardt Training", IEEE Transactions on Neural Networks, vol. 21, no. 6, pp. 930-937, 2010.
[16] G. Ciccarella, M. Dalamora and A. Germani. "A Luenberger-like observer for nonlinear systems", International Journal of Control, Vol. 57, no. 3, pp. 537-556, 1993.
[17] M.T. Hagan and M.B. Menhaj, "Training feedforward networks with the Marquardt algorithm", IEEE Transactions on Neural Networks, vol. 5, no. 6, pp. 989-993, 1994.
[18] Ghouati and J.C. Gelin, "Gradient based methods, genetic algorithms and the finite element method for the identification of material parameters", IEEE Theory Methods and Applications, Vol. 1, pp.157-162, 1996.
[19] M. Barkhordari Yazdi and M.R. Jahed-Motlagh, "Stabilization of a CSTR with two arbitrarily switching modes using modal state feedback linearization", Chemical Engineering Journal, Vol. 155, pp. 838–843, 2009.

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