Vector Median Filters: A Survey

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Abstract
In this paper, a comprehensive survey on vector median filters to remove the adverse effect due to impulse noise from color images is presented. Color images are nonstationary vectored value signals. Hence, nonlinear filters such as vector median filters are more effective than linear filters. A number of nonlinear filters are proposed in the literature. They have been categorized into 12 groups and discussed in details.

Keywords
Impulse noise, Vector median filter, Quaternion, Nonlinear, Sigma vector filter, Entropy vector filter.

1. Introduction

Color images are widely used on daily basis in printing, photographs, computer displays, television and movies and thus color image processing plays a crucial role in the field of advertising and dissemination of information [1]. The use of color images is increasing in medical images and remote sensing. Color information are being used in many color image processing applications like object recognition, image matching, content-based image retrieval, computer vision, color image compression, etc. [2]. Color images are corrupted by noise due to malfunctioning of sensors, electronic interference, imperfect optics, or fault in the data transmission process. Noise introduces color fluctuation making pixel values different from the ideal values and thus produces errors which complicate the subsequent stages of the image processing process [3]. A filter transforms a signal into a more suitable form for a specific purpose [4]. Filtering gives an estimate of signal degraded by noise. Since color images are nonstationary in nature due to the presence of edges and fine details, and also the human visual system is nonlinear, nonlinear filters are preferred more than linear filters. A color image can be treated as a mapping for 2D to 3D [5]. The three color channels exhibit strong spectral correlation. Marginal or component-wise filtering methods which process each channel independently produce images containing color shifts and other serious artifacts. In vector filtering techniques the input pixels are treated as a set of vectors and no new colors are introduced. Hence theyare able to preserve the correlation among the channel components [5-8]. Thus an efficient filter aims at processing of color images with respect to its trichromatic nature, nonlinear characteristics and noise corruption statistics.

Impulse noise is a high energy spikes having large amplitude with probability greater than that predicted by Gaussian density model occurring for a short duration. It is required to remove noise in the preprocessing stage to prevent degradation of image quality.Many nonlinear filters have been proposed in the literature for removing impulse noise. In this study, a large number of nonlinear filters are categorized into 12 families.

This paper aims at summarizing the recent developments in the vector median filters for removal impulse noise from color images. Section 2 describes the categories of vector median filters. In Section 3, a commonly used impulse noise model is described. Popular filtering performance criteria for evaluation filters are given in Section 4 followed by conclusion in Section 5.

2. Category of Filters

In this section, the commonly used filters used for removing impulse noise are grouped into 12 categories. They are

1. Basic Vector Filters
2. Weighted Vector Filters
3. Adaptive Vector Filters
4. Peer Group Vector Filters
5. Fuzzy Vector Filters
6. Hybrid Vector Filters
7. Sigma Vector Filters
8. EntropyVector Filters
9. Quaternion based Vector Filters
10. Morphological based vector median filters
11. Wavelet based median filters
12. Miscellaneous Filters

2.1 BasicVector Filters

The reduced vector or aggregated ordering technique is the most common filtering approach. In this method the aggregated distance of a sample pixel $\boldsymbol{x}_i$ inside a sliding filtering window W of length N, usually a finite odd number, is computed as follows:

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$D_i = \sum_{j=1}^{N} \rho(\boldsymbol{x}_i, \boldsymbol{x}_j),\ i = 1, \ldots, N$ (1)

in which ρ(.) represents the distance or dissimilarity function, $\boldsymbol{x}_i = (x_{i1}, x_{i2}, x_{i3})$ and $\boldsymbol{x}_j = (x_{j1}, x_{j2}, x_{j3})$ for three channels. The sorting of aggregated distance $s\ D_1, D_2 \ldots, D_N$ in ascending order represents same ordering of the associated vectors

$D_{(1)} \leq D_{(2)} \leq \cdots \leq D_{(N)} \Rightarrow \boldsymbol{x}_{(1)} \leq \boldsymbol{x}_{(2)} \leq \cdots \leq \boldsymbol{x}_{(N)}$ (2)

2.1.1 Vector Median Filter (VMF)

In [9] Vector Median filter (VMF), Generalized Vector Median Filter (GVMF) and Extended Vector Median Filter (EVMF) are introduced for processing vector-valued signals having properties similar with median filters operation such as zero impulse response and good smoothing ability while preserving sharp edges in the signal. They are based on the concept of nonlinear order statistics and derived as maximum likelihood estimates from exponential distributions. Since vectors which vary greatly from the data population correspond to the maximum aggregated magnitude difference, the VMF output is the lowest ranked vector with minimum aggregated distance to the input vectors present inside the window. If $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N$ represent the vectors inside the filtering window W, the vector median is computed as follows:

a) For each vector element $\boldsymbol{x}_i$ calculate the sum of distances to all other vectors inside the filtering window, using the Minkowski metric (either the L_1or L_2 norm) and add them together to get sum of distances S_i

$S_i = \sum_{j=1}^{N} \left\| \boldsymbol{x}_i - \boldsymbol{x}_j \right\|_{\gamma}$ (3)

where $\gamma = 1$ for city block distance and $\gamma = 2$ for Euclidean distance.

b) Find a parameter min such that S_{min} denotes the minimum S_i.

c) Corresponding to S_{min} , $\boldsymbol{x}_{min} = \boldsymbol{x}_{(1)}$ represents the vector median $\boldsymbol{x}_{VMF}$.

2.1.2 α-trimmed Vector Median Filter (α-VMF)

In α-VMF a trimming operation is incorporated in which (1+α) nearest samples to the vector median are given as input to an average filter. The output is defined as follows [9, 10]:

$\boldsymbol{x}_{\alpha\mathrm{VMF}} = \sum_{i=1}^{1+\alpha} \frac{1}{(1+\alpha)} \boldsymbol{x}_i, \quad \alpha \in [0, N-1]$ (4)

The trimming operation enhances in removing the long tailed or impulsive noise while the averaging filter performs well with Gaussian noise.

2.1.3 Extended Vector Median Filter (EXVMF)

EXVMF combines the vector median operation with an averaging filter. EXVMF of $\boldsymbol{x}_i, \ldots \boldsymbol{x}_N$ is denoted as $\boldsymbol{x}_{EXVMF}$ such that [9, 10]

$$\boldsymbol{x}_{EXVMF} = \begin{cases} \boldsymbol{x}_{AVE}, & if\ \sum_{i=1}^{N} \| \boldsymbol{x}_{AVE} - \boldsymbol{x}_i \|_2 < \sum_{i=1}^{N} \| \boldsymbol{x}_{VMF} - \boldsymbol{x}_i \|_2 \\ \boldsymbol{x}_{VMF}, & \text{otherwise} \end{cases} \quad (5)$$

where $\boldsymbol{x}_{AVE} = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{x}_i$ and $\boldsymbol{x}_{VMF}$ is the vector median output. It behaves like VMF near the edges while in smooth areas it behaves like the Arithmetic Mean Filter (AMF).

2.1.4 Generalized Vector Median Filter (GVMF)

The GVMF [11] of vectors $\boldsymbol{x}_i, \ldots \boldsymbol{x}_N$ is vector $\boldsymbol{x}_{GVMF}$ such that $\boldsymbol{x}_{GVMF} \in \{ x_i \mid i = 1, \ldots, N \}$ and for all $j = 1, \ldots, N$ satisfying the condition

$\sum_{i=1}^{N} d(\boldsymbol{x}_{GVMF} - \boldsymbol{x}_i) \leq \sum_{i=1}^{N} d(\boldsymbol{x}_j - \boldsymbol{x}_i)$ (6)

where $d(\boldsymbol{x}, \boldsymbol{y})$ is the distance between the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$.

2.1.5 Fast Modified Vector Median Filter (FMVMF)

In [12] a new filter similar with VMF is developed whose computational complexity is lower than that of VMF. The distance associated with central pixel $\boldsymbol{x}_{\frac{N+1}{2}}$ is denoted by $d_{(N+1)/2} = -\beta + (\sum_{j=1}^{\left(\frac{N+1}{2}\right)-1} d(\boldsymbol{x}_{\frac{N+1}{2}}, \boldsymbol{x}_j) + \sum_{j=\left(\frac{N+1}{2}\right)+1}^{N} d(\boldsymbol{x}_{\frac{N+1}{2}}, \boldsymbol{x}_j)$ where β is a threshold parameter and the distance associated with the neighbors of $\boldsymbol{x}_{\frac{N+1}{2}}$is given as $d_i = \sum_{j=1}^{N} d(\boldsymbol{x}_i, \boldsymbol{x}_j), i = 1, 2, 3, \ldots, N,$ excluding $(N+1)/2$. Then for some k, if d_k is smaller than $d_{(N+1)/2}$ i.e. $d_k = \sum_{j=1}^{N} d(\boldsymbol{x}_k, \boldsymbol{x}_j) < d_{(N+1)/2}$, then $\boldsymbol{x}_{\frac{N+1}{2}}$ is replaced by $\boldsymbol{x}_k$.

2.1.6 Directional Vector Median Filter (DVMF)

In this filter, four vector median are applied across the four main directions of the filtering window at $0^{\circ}, 45^{\circ}, 90^{\circ}and135^{\circ}$to obtain four vector median output $\boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{y}_3$ and $\boldsymbol{y}_4$. In the second stage, the final output is generated by applying another vector median on the four filtered results. Hence the Directional Vector Median Filter (DVMF) output is denoted as [13]

$\boldsymbol{x}_{DVMF} = \boldsymbol{y}_{(1)}$ (7)

where $\boldsymbol{y}_{(1)}$ is the vector median of $\boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{y}_3$ and $\boldsymbol{y}_4$. DVMF is effective in removing impulsive noise while preserving thin lines.

2.1.7 Rank Conditioned Vector Median Filter (RCVMF)

It incorporates a decision making process in which every pixels in the filtering window is assigned a rank depending on the ordered distance. In RCVMF [14] the output is the vector median when the rank of central pixel is larger than a predefined rank of uncorrupted vector pixels in the filtering window. Mathematically it is denoted as

$$\boldsymbol{x}_{RCVMF} = \begin{cases} \boldsymbol{x}_{VMF}, & if\ r_{(N+1)/2} > r_k \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \quad (8)$$

where $r_{(N+1)/2}$ denotes rank of center pixel and r_k is the rank of predefined healthy pixel.

