# RC Beams Behavior Retrofitted by FRP Subjected to Torsion, Shear and Flexure – a Review

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**Abstract**
Many reinforced concrete bridges and structures around the world are currently in need of repair or complete replacement as they approach the end of their service life. Increases in traffic volume, traffic loads, and corrosion-induced deterioration are necessitating significant expenditures to strengthen and rehabilitate existing structures [1]. Fiber Reinforced Polymer (FRP) as an external reinforcement is used extensively to address the strength requirements related to torsion, shear and flexure in structural systems. The noncorrosive properties and high strength-to-weight ratio of FRP materials make them a very viable repair material and can result in longer service life of structures [1]. Researches have shown that the RC beams retrofitted by FRPs, have more strength in crack and can carry more load and displacement. In this paper, the behavior and performance of reinforced concrete beams strengthened with externally bonded FRP sheets subjected to torsion, shear and flexure are presented.

***Keywords:***
*Reinforced Concrete Beam, Fiber Reinforced Polymer, Crack, displacement*

## 1. Introduction

Concrete industry increases daily in terms of infrastructure development applications; such manipulation mainly contributes to the environment pollution, due to CO2 emissions. Nowadays, several efforts are in progress to reduce the use of Portland cement in concrete to address the global warming issues. These searches include the use of supplementary cementing materials such as: fly ash, silica fume, clay and metakaolin, as well as the development of alternative binders to Portland cement [2-3]. In this respect, the modified eco-friendly concrete technology shows considerable promise for application in concrete industry as an alternative binder to the cement, and provides a very important progress in many engineering fields, since this alternative binder disposes of an interesting mechanical strength, adheres well to the other materials and provides high resistance to chemicals and corrosive agents [4]. In recent years, there are many reinforced concrete (RC) structures are suffering from various deteriorations: cracks, concrete spalling, large deflection, etc., which need to be reinforced to support the designed or even resist possible higher loading or to renovate existing cracks [5-7]. These deteriorations are caused by various factors such as aging, corrosion of steel reinforcement, environmental effects such as seawater and accidental impacts on the structure [8-10]. Especially, during the natural disasters such as the earthquake in Sichuan on 12th May, 2008, many concrete structures, if they were not collapsed, were damaged to some extent [11]. Commonly encountered engineering challenges such as increases in service loads, changes in use of the structure, design and/or construction errors, degradation problems, changes in design code regulations, and seismic retrofits are some of the causes that led to the need for rehabilitation of existing structures. Complete replacement of an existing structure may not be a cost-effective solution and it is likely to become an increasing financial burden if upgrading is a viable alternative. In such occasions, repair and rehabilitation are the most commonly used solutions. There are several options available for retrofitting or repairing structural members of the existing RC structures. The commonly used options are to bond thin steel and/or fibre reinforced polymer (FRP) sheets onto the damaged members to restrain cracks and to increase the load carrying capacity, ductility and stiffness of structures strengthened [12-13]. To externally bond FRP sheets on the tension and also lateral sides of RC beams and columns is a widely used method for repairing and strengthening of the RC structures. Depending on the application, fibre reinforcing polymer (FRP) sheets can be employed to increase the torsion strength, the shear strength [14], the flexural strength, the ductility and the serviceability performance of R/C structures, utilising these FRP sheets as additional external reinforcement, since FRP has better characteristics than the conventional strengthening material steel, in terms of the superior mechanical properties such as high tensile strength, lightweight, resistance to corrosion and fatigue, ease of installation, etc. [7,9,15-16]. Investigations [17,18] were undertaken in the past to evaluate reliability of such reinforcing technique related to static loading and showed that the RC structures strengthened would demonstrate a better performance in strength, ductility and retarding crack growth as long as an appropriate end anchorage was provided for the FRP sheet [7,16]. Recently, Fiber Reinforced Polymer (FRP) was used in many mechanical