2.1.8 Rank Conditioning and Threshold Vector Median Filter (RCTVMF)

In [14], a new improvement in the RCVMF is proposed in which the distance d between the central pixel and predefined healthy pixel is used as additional criteria for impulse detection.

$$\boldsymbol{x}_{RCTVMF} = \begin{cases} \boldsymbol{x}_{RCTVMF}, & if\ r_{\frac{N+1}{2}} > r_k \text{ and } D > T \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \quad (9)$$

where $D = \Delta(\boldsymbol{x}_{\frac{N+1}{2}}, \boldsymbol{x}_{(k)})$ denotes the distance between central pixel and neighboring healthy pixels in which $\boldsymbol{x}_{(k)}(1 < k < N)$ is a rank-ordered and healthy vector pixel. T is a pre-determined threshold. If the central pixel is detected as impulse, it is replaced by the vector median output.

2.1.9 Crossing Level Median Mean Filter (CLMMF)

Crossing Level Median Mean Filter [15] combines the idea of the VMF and Arithmetic Mean Filter (AMF) which is based on the vector ordering technique. If w_i denotes the weight of i^{th} elements of the ordered vectors $\boldsymbol{x}_{(1)}, \boldsymbol{x}_{(2)}, \dots \boldsymbol{x}_{(N)}$, the filtered output is given as follows:

$$\boldsymbol{x}_{CLMMF} = \sum_{i=1}^{N} w_{(i)}.\boldsymbol{x}_{(i)} \quad (10)$$

where

$$w_{(i)} = \begin{cases} 1 - \frac{N}{\sqrt{(N+1)(N+1+\gamma)}}, \text{for } i = 1 \\ \frac{1}{\sqrt{(N+1)(N+1+\gamma)}}, \text{ for } i = 2, \dots, N \end{cases} \quad (11)$$

where γ represents the parameter of the filter which resembles the standard VMF for γ→ ∞ and AMF for γ= 0.

2.1.10 Vector Filters based on Non-Causal (NC) linear prediction technique

A group of switching filters based on noncausal linear prediction is introduced in [16]. NC gives an estimate of the current pixel based on the past and future pixel values in the neighborhood of the current pixel by using a block-by-block autocorrelation function. The difference between the predicted pixel and the original current pixel is used as a measure for impulse detection.

The predicted pixel value at central location (r, c) is computed as

$$\hat{\boldsymbol{x}}(r,c) = \sum_{(i,j)\in W_2} a(i,j).\boldsymbol{x}(r-i, c-j) = \boldsymbol{X}_\eta \boldsymbol{a}_\eta \quad (12)$$

where $\boldsymbol{a}_\eta$ represents the vector obtained from the prediction coefficients, W_2 is the noncausal region for linear prediction, $\boldsymbol{X}_\eta$ denotes the matrix of vector pixels used for prediction and η is the order of prediction.

Then the predictor decides if the current sample is corrupted or not and is replaced by the vector median if the predicted error $e(r,c)$ exceeds a pre-defined threshold $\boldsymbol{T}$. Hence the output of the noncausal linear prediction based vector filter $\boldsymbol{x}_{NCVF}$ is denoted by

$$\boldsymbol{x}_{NCVF} = \begin{cases} \boldsymbol{x}_{VMF}, & \text{if } \| e(r,c) \| > T \\ \boldsymbol{x}(r,c), & \text{otherwise} \end{cases} \quad (13)$$

2.1.11 Basic Vector Directional Filter (BVDF)

It is a rank ordered filter in which the angle between two vectors is used as the distance measure. The vectors with atypical directions are regarded as an outlier and filtering is done similar with the VMF. The aggregated sum of angles between the vectors is given by

$$\theta_i = \sum_{j=1}^{N} A(\boldsymbol{x}_i, \boldsymbol{x}_j), i = 1, \dots, N \quad (14)$$

where

$$A(\boldsymbol{x}_i, \boldsymbol{x}_j) = \cos^{-1}\left(\frac{\boldsymbol{x}_i.\boldsymbol{x}_j}{\|\boldsymbol{x}_i\|\|\boldsymbol{x}_j\|}\right) \quad (15)$$

where $A(\boldsymbol{x}_i, \boldsymbol{x}_j)$ represents the angle between the vectors $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$. The angles are ordered which correspond to ordering of the input vectors as follows:

$$\theta_{(1)} \le \theta_{(2)} \le \cdots \theta_{(r)} \dots \le \theta_{(N)} \rightarrow \boldsymbol{x}_{(1)} \le \boldsymbol{x}_{(2)} \le \cdots \boldsymbol{x}_{(r)} \dots \le \boldsymbol{x}_{(N)}$$

The output of BVDF [17] is the vector $\boldsymbol{x}_i$ whose angular distance to all other vector in the window is minimum. BVDF preserves chromaticity better than the VMF since vector's direction corresponds to its chromaticity. BVDF considers only directional processing and is effective when vector magnitudes are less important than the vectors direction which is not suitable for multichannel signal processing.

2.1.12 Generalized Vector Directional Filter (GVDF)

The above mentioned filters do not consider both the unique features of a vector namely direction and magnitude together which may produce erroneous results. GVDF [18] is a generalization of BVDF. In the first step, a set of low- rank vectors that is the first r terms are selected from the ordered set of vectors based on the aggregated sum of angular distance as opposed to the

BVDF in which a single vector with the minimum aggregated angular sum is selected. Next these vectors are given as an input to an additional filter, e.g. alpha-trimmed average filter, multistage median filter or morphological filters which consider magnitude processing. Hence GVDF considers both the directional and magnitude processing.

2.1.13 Directional Distance Filter (DDF)

An improved filter is achieved by combining VMF and VDF which is known as Directional Distance Filter (DDF) [18,19]. DDF considers the vector's direction and magnitude in which both the vector's chromaticity and intensity are considered. The combined aggregated measure is defined as follows:

$$\Omega_i = \left(\sum_{j=1}^{N} \| \boldsymbol{x}_i - \boldsymbol{x}_j \|_\gamma\right)^{1-p} . \left(\sum_{j=1}^{N} A(\boldsymbol{x}_i, \boldsymbol{x}_j)\right)^p \quad (16)$$

where $p \in (0, 1)$ is a parameter which tunes the influence of magnitude and angle quantities.

2.1.14 Filters based on Hopfield Neural Network and Improved Vector Median Filter

In this filter, the noise detection in done using a Hopfield neural network (HNN) and in the second stage, the noisy pixels are replaced by an improved Vector Median Filter first in RGB space and then in HSI space [20]. For the improved VMF, the steps are given by

1. The vector median is computed in RGB space inside the filtering window.
2. All the pixels fit for being median inside the filtering window are collected.
3. If more than one pixel is fit for being median, then select that particular pixel which is nearest to the mean of the Hue in HSI space
4. In Step 3 if more than one pixel is qualified, then the pixel which is nearest to the mean of saturation in HSI space is selected.

2. 2 Weighted Vector Filters

Weighted Vector Filters are extension of Weighted Standard Median Filters in which a non-negative weight is assigned to every pixel inside the filtering window offering more flexibility.

2.2.1 Weighted Vector Median Filter (WVMF)

WVMF is a generalization of VMF in which each pixel x_i in the filter window is assigned a positive integer weight. The weight controls the filtering behavior while offering greater flexibility than the median-based filter. If $\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N$ are vectors inside the filtering window and $w_1, w_2, \dots, w_N$ are the corresponding nonnegative integer-valued weights, then WVMF is the vector $\boldsymbol{x}_{WVM}$ such that [10, 21] $\boldsymbol{x}_{WVM} \in \{\boldsymbol{x}_i; i = 1, \dots, N\}$ and for all $j = 1, \dots, N$

$$\sum_{i=1}^{N} w_i \| \boldsymbol{x}_{WVM} - \boldsymbol{x}_i \|_\gamma \le \sum_{i=1}^{N} w_i \| \boldsymbol{x}_j - \boldsymbol{x}_i \|_\gamma \quad (17)$$

It can be summarized as follows: sort the pixels inside the filtering window depending on the value of vector median, duplicate each pixel $\boldsymbol{x}_i$ to the number of their corresponding weight w_i and the median value from the new sequence represents the weighted vector median.

2.2.2 α-Trimmed Weighted Vector Median Filter (α-TWVMF)

α-TWVMF [21] of vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N$ having weights $w_1, w_2, \dots, w_N$ is defined as

$$\boldsymbol{x}_{\alpha-TWVM} = \begin{cases} \boldsymbol{x}_\alpha, \text{if} \sum_{i=1}^{N} w_i \| \boldsymbol{x}_\alpha - \boldsymbol{x}_i \| < \sum_{i=1}^{N} w_i \| \boldsymbol{x}_{WVM} - \boldsymbol{x}_i \| \\ \boldsymbol{x}_{WVM}, \quad \text{otherwise} \end{cases} \quad (18)$$

where $\boldsymbol{x}_\alpha = \frac{1}{|S_\alpha|}\sum_{\boldsymbol{x}_i \in S_\alpha} \boldsymbol{x}_i$ and $S_\alpha = \{\boldsymbol{x}_i; \text{having } S_i < S_{(N-\alpha)}\}$ S_i is the sum of weighted from vector $\boldsymbol{x}_i$ to all other vectors $\boldsymbol{x}_j, j = 1, \dots, N$. $|S_\alpha|$ represents the number of elements in S_α and $S_{(i)}$ is the i^{th} smallest of $S_1, \dots S_N$. α can have any value 0, 1, ………., N-1.

2.2.3 Extended Weighted Vector Median Filter (EXWVMF)

The EXWVMF [21,22] is an extension of WVMF which is defined as

$$\boldsymbol{x}_{EXWVMF} = \begin{cases} \boldsymbol{x}_{WAVE}, \ \text{if} \sum_{i=1}^{N} w_i \| \boldsymbol{x}_{WAVE} - \boldsymbol{x}_i \| < \sum_{i=1}^{N} w_i \| \boldsymbol{x}_{WVM} - \boldsymbol{x}_i \| \\ \boldsymbol{x}_{WVM}, \ \text{otherwise} \end{cases}$$

(19)

where $\boldsymbol{x}_{WAVE} = \frac{1}{\sum_{i=1}^{N} w_i}\sum_{i=1}^{N} w_i \boldsymbol{x}_i$ (20)

EWVMF chooses the average as output in the smooth areas while it chooses weighted vector median (WVM) near edges.

2.2.4 Rank Order Weighted Vector Median Filter (ROWVMF)

In [23] an adaptive noise attenuating and edge enhancing filter based on the minimization of aggregated weighted distances among pixels in window is proposed. The distance between a pixel $\boldsymbol{x}_i$ and all other pixels inside the filtering window is ordered to obtain $d_{i(r)}$ by assigning a rank r

$$d_{i1}, d_{i2}, \dots, d_{iN} \to d_{i(1)},\ d_{i(2)}, \dots, d_{i(N)}$$

A weighted sum of distances is computed by utilizing the distance ranks denoted as follows:

$$\Lambda_i = \sum_{r=1}^{N} f(r). d_{i(r)} \quad (21)$$

where $f(r)$ denotes a constantfunction associated with the distance rank r. All the weighted aggregated distances Λ_i are sorted forming a new order of vectors

$\Lambda_{(1)}, \Lambda_{(2)}, \dots, \Lambda_{(N)} \rightarrow \boldsymbol{x}^*_{(1)}, \boldsymbol{x}^*_{(2)}, \dots, \boldsymbol{x}^*_{(N)}$

The output of the ROWVMF is the vector $\boldsymbol{x}^*_{(1)}$. Another filter called Rank-based Vector Median Filter having similar concept is also designed in [24].

2.2.5 Weighted Vector Directional Filters (WVDF)

It employs a nonnegative real weighing coefficient $\{w_1, w_2, \dots w_N\}$ corresponding to vector elements $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots \boldsymbol{x}_N\}$ with filter output $\boldsymbol{x}_{WVDF} = \boldsymbol{x}_i \in W$ which minimizes the aggregated weighted angular distance with other vector pixels given by [25, 26]

$$\boldsymbol{x}_{WVDF} = \underset{x_i \in W}{argmin} \sum_{j=1}^{N} w_j A(\boldsymbol{x}_i, \boldsymbol{x}_j) \quad (22)$$

where $A(\boldsymbol{x}_i, \boldsymbol{x}_j)$ represents the angle between two vectors. Similarly Weighted Directional Distance Filter (WDDF) is also obtained using both the magnitude and angular distance criteria.

2.2.6 Genetic Algorithm based Weighted Vector Directional Filter (GA WVDF)

An optimized WVDF based on Genetic Algorithm is designed in [27] which adapts the filter weights in order to match the varying image and noise characteristics. Since GA-based methods search the entire solution space, they are able to provide a globally optimal (or very close) solution as compared with other optimization techniques. Another filter based on Genetic Programming (GP) is developed in [28] aiming at the removal of mixed noise. The estimator is based on the global learning capability of GP and measurement of local statistical properties of the healthy pixels present in the surrounding of the corrupted pixel.

2.2.7 Center-Weighted Vector Median Filter (CWVMF)

In WVMF when the center weight is varied and the others remain fixed, a new class of filter is formed called the Center weighted Vector Median Filter (CWVMF) [29,30] is formed. It is defined as

$$\boldsymbol{x}_{CWVMF} k = \underset{x_i \in W}{\operatorname{argmin}} \left(\sum_{j=1}^{N} w_j(k). \parallel \boldsymbol{x}_i - \boldsymbol{x}_j \parallel \right) \quad (23)$$

with $w_j(k) = \begin{cases} N - 2\kappa + 2, & \text{for } j = (N+1)/2 \\ 1, & \text{otherwise} \end{cases}$

where only the central weight $w_{(N+1)/2}$ is varied with smoothing parameter κ.

2.3 Adaptive Vector Filters

VMF and its variants result in fixed amount of smoothing leading to blurring of edges and fine details since they perform filtering operation on all pixels which may not be noisy. Also noise characteristics varies in the image and hence nonadaptive filters have low performance. Adaptive filters are introduced to handle the difficulty of varying noise characteristics by implementing estimation procedures based on the nature of data on local image statistics [3]. The coefficients of filter kernel change values depending on the image structure which is to be smoothed.

2.3.1 Adaptive Vector Median filter (AVMF)

In this filter desired features are made invariant to filtering operation while the noisy pixels are effected by altering between VMF and the identity operation. This is based on the decision rule expressed as follows [31]

if $Val \geq Tol$, then $\boldsymbol{x}_{\frac{N+1}{2}}$ is impulse

else $\boldsymbol{x}_{\frac{N+1}{2}}$ is noise free

where Val is the vector distance between the central pixel $\boldsymbol{x}_{\frac{N+1}{2}}$ and the mean of the first r vector order statistics $\boldsymbol{x}_{(1)}, \boldsymbol{x}_{(2)}, \dots, \boldsymbol{x}_{(r)}$ associated with the ordered distances $L_{(1)}, L_{(2)}, \dots, L_{(r)}$, for $r \leq N$. Tol is a prespecified threshold value. Mathematically, Val is denoted by

$$Val = \left\| \boldsymbol{x}_{\frac{N+1}{2}} - \frac{1}{r} \sum_{i=1}^{r} \boldsymbol{x}_{(i)} \right\|_{\gamma} \quad (24)$$

where γ characterizes the norm used in Minkowski metric. If noise is detected the central pixel is replaced by the vector median output.

Another adaptive filter is the Adaptive Basic Vector Directional Filter (ABVDF) which is the angular counterpart of AVMF.

2.3.2 Adaptive based Impulsive Noise Removal Filter

In [32], a new adaptive filtering scheme is proposed. The impulse is detected based on the difference between the aggregated distance assigned to central pixel and the vector median output. The output of the filter is given as follows:

$$\boldsymbol{x}_{output} = \alpha \boldsymbol{x}_{\frac{N+1}{2}} + (1 - \alpha) \boldsymbol{x}_{(1)} \quad (25)$$

where α is a filter parameter and $\boldsymbol{x}_{(1)}$ is the VMF output. The value of α is 0 when an impulse is present otherwise it is 1.

2.3.3 Multiclass Support Vector Machine based Adaptive Filter (MSVMAF)

A new filter called Multiclass Support Vector Machine based Adaptive Filter (MSVMAF) is developed in [33] for reducing high density impulse noise. It takes the advantages of both multiclass Support Vector Machine (SVM) as well as adaptive vector median filtering.

2.3.4 Adaptive Threshold and Color Correction (ATCC) Filter

For removing random-valued impulse noise, a new filter named Adaptive Threshold and Color Correction (ATCC) filter is proposed in [34]. It has an adaptive threshold which is computed on the basis of local pixel statistics within the sliding window.

2.3.5 Robust Switching Vector Filter (RSVF)

In this filter, the pixels in the window are ordered according to the distance measure used in VFM, VDF and DDF. A pixel is detected as corrupted if the cumulative distance $d_{(N+1)/2}$ of the central pixel is larger than the median cumulative distance of the neighborhood. The output of the filter is one of the outputs of VMF, VDF and DDF as follows:

$$\boldsymbol{x}_{RSVF} = \begin{cases} \boldsymbol{x}_{(N+1)/2}, & \text{if } d_{(N+1)/2} \leq \text{med } \alpha.(d_1, \dots, d_N) < 0 \\ \boldsymbol{x}_{RSVF}, & \text{otherwise} \end{cases} \quad (26)$$

where $med(.)$ is a robust univariate median operator and α is a filter parameter used for preserving image details and smoothing. If $d_i = \sum_{j=1}^{N} L_\gamma(\boldsymbol{x}_i, \boldsymbol{x}_j)$ the output is denoted by the VMF ($\boldsymbol{x}_{RSVF} = \boldsymbol{x}_{RSVMF}$) to obtain Robust Switching Vector Median Filter (RSVMF). If $d_i = \sum_{j=1}^{N} A(\boldsymbol{x}_i, \boldsymbol{x}_j)$, $\boldsymbol{x}_{RSVF} = \boldsymbol{x}_{RSBVDF}$ to represent Robust Switching Basic Vector Directional Filter (RSBVDF) while for $d_i = (\sum_{j=1}^{N} A(\boldsymbol{x}_i, \boldsymbol{x}_j))^{\gamma} (\sum_{j=1}^{N} L_p(\boldsymbol{x}_i, \boldsymbol{x}_j))^{1-\gamma}$, $\boldsymbol{x}_{RSVF} = \boldsymbol{x}_{RSDDF}$ which is obtained by the Directional Distance filter output to represent Robust Switching Directional Distance filter (RSDDF) [35,36].

2.3.6 Adaptive Marginal Median filter (AMMF)

A new modification to Vector Marginal Median Filter (VMMF) is designed in [4] which aims at integrating the noise reduction capability of VMMF as well as preserving the vector correlation resulting from VMF. From the ordered aggregated distance used in VMF $d_{(1)}, d_{(2)}, \dots d_{(N)} \rightarrow \boldsymbol{x}_{(1)}, \boldsymbol{x}_{(2)}, \dots \boldsymbol{x}_{(N)}$, select a set of vectors S constituted by m vectors which are most similar to the Vector Median $\boldsymbol{x}_{(1)}$ such that $S = \{\boldsymbol{x}_{(1)}, \boldsymbol{x}_{(2)}, \dots \boldsymbol{x}_{(m)}\}$ for $m \leq N$. Then the Vector Marginal Median filter is applied to this set to achieve high noise reduction. The output of the marginal median filter is given as follows [37]

$$\boldsymbol{x}_{AMM} = \left(\left(med(\{x_{(1)}^R, \dots x_{(N)}^R\}) \right), \left(med(\{x_{(1)}^G, \dots x_{(N)}^G\}) \right), \left(med(\{x_{(1)}^B, \dots x_{(N)}^B\}) \right) \right) \quad (27)$$

2.3.7 Adaptive Center-Weighted Vector Filter

To provide more flexibility for modification in the size and shape of the window, adaptive center weighted vector filters are designed in [29,38,39]. They are based on user-defined threshold for detection of impulses. If the central pixel is detected as corrupted, it is replaced by one of the output of VMF, BVDF and DDF forming ACWVMF, ACWBVDF and ACWDDF. The mathematical expressions of the corresponding filters are given below

$$\boldsymbol{x}_{ACWVMF} = \begin{cases} \boldsymbol{x}_{VMF}, & \text{if } \sum_{k=\lambda}^{\lambda+2} \| \boldsymbol{x}_{CWVMF} k - \boldsymbol{x}_{\frac{N+1}{2}} \| > T \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \quad (28)$$

$$\boldsymbol{x}_{ACWBVDF} = \begin{cases} \boldsymbol{x}_{BVDF}, & \text{if } \sum_{k=\lambda}^{\lambda+2} A(\boldsymbol{x}_{CWBVDF} k - \boldsymbol{x}_{\frac{N+1}{2}}) > T \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \quad (29)$$

$$\boldsymbol{x}_{ACWDDF} = \begin{cases} \boldsymbol{x}_{DDF}, & \text{if } \sum_{k=\lambda}^{\lambda+2} A^{\gamma}(\boldsymbol{x}_{CWDDF} k - \boldsymbol{x}_{\frac{N+1}{2}}) \| \boldsymbol{x}_{CWDDF} k - \boldsymbol{x}_{\frac{N+1}{2}} \|^{1-\gamma} > T \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \quad (30)$$

where $\lambda \in [1, \frac{N+1}{2} - 1]$

2.3.8. Modified switching median filter (MSMF)

It is an extension of the VMF and AVMF consisting of two-stage noise detector [40]. In the first stage, the probably noisy candidates are detected using the AVMF detection procedure as follows

$$\boldsymbol{y}^{MSMF} = \begin{cases} \boldsymbol{y}^{MSMF}_{step\ 2}, & \left\| \boldsymbol{x}_{\frac{N+1}{2}} - \frac{1}{r} \sum_{i=1}^{r} \boldsymbol{x}_{(i)} \right\|_{\gamma} \geq Tol \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \quad (31)$$