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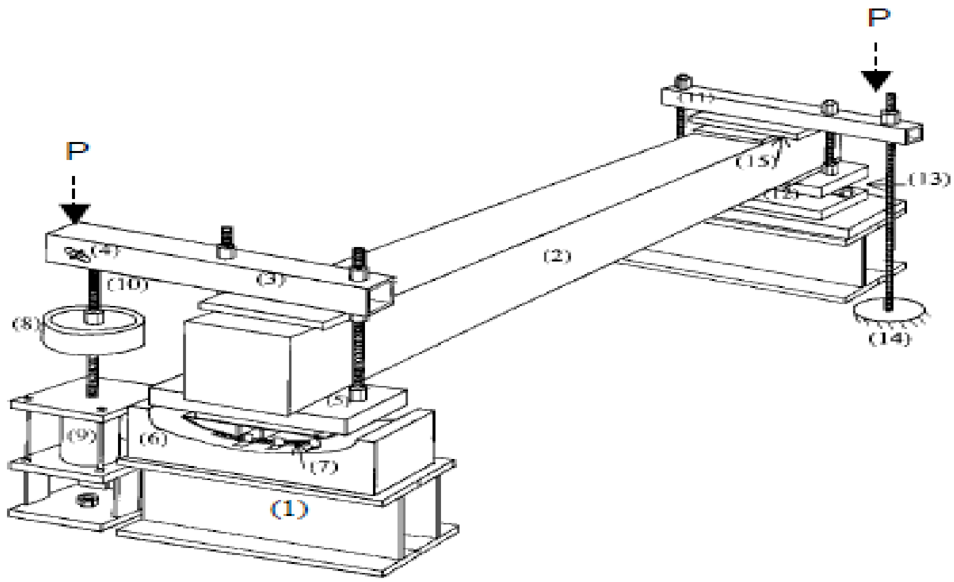
engineering fields, aeronautics, and especially in civil engineering. The popular types of FRP identified by researchers include aramid fiber reinforced polymers (AFRP), carbon fiber reinforced polymers (CFRP) and glass fiber reinforced polymers (GFRP) in the form of rods, strips, plates and laminates. All of these materials have been investigated under a variety of strengthening techniques that include near surface mounted (NSM), externally bonded reinforcement (EBR) and external post-tensioning (EPT) using anchorage and non-anchorage systems. In this respect, the enhancement in terms of mechanical performances of strengthened structures with composite materials was experimentally demonstrated by several studies and researches [19-22]. Recently, several approaches for confined and reinforced concrete members by FRP composites have been developed, whereas, one of the most used strengthening methods was the bonding of external composite sheets on the exposed faces to failure [23]. Reinforcement corrosion and structural deterioration in reinforced concrete (RC) structures are common and prompted many researchers to seek alternative materials and rehabilitation techniques. While many solutions have been investigated over the past decades, there is always a demand to search for use of new technologies and materials to upgrade the deficient structures. In this context, strengthening with Fiber Reinforced Polymers (FRP) composite materials in the form of external reinforcement is of great interest to the civil engineering community.

## Discussion

## 2. Torsion

Reinforced concrete members in a structure may be subjected to loads with magnitudes higher than those considered as design loads. Axial forces, shear forces, bending moments, torsion, or a combination of these effects, are considered to design a safe structural member. For most design situations, bending moments and shear forces are considered as primary effects, whereas torsion is regarded as secondary [24]. For this reason, the torsional behavior of reinforced concrete beams is not studied as much in depth as their behavior under bending and shear stresses [24]. Torsion becomes a primary effect, however, for situations such as spandrel or curved beams [25]. Ghobarah et al. [25] evaluated the effectiveness of FRP strengthening of steel-reinforced concrete beams and columns subjected to torsion. They conducted experiment on 11 beams with different orientation of CFRP and GFRP wrap. Complete wrap of torsional zone of RC beam was found to be more effective. The 450 orientation of the fibers showed more efficiency of material. In RC torsional members, diagonal cracks are formed due to the same mechanism that is responsible for the formation of shear cracks [26-27]. Since the diagonal tension cracks are found to be common in both shear and torsion, the strengthening schemes considered in the shear strengthening situations [28] can be considered as possible options for torsional strengthening of RC beams. The main difference between shear cracking and torsional cracking lies in the crack pattern [27]. Spiral-like crack pattern are found in torsional members. Structural members curved in plan, members of a space frame, eccentrically loaded beams, curved box girders in bridges, spandrel beams in buildings, T-shape, inverted L-shape, and spiral stair-cases are typical examples of the structural elements subjected to torsional moments and torsion cannot be neglected while designing such members. These different configurations make the understanding of torsion in RC members a complex task. In addition, torsion is usually associated with bending moments and shearing forces, and the interaction among these forces is important.