In the second phase for preserving the edge pixels, these noise candidates are again judged by four Laplacian operators in which the central pixel is convolved with four convolution kernals. For edge detection, the minimum difference of the four convolutions denoted by Z is used and the noisy samples are replaced by the VMF. The output is defined as follows:

$$\boldsymbol{Z} = \min \left\{ \boldsymbol{x}_{\frac{N+1}{2}} * W_c \mid c = 1, \dots, 4 \right\} \quad (32)$$

$$\boldsymbol{y}^{MSMF}_{step2} = \begin{cases} \boldsymbol{y}^{VMF}, & Z \geq T \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \quad (33)$$

2.3.9 Sharpening Vector Median Filter

In [23] if the function $f(r)$ is a step-like function defined by

$$f(r) = \begin{cases} 1, & \text{for } r \leq \alpha \text{ and } \alpha \leq N \\ 0, & \text{otherwise} \end{cases} \quad (34)$$

then a new vector filter is obtained known as the Sharpening Vector Median Filter [41].

2.3.10 Adaptive rank weighted switching filter (ARWSF)

ARWSF [42] is a modification of the Rank Order Weighted Vector Median Filter which incorporates an adaptive scheme. If the rank weighted distance assigned to the central pixel $\boldsymbol{x}_{\frac{N+1}{2}}$ is denoted by $\Lambda_{\frac{N+1}{2}}$, then the difference $\delta = \Lambda_{\frac{N+1}{2}} - \Lambda_{(1)}$ is used for measuring impulse noise corruption. The output of ARWSF is given as follows:

$$\boldsymbol{y}_{ARWSF} = \begin{cases} \boldsymbol{x}_{AMF}, & \text{if } \delta > T \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \qquad (35)$$

where $\boldsymbol{x}_{AMF}$ is the output of the Arithmetic Mean Filter which is calculated using the non-corrupted pixels declared by the detector.

2.4. Peer Group Vector Filters

Peer Group Filters use the neighborhood of each pixel while building its peer group which is defined as a set constituted by the central pixel with neighboring pixels which are similar to it [43-45].

2.4.1 Peer Group Averaging (PGA) Filter

For an image I, the peer group associated with a pixel i comprises of pixels in a predefined $l \times l$ window centered at i, whose intensity is nearest with i. This pixel is then replaced by the intensity of average of the peer group. This concept is referred to as Peer Group Averaging (PGA) [44].

2.4.2 Peer Group Vector Filter

For color images, Peer Group filters [43] use the vector filter such as VMF. To develop the peer group, first the pixels in window are sorted in ascending order according to the distance between the central pixel and the neighboring pixels as follows:

$$c_i = \| \boldsymbol{x}_{\frac{N+1}{2}} - \boldsymbol{x}_i \|_\gamma \text{ for } i = 1,2, \dots N \qquad (36)$$

Then the peer group of the central pixel is computed as m pixels that rank lowest in the ordered sequence with m given by

$$m = \frac{(\sqrt{N} + 1)}{2}$$

To check the presence of impulse, the first order difference of the peer group is calculated

$$\delta_i = c_{i+1} - c_i \text{ for } i = 1,2, \dots m$$

The central pixel is declared as noisy if any one of these differences is larger than a pre-specified threshold and replaced by VMF.

In [46], a similar peer group switching filter is proposed which utilizes the statistical properties of the sorted sequence of the aggregated distance of pixels inside filtering window. Noise detection is based on the Fisher's Linear Discriminant computed on the aggregated distance and the outliers are replaced by the VMF. A modification to peer group filter is the Fast Peer Group Filters (FPGF) [47] in which the central pixel is regarded as noise free if m pixels are found to be similar. If the noise density is low, m is kept low which reduces the number of distance calculation. In [48] a new Fast Averaging Peer Group Filter (FAPGF) is designed in which a pixel is considered as noisy if the peer group consists of at least two close pixels. If this condition is not satisfied, the central pixel is replaced by the weighted average of the uncorrupted pixels from the neighborhood. In order to improve the efficiency and detection, fuzzy metric can be used for defining the peer group concept.

A two stage filter based on the fuzzy peer group concept is developed in [49] for removing Gaussian and impulse noise as well as mixed Gaussian impulse noise. It consists of a fuzzy rule –based switching impulse noise filter followed by a fuzzy averaging filter. A fuzzy peer group is defined as a fuzzy set which considers a peer group as support set and membership degree of each peer group member is given by its fuzzy similarity with respect to the central pixel. A Novel Peer Group Filter (NPGF) is proposed in [50] in which the noise detection is done in the CIELab instead RGB color space in order to enhance the noise detection ability. The peer group is formed by two different sized windows aiming at reducing the false detection of non-corrupted pixels.

2.5. Fuzzy Vector Filters

Fuzzy set concepts are suitable for dealing with ambiguity resulting from inexactness and imprecision in an image such as shape of objects, continuity of line segment and similarity of regions [2]. Also based on the fact that the human vision system is a fuzzy system, many fuzzy based filters have been designed for image enhancement. The weights of Fuzzy filtersare determined from the features of datainside the window by applying fuzzy transformation thus utilizing the local correlation [51]. This transformation can be modeled as a membership function in accordance to a specific window component.A fuzzy set is defined by the membership function $\mu_F: I \to [0,1]$ that transforms the pixels in image I to fuzzy set with degree of membership ranging between 0 and 1.

2.5.1 Fuzzy Weighted Average Filter [FWAF]

The general form of fuzzy based filters is a fuzzy weighted average [52,53] of the pixel values inside the filtering window. The output is estimated as the centroid given below

$$\boldsymbol{x}_{FWAF} = \sum_{i=1}^{N} w_i \boldsymbol{x}_i = \frac{\sum_{i=1}^{N} f(\mu_i) \boldsymbol{x}_i}{\sum_{i=1}^{N} f(\mu_i)} \qquad (37)$$

where $f(\mu_i) = \mu_i^\lambda$ denotes an adaptive function determined from the local context with membership function μ_i of the pixel x_i and λ is a parameter such that $\lambda \in [0, \propto)$. This filter should satisfy the following two constraints

(i) Each weight is a positive number i.e. $w_i \geq 0$ and

(ii) The sum of all the weights is equal to unity $\sum_{i=1}^{N} w_i = 1$.

2.5.2 Fuzzy Stack Filter

In [54] Fuzzy Stack Filters are proposed to extend the smoothing characteristics of stack filters which is based on the application of fuzzy positive Boolean function.

2.5.3 Fuzzy Vector Median Filter (FVMF)

In Fuzzy Vector Median Filter the dissimilarity distance measure used is the Minkowski metric L_γ which is fed as an input to the membership function for determining the fuzzy weights. The membership function is the exponential (Gaussian-like) form [52, 53]:

$$\mu_i = \exp[-\frac{L_\gamma(i)^r}{\xi}] \qquad (38)$$

where r denotes a positive constant and ξ is a distance threshold which control the amount of fuzziness in the weights.

2.5.4 Fuzzy Vector Directional Filter (FVDF)

In Fuzzy directional filter the vector angle criterion (angular distance) is used as the distance function and has asigmoidal membership function. The fuzzy weight associated with the vector $\boldsymbol{x}_i$ is given by [52, 53]

$$\mu_i = \frac{\xi}{(1+\exp(\alpha_i))^r} \qquad (39)$$

where α_i is the angular distance measure.

2.5.5 Fuzzy Ordered Vector Filter (FOVF)

Fuzzy Ordered Vector Filters use only a part of the vector-valued pixels which are ordered according to their corresponding fuzzy membership strengths. It is given as follows:

$$\boldsymbol{x}_{FOVDF} = \frac{1}{Z}\sum_{i=1}^{\tau} w_{(i)}\boldsymbol{x}_{(i)} \qquad (40)$$

where $Z = \sum_{i=1}^{\tau} w_{(i)}$

$w_{(i)}$ denotes the i^{th} ordered fuzzy membership function such that $w_{(\tau)} \leq w_{(\tau-1)} \leq \cdots \leq w_{(1)}$ with $w_{(1)}$ being the fuzzy coefficient having the largest membership value [52].These filters resemble fuzzy generalization of α-trimmed filters.

2.5.6 Fuzzy Hybrid Filter (FHF)

Hybrid filters combine a nonlinear filter used for suppression of noise with a fuzzy weighted linear filter. One kind of these filters proposed in [15,55] can be described as follows:

Perform a pre-processing activity for removing impulse noise from the set of pixels in the filtering window W by forming a new set $W' = \{\boldsymbol{x}'_i; i = 1, \ldots, N\}$. This is obtained by replacing the minimum and maximum luminance values by the median pixel value $\boldsymbol{x}_{VMF}$. The output of the filter is given by

$$\boldsymbol{x}_{FHF} = \frac{\sum_{i=1}^{N} \mu_\Pi(\Delta\boldsymbol{x}'_i)\boldsymbol{x}'_i}{\sum_{i=1}^{N} \mu_\Pi(\Delta\boldsymbol{x}'_i)} \qquad (41)$$

where $\Delta\boldsymbol{x}'_i = \boldsymbol{x}'_i - \boldsymbol{x}_{VMF}$ and μ_Π is the membership function which describes a Π-type (i.e. a bell-shaped) fuzzy set aiming at removing the pixels with large luminance values.

2.5.7 Adaptive Nearest-Neighbor Filter (ANNF)

Adaptive nearest-neighbor filter (ANNF) is based on the nearest neighbor rule in which the fuzzy weights are calculated as follows [56]:

$$w_i = \frac{a_{(N)} - a_{(i)}}{a_{(N)} - a_{(1)}} \text{ for } i = 1, 2, \ldots N. \qquad (42)$$

where $a_{(n)}$ is the maximum angular distance and $a_{(1)}$ represents the minimum angular distance associated with the center-most pixel inside the filtering window.

2.5.8 Adaptive Nearest-Neighbor Multichannel Filter (ANNMF)

An improvement in the ANNF is the Adaptive nearest-neighbor multichannel filter (ANNMF) which combines vector directional with vector magnitude filtering. The distance measure for noisy vector $\boldsymbol{x}_i$ is given by the following formula [57]

$$d_i = \sum_{j=1}^{N}(1 - S(\boldsymbol{x}_i, \boldsymbol{x}_j)) \qquad (43)$$

$$S(\boldsymbol{x}_i, \boldsymbol{x}_j) = \left(\frac{\boldsymbol{x}_i\boldsymbol{x}_j^t}{|\boldsymbol{x}_i||\boldsymbol{x}_j|}\right)\left(1 - \frac{||\|\boldsymbol{x}_i\| - \|\boldsymbol{x}_j\|||}{\max(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|)}\right) \qquad (44)$$

The first part of the equation represents the vector angular criteria and second part is the normalized magnitude difference. According to this equation the directional information is used when the vectors have same length while the magnitude difference part is used when the vectors have direction.

2.5.9Adaptive Hybrid Multichannel Filter (AHMF)

To achieve three objectives such as noise attenuation, chromaticity retention and edges or detail preservation a new filter named Adaptive Hybrid Multichannel Filter (AHMF) is introduced in [58]. The structure of AHMF consists of three parts: a Hybrid Multichannel (HM) filter, a fuzzy ruled-based system and a learning algorithm.HM comprises of four components: VMF, BVDF, Identity Filter (IF) and a summation combinatory.

2.5.10 Fuzzy Impulse Detection and Reduction Method (FIDRM)

For images corrupted with mixture of impulse noise and other types of noise, Fuzzy Impulse Detection and Reduction Method [59]is developed based on the concept of fuzzy gradient values which constructs a fuzzy set impulse noise. This fuzzy set is denoted by a fuzzy membership function which is used by filtering method usually a fuzzy averaging of neighboring pixels. It results an output image quasi without or with very little impulse noise such that other filters can be applied afterwards.

2.6. Hybrid Vector Filters

Hybrid Filters are combination of sub filters which belong to different types giving an output being a linear or non-linear combination of the vector samples.

2.6.1 Vector Median Rational Hybrid filter (VMRHF)

Vector Median Rational Hybrid filters [60] are multispectral image processing filters based on rational functions. VMRHF comprises of three sub filters (two vector median filters and one centre weighted vector median filter) with a vector rational operation.
The output $\boldsymbol{x}_{VMRHF}$ of VMRHF results from the vector rational function considering three input sub functions $\{\underline{\phi}_1, \underline{\phi}_2, \underline{\phi}_3\}$ where $\underline{\phi}_2$ is fixed as center weighted vector sub filter and is given as

$$\boldsymbol{x}_{VMRHF} = \underline{\phi}_2(n) + \frac{\sum_{j=1}^{3} a_j \underline{\phi}_j(n)}{h + k\|\underline{\phi}_1(n) - \underline{\phi}_3(n)\|_2} \qquad (45)$$

with $\alpha = [\alpha_1, \alpha_2, \alpha_3]^T$ represents the constant vector coefficient used to weight the output of the three sub filters. h and k are positive constants, the former ensures numerical stability whereas the nonlinearity is regulated by the latter.

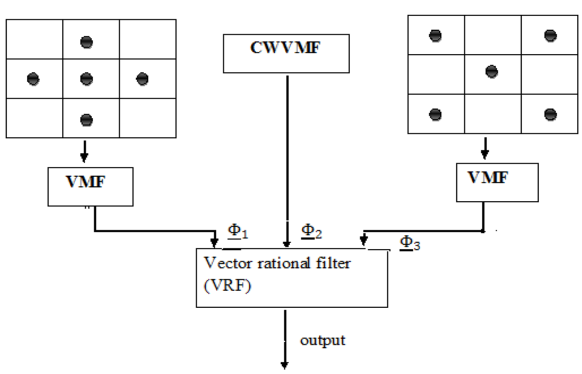

Fig. 1 Structure of VMRHF

2.6.2 Structure-Adaptive Hybrid Vector Filter (SAHVF)

In [61], another hybrid filter is designed. At each pixel location, noise adaptive preprocessing and modified quad tree decomposition techniques are used for classifying the corrupted image into several signal
activity categories. Then according to the structure classification, window adaptive hybrid filter is designed. The hybrid filter consists of three sub filters as follows

a) Modified Peer Group (MPG) which reduces image degradation in high-activity regions
b) Adaptive Nearest Neighbor Filter (ANNF) which removes noise by preserving edge structures in medium-activity regions
c) Structure Weighted Average Filter (SWAV) which smoothes small distortions in low-activity regions

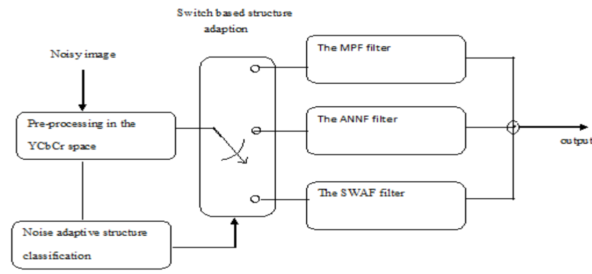

Fig. 2 Schematic diagram of Structure-Adaptive Hybrid Vector Filter

2.7 SigmaVector Filters

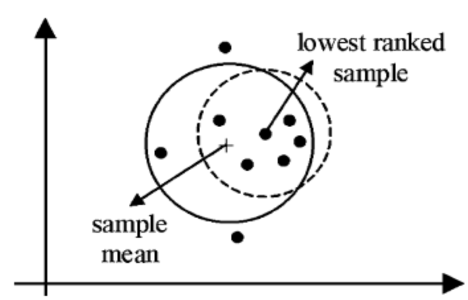

Fig. 3 Concept of sigma filtering in which radius indicates the variance or variance multiplied by a tuning parameter.

Sigma filters combine the order statistics theory and standard deviation or approximation of multivariate variance. In the figure, the radius of the circles represents the approximation of the variance and if the central pixel lies inside the circle, it is healthy otherwise it is regarded as an outlier.

2.7.1 Standard Sigma Filters

Sigma filters are adaptive switching filters which are based on the concept of standard deviation used for gray-scale images described in [62, 63]. Mathematically, the standard deviation is given by

$$\sigma = \sqrt{\left(\frac{1}{N}\sum_{i=1}^{N}(\boldsymbol{x}_i - \overline{\boldsymbol{x}})^2\right)} \qquad (46)$$

where $\overline{\boldsymbol{x}}$ represents the mean of the pixels inside the sliding window. The output of the sigma filter is described as

$$\boldsymbol{x}_{sigma} = \begin{cases} f(\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N), & if \ \mid \boldsymbol{x}_{\frac{N+1}{2}} - \overline{\boldsymbol{x}} \mid \ k \geq \ k\sigma \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases}$$

(47)

where $(.)$ is the smoothing function usually a median filter and k is the smoothing factor.

2.7.2 Vector Sigma Filters

The concept of standard sigma filter can be extended to multichannel images. For calculating the standard deviation, the Minkowski metric, the angular distance or the combination of both can be used. If the central pixel is detected as noisy, it is replaced by one of the output of VMF, BVDF or DDF otherwise it is left unchanged. These filters require a tuning parameter λ for determining the switching threshold. These filters are described below [64-66]

(a)Sigma Vector Median Filter (SVMF)

The output of sigma filter is described by

$$\boldsymbol{y}_{SVMF} = \begin{cases} \boldsymbol{x}_{(1)}, & \text{for } L_{(N+1)/2} \geq Tol \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \qquad (48)$$

where Tol is a threshold determined from the approximation of the multivariate variance of the vectors ψ_γ present in the window.

$$Tol = L_{(1)} + \lambda\psi_\gamma = \frac{N-1+\lambda}{N-1}L_{(1)} \qquad (49)$$

with

$\psi_\gamma = \frac{L_{(1)}}{N-1}$, $L_{(1)}$ is the aggregated distance associated with the vector median $\boldsymbol{x}_{(1)}$ and λ represents the tuning parameter.

(b) Sigma Basic Vector Directional Filter (SBVDF)

The output of SBVDF is given by

$$\boldsymbol{y}_{SBVDF} = \begin{cases} \boldsymbol{x}_{(1)}, & \text{for } \alpha_{(N+1)/2} \geq Tol \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \qquad (50)$$

where $Tol = \alpha_{(1)} + \lambda\psi_A$ with $\psi_A = \frac{\alpha_{(1)}}{N-1}$ in which $\alpha_{(1)}$ is the smallest aggregated angular distance and ψ_A represents the approximated variance calculated using the angular distance.

(c)Sigma Directional Distance Filter (SDDF)

The output of SDDF is given by

$$\boldsymbol{y}_{SDDF} = \begin{cases} \boldsymbol{x}_{(1)}, & \text{for } \Omega_{(N+1)/2} \geq Tol \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \qquad (51)$$

where $Tol = \Omega_{(1)} + \lambda\psi_{\gamma A}$with $\psi_{\gamma A} = \frac{\Omega_{(1)}}{N-1}$in which $\Omega_{(1)}$ is the smallest hybrid measureconsidering both magnitude and angular distance and ψ_A represents the approximated variance.

2.7.3 Adaptive Vector Sigma Filters

In Adaptive Vector Sigma Filters [67] the threshold is determined adaptively which employ the approximations of the multivariate variance based on the sample mean or the lowest-ranked vector.