The torsional strengthening of RC beams has been conducted by some techniques, such as steel plate jacketing, increasing cross sections, and adding extra steel bars. The advantages of Fiber-Reinforced-Polymers (FRPs), such as their relatively high strength to weight ratio, high resistance in corrosive regions, and their easy-to-apply character, caused engineering interest in extending the application of this material for strengthening demands [29]. While the literature on torsional strengthening is quite limited, initiated just in 2001, the flexural and shear strengthening of RC beams with FRP materials has been studied since the early 1990s [29]. Since then, flexural and shear capacity has only been included in design guides and recommendations (FIB [30], JSCE [31], ACI [32] and CSA [33]). In the case of torsional strengthening with FRP materials, only FIB proposed design equations based on shear strengthening models have been proposed [29]. Most of the tested specimens available in the literature encompass solid rectangular beams, steel reinforced with longitudinal bars and transverse stirrups, strengthened with externally bonded carbon or glass FRP (CFRP or GFRP) materials in the transverse direction and tested under monotonic torsion [34,25,35,36]. Ameli and Ronagh [35] introduced an analytical method for evaluating the ultimate torque of FRP-strengthened reinforced concrete beams based on the compression field theory. Their computational procedure only capable of predicting the torsional strength of strengthened beams and, consequently, an entire response is not obtained. They carried out an experimental and numerical study of twelve rectangular RC beams strengthened by CFRP / GFRP wrap with different configuration. Numerical modeling of FRP strengthened beam was done with ANSYS. Significant improvement in ductility was observed with GFRP wrapping as compared to CFRP wrapping. Box-section RC beams strengthened with CFRP strips were also tested under monotonic torque by Hiii and Al-Mahaidi [37-38] and under cyclic torque by Jing et al. [39]. Moreover, torsional tests of retrofitted beams with GFRP in the transverse or/and longitudinal direction were conducted by Panchacharam and Belarbi [40]. They had a study on Torsional Behavior of Reinforced Concrete Beams Strengthened with FRP Composites. All beams were tested under pure torsion using the test setup shown in Fig. 1.




(1) Supporting beam, (2) Test beam (3) Loading arm, (4) Inclined cut, (5) Spherical bearing seat (6) Concrete pedestal, (7) Steel rollers, (8) Load cell, (9) Hydraulic Jack, (10) 25-mm diameter threaded rod, (11) Reaction arm, (12) Bearing plate, (13) 25-mm diameter steel roller, (14) Reaction floor, (15) Steel plates with rollers

Fig 1. Schematic diagram of the torsion test set-up [40]

Total of eight beams were included in this investigation and different strengthening schemes were adopted. Combined FRP sheets in the longitudinal direction of the beam followed by all-around wrapped strips, showed an increase in both the ultimate strength and ductility of the beam. This experimental study on the effect of fiber orientation revealed that FRP sheets with fibers oriented parallel to the longitudinal axis of the beam would not contribute to the post-cracking stiffness of the beam. As the conclusions obtained from the experimental and analytical program done by S. Panchacharam and A. Belarbi, the following summarized points can be mentioned:

1. Torsional reinforced concrete beams strengthened with GFRP sheets exhibited significant increase in their cracking and ultimate strength as well as ultimate twist deformations.
2. Strengthening schemes with complete wraps in 90-degree fiber orientation with respect to beam axis provided an effective confinement and therefore resulted in a significant increase (about 150%) in the ultimate torsional strength.
3. Strengthening with FRP sheets in the longitudinal direction of the beam on three faces or four faces of the cross-section provided similar behavior.
4. U-wrapped strengthening showed the least twist capacity due to peeling of FRP sheets along the side of the beam. However, anchoring the wraps to concrete enhanced the twist capacity and failure was mainly due to crushing of concrete and lateral separation of anchor bars along with FRP sheets.
5. When combining FRP sheets in the longitudinal direction of the beam followed by all-around wrapped strips, the results showed that there was an increase in both the ultimate strength and post-cracking torsional twist and ductility of the beam.

There are only a few torsional tests of nonrectangular beams strengthened with FRP materials [41,42]. Ghobarah et al. [25] experimentally investigated the effectiveness of carbon and glass FRP sheets and strips as additional external reinforcement to rectangular beams with bars and stirrups under torsion, and simple design approaches were also discussed. Salom et al. [41-42] studied experimentally and analytically the torsional behavior of six spandrel beams with bars and stirrups, which had been strengthened with FRP laminates using a special anchoring system. Both studies addressed that, in general, FRP materials caused a significant increase on the torsional capacity of the tested beams. Previous research on the torsional behavior of FRP strengthened reinforced concrete beams encompassed experimental, analytical and numerical investigations. Experimental investigations on the torsional behavior of concrete beams with and without stirrups, including rectangular and T-shaped sections strengthened using FRP sheets and strips as external transverse reinforcement, have been conducted by Chalioris [43]. Constantin E. Chalioris, did an experimental study to evaluate the effectiveness of the use of epoxy-bonded carbon FRP fabrics as external transverse reinforcement to under-reinforced concrete beams with rectangular and flanged cross-section subjected to pure torsion. His experimental investigation deals with the torsional strengthening of concrete beams without stirrups using epoxy-bonded carbon fiber-reinforced-polymer (FRP) sheets and strips as external transverse reinforcement.

The experimental setup is shown in Fig. 2.

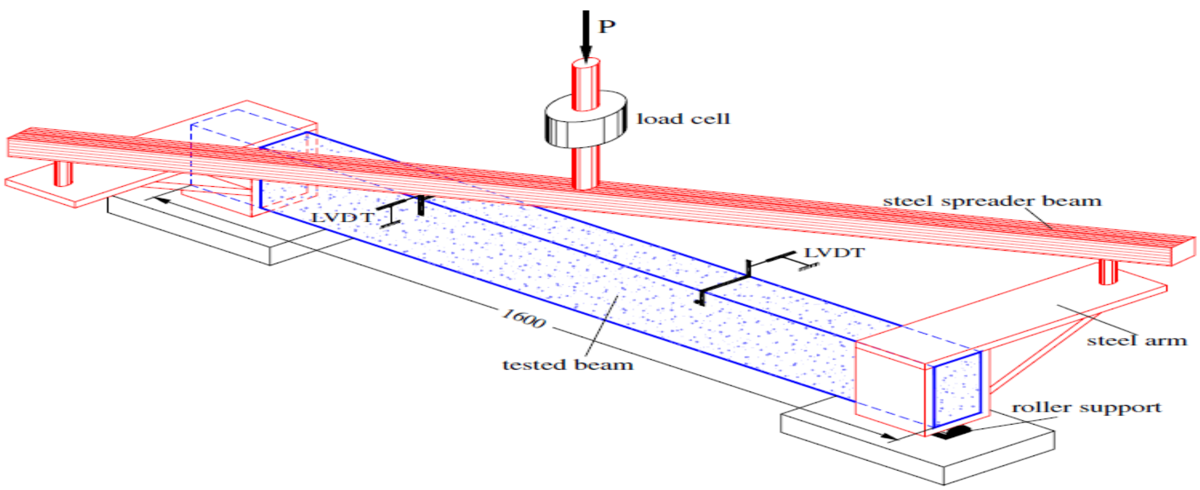



Fig.2. Schematic of the test set-up [43]

In order to evaluate the effectiveness of epoxy bonded FRP fabrics as external transverse reinforcement on the torsional strength, curves of the percentage increase of the measured torque capacities versus the FRP ratios are shown in Fig.3.