(i) Design based on the sample mean

(a) Adaptive Sigma Vector Median Filter (ASVMF-mean)

The output of ASVMF-mean is given by

$$\boldsymbol{y}_{ASVMF-mean} = \begin{cases} \boldsymbol{x}_{VMF}, & \text{if } \parallel \boldsymbol{x}_{\frac{N+1}{2}} - \overline{\boldsymbol{x}} \parallel_\gamma \geq \sigma_\gamma \\ \boldsymbol{x}_{\frac{N+1}{2}}, & otherwise \end{cases} \qquad (52)$$

where $\overline{\boldsymbol{x}}$ is the mean of the samples inside the filtering window and the variance σ^2 for chosen norm γ is given by

$$\sigma_\gamma^2 = \frac{1}{N}\sum_{i=1}^{N}((\parallel \boldsymbol{x}_i - \overline{\boldsymbol{x}} \parallel)_\gamma)^2 \qquad (53)$$

(b) Adaptive Sigma Basic Vector Directional Filter (ASBVDF-mean)

The output of ASBVDF-mean is given by

$$\boldsymbol{y}_{ASBVDF-mean} = \begin{cases} \boldsymbol{x}_{BVDF}, & \text{if } A(\boldsymbol{x}_{\frac{N+1}{2}}, \overline{\boldsymbol{x}}) \geq \sigma_A \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases} \qquad (54)$$

with angular definition of multichannel variance σ_A^2 defined by

$$\sigma_A^2 = \frac{1}{N}\sum_{i=1}^{N}A^2(\boldsymbol{x}_i, \overline{\boldsymbol{x}}) \qquad (55)$$

(c) Adaptive Sigma Directional Distance Filter (ASDDF-mean)

The output of the ASDDF-mean is given by

$$\boldsymbol{y}_{ASDDF-\overline{x}} = \begin{cases} \boldsymbol{x}_{DDF}, & if \ \ (A(\boldsymbol{x}_{\frac{N+1}{2}}, \overline{\boldsymbol{x}}))^p . (\parallel \boldsymbol{x}_i - \overline{\boldsymbol{x}} \parallel)_\gamma^{1-p} \geq \sigma_{pA} \\ \boldsymbol{x}_{\frac{N+1}{2}}, & \text{otherwise} \end{cases}$$

(56)

with variance σ_{pA}^2 is the combination of σ_γ^2 and σ_A^2 given by

$$\sigma_{pA}^2 = (\sigma_\gamma^2)^{1-p}(\sigma_A^2)^p$$

(ii) Design based on the lowest-ranked vector

In this family, the lowest-ranked vector is used in calculation of the variance.

(a) Adaptive Sigma Vector Median Filter (ASVMF-rank)

The output of ASVMF-rank is given by

$$\boldsymbol{y}_{ASVMF-rank} = \begin{cases} \boldsymbol{x}_{VMF}, & \text{if } \parallel \boldsymbol{x}_{\frac{N+1}{2}} - \boldsymbol{x}_{(1)} \parallel_\gamma \geq \sigma_\gamma \\ \boldsymbol{x}_{\frac{N+1}{2}}, & otherwise \end{cases} \qquad (57)$$

where $\boldsymbol{x}_{(1)}$ is the lowest-ranked vector inside the filtering window and the variance σ^2 for chosen norm γ is given by

$\sigma_\gamma^2 = \frac{1}{N-1}\sum_{i=1}^{N}((\| \boldsymbol{x}_i - \boldsymbol{x}_{(1)} \|)_\gamma)^2$ (58)

(b) Adaptive Sigma Basic Vector Directional Filter (ASBVDF-rank)

The output of ASBVDF-rank is given by

$$\boldsymbol{y}_{ASBVDF-rank} = \begin{cases} \boldsymbol{x}_{BVDF}, \; if \; A(\boldsymbol{x}_{\frac{N+1}{2}}, \boldsymbol{x}_{(1)}) \geq \sigma_A \\ \boldsymbol{x}_{\frac{N+1}{2}}, \quad \text{otherwise} \end{cases} \quad (59)$$

with angular definition of multichannel variance σ_A^2 defined by

$\sigma_A^2 = \frac{1}{N-1}\sum_{i=1}^{N} A^2(\boldsymbol{x}_i, \boldsymbol{x}_{(1)})$ (60)

(c) Adaptive Sigma Directional Distance Filter (ASDDF-rank)

The output of the ASDDF-rank is given by

$$\boldsymbol{y}_{ASDDF-rank} = \begin{cases} \boldsymbol{x}_{DDF}, \; if \; (A(\boldsymbol{x}_{\frac{N+1}{2}}, \boldsymbol{x}_{(1)}))^p . (\| \boldsymbol{x}_i - \boldsymbol{x}_{(1)} \|)_\gamma^{1-p} \geq \sigma_{pA} \\ \boldsymbol{x}_{\frac{N+1}{2}}, \; \text{otherwise} \geq 0 \end{cases} \quad (61)$$

with variance σ_{pA}^2 is the combination of σ_γ^2 and σ_A^2 given by

$\sigma_{pA}^2 = (\sigma_\gamma^2)^{1-p}(\sigma_A^2)^p$ (62)

2.8 Entropy Vector Filters

Entropy Vector Filters are adaptive multichannel filters based on the local contrast entropy introduced by Beghdadi and Khellaf in [68] for gray scale image. Each pixel x_i inside the filtering window is associated with its contrast C_i defined by

$C_i = \frac{|x_i - \bar{x}|}{\bar{x}} = \frac{\Delta_i}{\bar{x}}$ (63)

with Δ_i is the gradient and $\bar{x}$ represents the mean of the input $\{x_1, x_2, \dots x_N\}$. The local contrast probability is given by

$P_i = \frac{\Delta_i}{\sum_{j=1}^{N}\Delta_i}$ (64)

Any pixel is considered as noisy if the local contrast probability is too high. In case of multichannel image, the local contrast probability is given by

$$P_i = \frac{(\sum_{k=1}^{m}(|x_{ik} - \bar{x}_k|)^\gamma)^{\frac{1}{\gamma}}}{\sum_{j=1}^{N}(\sum_{k=1}^{m}|x_{ik} - \bar{x}_k|^\gamma)^{\frac{1}{\gamma}}}, \quad (65)$$

where μ_k represents the k^{th} component of the mean. Noisy pixels contribute heavily to the entropy defined by

$H_i = -P_i log P_i$ (66)

It is assumed that each sample is associated with an adaptive threshold β_i defined as the rate of change of local contrast entropy H_i and the overall entropy H expressed as

$\beta_i = \frac{-P_i log P_i}{-\sum_{j=1}^{N} P_j log P_j}$ (67)

The output of the entropy vector median filter (EVMF) [69] is given by

$$\boldsymbol{y}_{EVMF} = \begin{cases} \boldsymbol{x}_{(1)}, \; \text{if } P_{(N+1)/2} \geq \beta_{(N+1)/2} \\ \boldsymbol{x}_{\frac{N+1}{2}}, \; \text{otherwise} \end{cases} \quad (68)$$

$$\boldsymbol{y}_{EVMF} = \begin{cases} \boldsymbol{x}_{(\mathbf{1})}, \; if \; P_{(N+1)/2} \geq \beta_{(N+1)/2} \\ \boldsymbol{x}_{\frac{N+1}{2}}, \; otherwise \end{cases} \quad (69)$$

where $\beta_{(N+1)/2}$ is the adaptive threshold of the central pixel and $x_{(1)}$ is the vector median output. Similarly other entropy vector filters such as Entropy Basic Vector Directional Filter (EBVDF) and Entropy Directional Distance Filter (EDDF) are also developed according to their corresponding distance and angular measure [70].

2.9 Quaternion based Vector Filters

Another approach for impulsive noise removal is based on the quaternion theory for improving the evaluation of color dissimilarity. A quaternion number q is a four dimensional number that consists of a real part a and three imaginary parts b, c and d[71-73]. In the hyper complex form, it is represented as

$q = a + b\hat{\imath} + c\hat{\jmath} + d\hat{k}$ (70)

where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the complex operators, which obey the following rules

$\hat{\imath}^2 = \hat{\jmath}^2 = \hat{k}^2 = -1$

$\hat{\imath}\hat{\jmath} = \hat{k}, \hat{\jmath}\hat{k} = \hat{\imath}, \hat{k}\hat{\imath} = \hat{\jmath}$

$\hat{\jmath}\hat{\imath} = -\hat{k}, \hat{k}\hat{\jmath} = -\hat{\imath}, \hat{\imath}k = -\hat{\jmath}$

Multiplication of the quaternion is not commutative. A pure quaternion has $a = 0$. The modulus and conjugate of a quaternion are

$|q| = \sqrt{a^2 + b^2 + c^2 + d^2}$ (71)

$\bar{q} = a - b\hat{\imath} - c\hat{\jmath} - d\hat{k}$ (72)

A unit quaternion has unit modulus. A quaternion in the polar form is represented as

$q = |q|e^{\mu\theta}$

$= (cos\theta + \mu sin\theta)$ (73)

where μ is a unit pure quaternion and $\mu = (\hat{\imath} + \hat{\jmath} + \hat{k})/\sqrt{3}$; μ and θ are referred to as the eigenaxis and eigenangle respectively.

An RGB color pixel can be represented in quaternion form as

$q_1 = r_1\hat{\imath} + g_1\hat{\jmath} + b_1\hat{k}$ (74)

where r_1, g_1 and b_1 denote the pixel values of red, green and blue channels respectively. Any unit quaternion U can be represented as $U = |U|e^{\mu\theta}$. The quaternion unit transform of a color pixel q_1 can be expressed as follows [74]

$$Y = Uq_1\bar{U} = [cos\theta + \mu sin\theta](r_1\hat{\imath} + g_1\hat{\jmath} + b_1\hat{k})[cos\theta - \mu sin\theta] = (r_1\hat{\imath} + g_1\hat{\jmath} + b_1\hat{k})cos2\theta + \frac{2}{\sqrt{3}}\mu(r_1\hat{\imath} + g_1\hat{\jmath} + b_1\hat{k})sin^2\theta + \frac{1}{\sqrt{3}}[(b_1 - g_1)\hat{\imath} + (r_1 - b_1)\hat{\jmath} + (g_1 - r_1)\hat{k}]sin2\theta$$

$$= Y_{RGB} + Y_I + Y_\Delta \quad (75)$$

where $Y_{RGB} = (r_1\hat{\imath}i + g_1\hat{\jmath}j + b_1\hat{k}k)cos2\theta$, $Y_I = \frac{2}{\sqrt{3}}\mu(r_1\hat{\imath} + g_1\hat{\jmath} + b_1\hat{k})sin^2\theta$,

$Y_\Delta = \frac{1}{\sqrt{3}}[(b_1 - g_1)\hat{\imath} + (r_1 - b_1)\hat{\jmath} + (g_1 - r_1)\hat{k}]sin2\theta$; Y_I represents the intensity of the color image; Y_Δ is the projection of the tristimuli in Maxwell triangle rotated 90^0, and it represents the chromaticity [3]. When $\theta = \frac{\pi}{4}$, $T = U|_{\theta=\frac{\pi}{4}} = (1/\sqrt{2}) + (1/\sqrt{6})(\hat{\imath} + \hat{\jmath} + \hat{k})$ and $Y_{RGB} = 0$, $Y_I = (1/3)(r_1 + g_1 + b_1)(\hat{\imath} + \hat{\jmath} + \hat{k})$ and $Y_\Delta = (1/\sqrt{3})\,[\,(b_1 - g_1)\hat{\imath} + (r_1 - b_1)\hat{\jmath} + (g_1 - r_1)\hat{k}]$ .Similarly, $\bar{Y}$ can be defined as follows

$$\bar{Y} = \bar{T}q_1T = \frac{1}{3}(r_1 + g_1 + b_1)(\hat{\imath} + \hat{\jmath} + \hat{k}) - \frac{1}{\sqrt{3}}[(b_1 - g_1)\hat{\imath} + (r_1 - b_1)\hat{\jmath} + (g_1 - r_1)\hat{k}] = Y_I - Y_\Delta \quad (76)$$

Y_I and Y_Δ can be expressed as $Y_I = \frac{1}{2}(Tq_1\bar{T} + \bar{T}q_1T)$ and $Y_\Delta = \frac{1}{2}(Tq_1\bar{T} - \bar{T}q_1T)$.

The color pixel difference between two pixels q_i and q_j can be written as the sum of the intensity and chromaticity differences as follows:

$$d(q_i, q_j) = d_I(q_i, q_j) + d_\Delta(q_i, q_j) \quad (76)$$

where $d_I(q_i, q_j) = (1/2)|(Tq_i\bar{T} + \bar{T}q_iT) - (Tq_j\bar{T} + \bar{T}q_jT)|$ is the color intensity difference and $d_\Delta(q_i, q_j) = (1/2)|(Tq_i\bar{T} - \bar{T}q_iT) - (Tq_j\bar{T} - \bar{T}q_jT)|$ is the color chromaticity difference.The intensity difference and chromaticity difference approach zero when q_i is very similar to q_j.

In [75-76] a switching VMF based on quaternion impulse detector is proposed using 5×5 window. The quaternion color difference between pixels along the four directional operators at $0^\circ, 45^\circ, 90^\circ$ and 135° are computed. An average color difference Val_h between the central pixel and other pixel values in the direction $h = (1, 2, 3, 4)$ for respective degrees are computed as follows:

$$Val_h = \frac{1}{4}\sum_{j=1}^{4} \mid d(\boldsymbol{x}_i, \boldsymbol{x}_j) \mid \quad (77)$$

$$Val = \min(Val_1, Val_2, Val_3, Val_4)$$

The above two equations are used for impulse detection. If the central pixel is an impulse, its Val value will be large otherwise it is small. Hence the output of the filter is given by

$$\boldsymbol{x}^{QVMF} = \begin{cases} Q^{(VMF)}, & \text{if } Val \geq T \\ \boldsymbol{x}_{(N+1)/2}, & \text{otherwise} \end{cases} \quad (78)$$

where $Q^{(VMF)}$ represents the VMF calculated in Quaternion form and T being a pre-specified threshold.

In [77] another Quaternion Switching Vector Median Filter (QSVMF) is developed based on both the intensity and chromaticity differences described above.

A modification of QSVMF is designed in [78]. It is different from other related works in the impulse detection which utilizes the pixels along only a direction with maximum number of similar pixels while other utilize the color differences between the central pixel and its neighboring pixels in the four-edge direction.

A two-stage filter using both the Quaternion based Switching filter and a local mean filter is designed in [79] for removing mixture noise. In [80] a new two-stage filter is proposed which incorporates the peer group concept along with the quaternion based distance measure for impulse detection. The probably noisy pixels are replaced by Weighted Vector Median Filter in the second stage. The concept of quaternion is extended in video sequences for removal of random-valued impulse noise in [81]. The luminance and chromaticity difference are represented in quaternion form which are combined together for measuring color difference between color samples. Based on this color difference, the color vectors along the horizontal, vertical and diagonal directions in current frame and adjacent frames on motion trajectory are utilize for the detection of presence of impulse. For noisy pixels, a 3-D weighted vector median operation is carried out while the other pixels are left intact. Another filter for removal of random-valued impulse noise from video sequences is also designed in [82].

2.10 Morphology based filters

Morphological filters (MF) are designed by parallel or sequential combination of the basic fundamental morphological operations such as erosion, dilation, opening and closing [83-85]. The common structures of MF are defined as

$$\text{OC Filter} \rightarrow \text{Opening followed by Closing}$$
$$\text{CO Filter} \rightarrow \text{Closing followed by Opening} \quad (79)$$

Erosion distributes the local minima while dilation distributes the local maxima within the sliding window defined by the structuring element which resembles a min/max filtering action for suppression of impulse noise.

In [86], an extension of MF to multichannel images on learning-based morphological color operations is developed. The color pixel ordering scheme is learned in accordance to pre-estimation of healthy and corrupted pixels where the corrupted samples are ordered either as maximum in erosion or minimum in dilation. The SVM is used as the classification technique of such learning-based multichannel MF in the RGB color domain. After each morphological operation, the image reconstruction step is carried out for restoring the original features.

Extension of mathematical morphology to multivariable data such as multichannel image is performed in [87-89]. In order to develop multivariate morphological operators, various complex mathematical tools such as machine learning [90], principal component analysis [91], hyper complex [92], rand projection depth function [93], group-invariant frames [94], and probabilistic extrema estimation [95] are used.

Based on the self-duality property of morphological operator, a multivariate self-dual morphological operator is developed in [96] which is applicable for noise removal and segmentation purposes. A pair of symmetric vector ordering is introduced in order to develop multivariate dual morphological operators.

Quantile filters and a group of morphological gradient filters are proposed in [97]. Their properties and links with dilation and erosion operators are also investigated. Another quantile filter is also designed in [98].

In [99], a new hybrid filter based on mathematical morphology and trimmed standard median filter is developed. For impulse detection, mathematical morphology such as erosion and dilation are utilized. Erosion refers to the computation of local minima while dilation estimates the local maxima of the neighboring pixels. The existence of an inverse relationship between erosion and dilation is used for detection for salt & pepper noise.

2.11 Wavelet-Transform based Filters

Many nonlinear filtering techniques are designed based on wavelet transform. Haci Tasmaz et al., [100] proposes a satellite image enhancement system comprising of image denoising and illumination enhancement technique. It is based on dual tree complex wavelet transform (DT-CWT). Based on the combined effect of Haar wavelet transform and median filter, a denoising technique is also proposed [101]. In [102], another denoising algorithm based on combined effect of the bi-orthogonal wavelet transform and median filter is designed which removes noise effectively. In [103], a wavelet based denoising technique for suppression of noise in Magnetic Resonance images (MRI) is proposed. Shalini and Godwin in [104] present a comparative analysis of denoising algorithms based on different wavelet transform such as Bior, Surelet, Haar and Curvelet.

2.12 Miscellaneous Filters

Miscellaneous Filters consist of those filters which cannot be included into any of the filter families described above although some filters may have common properties.

2.12.1 Selective Vector Median Filter

In [105] Selective Vector Median Filter is introduced which has two steps: noise detection and noise removal. For every pixel in the window aggregated sum of distance between other pixels is computed

$$S_i = \sum_{j=1}^{N} \parallel \boldsymbol{x}_i - \boldsymbol{x}_j \parallel, \; i = 1, \dots N \qquad (80)$$

Then the mean S value for that neighborhood is calculated as follows

$$\bar{S} = \frac{1}{N}\sum_{j=1}^{N} S_j \qquad (81)$$

The pixels which have S value higher than $k\bar{S}$ in the neighborhood are flagged as outliers. k represents a constant used to increase or decrease the sensitivity of the threshold. If the central pixel is an outlier, then the remaining pixels which are not flagged as outliers are used for calculating the Vector median and the noisy pixel is replaced by the corresponding vector median.

2.12.2 Similarity based Impulsive Noise Removal Filter

In [106] a filter based on similarities between the pixels in a predefined window is developed. If $\{\boldsymbol{x}_i, \dots, \boldsymbol{x}_N\}$ denote the pixels in a filtering window, then a convex similarity function is used to find the similarity between pixels. For the central pixel and its neighboring pixels, the cumulated sum of similarities M are computed as follows:

$$M_1 = \sum_{j=2}^{N} \mu(\boldsymbol{x}_1, \boldsymbol{x}_j),$$

$$M_k = \sum_{j=2, j \neq k}^{N} \mu(\boldsymbol{x}_k, \boldsymbol{x}_j) \qquad (82)$$

$\boldsymbol{x}_1$ is the central pixel and $\mu(\boldsymbol{x}_i, \boldsymbol{x}_j)$is a convex similarity function. The central pixel is detected as noisy if $M_1 < M_k, k = 2, \dots, N$and is replaced by that $\boldsymbol{x}_k$for which $k = argmax M_{i,} \, k = 2, \dots, N$.This filter is faster than VMF. A similar filter based on this idea is also developed in [107].

2.12.3 Filters based on Digital Path Approach

In [108] noise filtering approach based on the connections between image samples using the digital geodesic path is developed instead of using a fixed window. In this filter the image pixels are grouped togetherthrough the fuzzy membership function defined over vectorial inputs by forming digital paths revealing the underlying structure dynamics of the image. It shows better performance in suppressing impulsive, Gaussian as well as mixed-type noise.

2.12.4 Filters based on Long Range Correlation

Traditional method of noise filtering utilizes information based on local window centered around the corrupted pixel. In [109] a new class of filter based on information on both the local window and also some remote regions in the image is developed. This is due to the fact that there exists a long range correlation within natural images and also the human visual system can use such correlations to interpret and restore image information [110]. This filter comprises of two steps: impulse detection and noise cancellation. In the impulse detection stage any impulse detector as described in [111] and [112] can be used for identifying the corrupted pixel in the local window. If the central pixel is corrupted, then a remote window centered around the impulse pixel is defined in the search range which is larger than the local window. All the pixels in the remote window will compete for the perfect match with the local window based on the mean square error of uncorrupted pixels in local and remote window. Finally, the corrupted pixel is replaced by the central pixel of the remote window with minimum mean squared error.

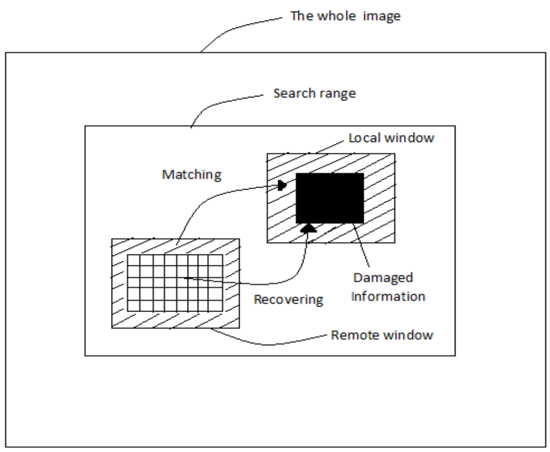

Fig. 4 Demonstration of the general approach

2.12.5 Vector Rational Filters (VRF)

Vector rational filters are extension of rational filters. In [113], two algorithms are proposed VRF_L and VRF_d based on L_p -norm and directional processing using two decimation schemes such as rectangular and quincunx decimation for down-sampling. Merits of these filters are better edge-preserving properties and absence of artifacts.

2.12.6 Robust Local Similarity Filters (RLSF)

A new algorithm for suppression of mixed Gaussian and impulsive noise based on concept of Rank ordered Absolute Difference (ROAD) [114] is proposed in [115]. Noise is detected by computing ROAD between samples of the processing block and a small window which is centered at block's central pixel. Main contribution is that the similarity measure is not affected by outliers due to impulses and the averaging operation reduces the Gaussian noise. The output of this filter is given as

$$\boldsymbol{x}_{RLSF} = \frac{\sum_{j=1}^{N} w_j . x_j}{\sum_{j=1}^{N} w_j} \qquad (83)$$

with $w_j = Ҡ\left(\frac{1}{\alpha}\sum_{k=1}^{\alpha} d_j(k)\right)$ (84)

where Ҡ represents the kernel function (Gaussian) and $d_j(k)$ denotes the k^{th} smallest Euclidean distance between pixel x_j of the processing block and pixels of W of the small window. α is the number of close neighbors used in calculation of ROAD.

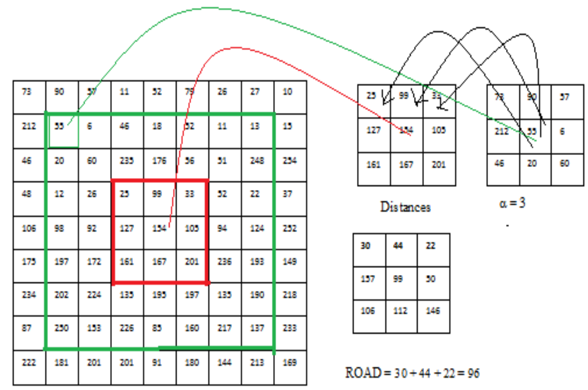

Fig. 5 Calculation of similarity measure between pixel in the processing block (Green Square) and small window (red square) by ROAD.

2.12.7 Vector Marginal Median Filters (VMMF)

In [3] a vector filter named Vector Marginal Median Filter (VMMF) based on the median operation is presented. It calculates the median value of each channel separately and the central pixel is replaced by the median value of the respective channel. The output of VMMF is given below

$$\boldsymbol{x}_{VMMF} = \left((med(\{x_1^R, \dots x_N^R\})), (med(\{x_1^G, \dots x_N^G\})), (med(\{x_1^B, \dots x_N^B\}))\right) \qquad (85)$$

where R, G and B are the red, green and blue channel of the pixels in the window. Its noise reduction capability is highest but since it does not consider the correlation among the vector channel, it leads to color artifacts.

2.12.8 Vector Signal-Dependent Rank Order Mean Filters

Multichannel extension of the grayscale Signal Dependent Rank Order Mean (SDROM) filter [116] is the Vector signal-dependent rank order mean (VSDROM) filter [117]. For noise detection, first the vector pixels in the window are sorted according to their aggregated distance to all other samples. The distance between the four lowest-ranked vectors and the central pixel are compared against respective increasing thresholds. The central pixel is regarded as noisy if any of these distances is greater than their threshold and replaced by the VMF.

2.12.9 Decision based Couple Window Median Filters (DBCWMF)

They use an increasing window size in order to maximize the probability of finding noise-free pixels [118]. The steps of DBCWMF is given below

1. Define a 2-D local window W_n of dimension $(2n+1) \times (2n+1)$ (initialize algorithm by choosing $n = 1$).

2. Identify salt & pepper noise by checking if the central pixel $\boldsymbol{x}_{\frac{N+1}{2}}$ is either 0 or 255. If 0< $\boldsymbol{x}_{\frac{N+1}{2}} <$ 255 it is left intact.

3. If $\boldsymbol{x}_{\frac{N+1}{2}}$ is noisy, then trim all 0's and 255's present in the W_n and follow the two cases

Case 1. If the number of non-noisy pixels is non-zero, then replace the central pixel by the median value calculated from the remaining uncorrupted pixels.

Case 2. If the number of non-noisy pixels is zero, then update the window size by increasing $n = n + 1, (n < 5)$ and goto Step 1.

4. If the number of non-noisy pixels in W_4 is zero, then replace the value of central pixel by mean of W_1.

5. Repeat Steps 1 to 4 until all the pixels are processed.

2.12.10 Improved Bilateral Filter for reducing mixed noise

A Bilateral Filter (BF) [119] is a combination of two low-pass Gaussian filters operating in spatial and color (range) domain for reducing both impulse and additive noise while preserving edge structures. The spatial low-pass filter gives larger weights to those samples closer to the central pixel while the range low pass filter assigns larger weights to those pixels having similar color distributions with the central pixel. Here the filter's output is mainly dependent on the central pixel and neighboring pixels close in both spatial and range domain with the central pixel.An improvement in the traditional Bilateral Filter (BF) is designed in [120] in which a new weighting function to the bilateral filtering mechanism is introduced. For an impulse, the vector median (as opposed to the traditional BF which always uses the central pixel) is considered as base for bilateral filtering operation for replacement of the central pixel otherwise the normal bilateral filtering action is continued.

3. Impulse Noise Model

In the real life, the form of impulse noise varies. For example, the value of impulse noise caused by malfunction of sensor is expected to be fixed-valued impulse noise, while the value of impulse caused by electronic interference is random-valued impulse noise [77]. Impulse noise can be divided into two types: Correlated impulse noise and Uncorrelated impulse noise. The impulse noise model proposed by Viero et al [10] is correlated type impulse noise and it has the following form

$$\boldsymbol{q}' = \begin{cases} \{q_1 \quad q_2 \quad q_3\} \text{with probability } 1-p \\ \{n_1, \quad q_2, \quad q_3\} \text{with probability } p_1 p \\ \{q_1, \quad n_2, \quad q_3\} \text{with probability } p_2 p \\ \{q_1, \quad q_2, \quad n_3\} \text{with probability } p_3 p \\ \{n_1, \quad n_2, \quad n_3\} \text{with probability } p_a p \end{cases} \qquad (86)$$

where $\boldsymbol{q} = (q_1, q_2, q_3)$ is the original uncontaminated vector pixel, $\boldsymbol{q}'$ may be either contaminated or uncontaminated, $n_k (k = 1,2,3)$ equals 0 or 255 with equal probability for fixed-valued impulse noise, or takes any value in the range $[0,255]$ for random-valued impulse noise; p is the sample corruption probability; p_1, p_2 and p_3 are the channel corruption probabilities and $p_a = 1 - p_1 - p_2 - p_3$.

The uncorrelated impulse noise has the following form [46]

$$q'_k = \begin{cases} n_k & \text{with probability } p \\ q_k & \text{with probability } 1-p \end{cases} \qquad (87)$$

where $k = 1,2,3$ denotes the three channels in RGB color space; p is the channel corruption probability; q_k and n_k denote the original component and contaminated component respectively. n_k can take either 0 or 255 for fixed-valued impulse noise and can take any discrete value in [0,255] for random-valued impulse noise.

4. Performance Measurement of Filters

The performance of filter is evaluated by the following parameters

(a) Mean Absolute Error (*MAE*)

$$MAE = \frac{1}{M_1 \times N_1} \sum_{i=1}^{M_1} \sum_{j=1}^{N_1} |q(i,j) - f(i,j)| \qquad (88)$$

where $M_1 \times N_1$ is the size of the image; $q(i,j)$ and $f(i,j)$ are the original and filtered pixel values at (i,j) location.

(b) Mean Squared Error *(MSE)*

$$MSE = \frac{1}{M_1 \times N_1} \sum_{i=1}^{M_1} \sum_{j=1}^{N_1} (q(i,j) - f(i,j))^2 \qquad (89)$$

(C) Peak Signal to Noise ratio *(PSNR)*

$$PSNR = 10\log_{10}\left(\frac{I_{max}^2}{MSE}\right) \qquad (90)$$

where I_{max} is the maximum pixel value of the original image.

(d) Normalized Color Distance *(NCD)*

The NCD is defined in the Lu*v* color space by

$$NCD = \frac{\sum_{i=1}^{M_1} \sum_{j=1}^{N_1} \sqrt{\left(L^o(i,j) - L^x(i,j)\right)^2 + \left(u^o(i,j) - u^x(i,j)\right)^2 + \left(v^o(i,j) - v^x(i,j)\right)^2}}{\sum_{i=1}^{M_1} \sum_{j=1}^{N_1} \sqrt{\left(L^o(i,j)\right)^2 + \left(u^o(i,j)\right)^2 + \left(v^o(i,j)\right)^2}}$$

(91)

where $L^o(i,j), u^o(i,j)$, $v^o(i,j)$ and $L^x(i,j), u^x(i,j)$, $v^x(i,j)$ are values of the lightness and two chrominance components of the original image sample $q(i,j)$ and filtered image sample $f(i,j)$ respectively.

(E) Structural Similarity Index *(SSIM)*

Structural similarity index measures the similarity between two images and is given below

$$SSIM = \frac{(2\mu_x \mu_y + C_1)(2\mu_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \qquad (92)$$

where μ_x and μ_y are mean of the original and filtered image, σ_{xy}, σ_x^2 and σ_y^2 represent the corresponding covariance and variance of the original and filtered images. C_1 and C_2 are the constants.

MAE is used to evaluate detail preservation; MSE and PSNR are used to evaluate noise suppression capability; NCD is used to measure perceptual error in the CIELab color space [3].
For an efficient filter, it is expected to have high PSNR and SSIM, while the other parameters like MAE, MSE and NCD are minimum.

5. Conclusions

A comprehensive survey of various vector median filters for the removal of impulse noise from color images is presented. These filters have been categorized into 12 different families. Their properties have been summarized and presented. Some recently proposed algorithms have been added and studied.

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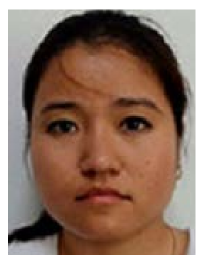

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