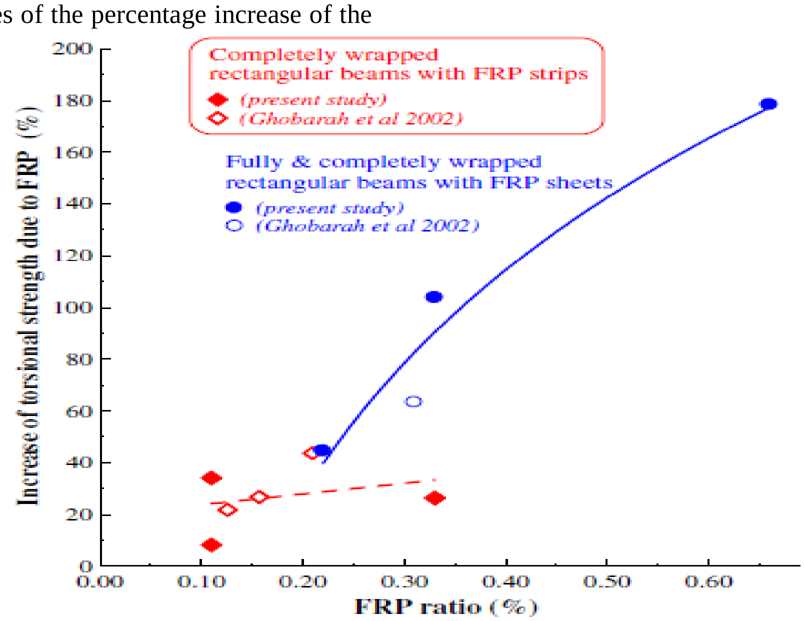



Fig.3. Influence of FRP fabrics on the torsional strength of rectangular beams [43]

Based on the test results, the following concluding remarks are drawn:

1. Strengthened beams using FRP sheets and strips as the only transverse reinforcement exhibited better overall torsional performance than the non-strengthened control beams.
2. It is emphasized that full wrapping with continuous sheets is far more efficient for torsional upgrading than the use of wrapping with the same volume of discrete strips.

An analytical model based on the softened truss model in conjunction with the smeared crack theory was then proposed by Chalioris [29] to obtain the entire behavior of RC beams strengthened by FRP under torsion. The Chalioris analytical model is comprised of two distinctive steps to obtain the torsional behavior of strengthened beams in pre and post cracking stages. Ghobarah et al. [25] conducted experimental tests on FRP-strengthened reinforced concrete beams subjected to torsion and a small value of flexure, to study the effect of various strengthening configurations with carbon and glass FRP sheets and strips. To validate their experimental tests with an analytical approach, Deifalla and Ghobarah [44] utilized the compression field theory and established a series of equations for computing the contribution of FRP sheets to the torsional resistance of beam. They also adapted the model by Rahal and Collins [45] to: (1) predict the behavior of rectangular RC beams up to failure using a displacement control solution scheme rather than a force control solution scheme; (2) include the FRP material modeling; (3) model external bonded reinforcements with different arrangements; and (4) improved the concrete constitutive modeling [44-45]. According to experimental tests, strengthening with continuous FRP sheets provides more efficient confinement than that of the strip, due to less crack openings [25,35,40,43].

## 2. Shear

Several theoretical and experimental studies [46-48] have been carried out to analyze the phenomenon of the shear failure of reinforced concrete (RC) beams. Russo et al. [49] presents a comprehensive review of various proposed methods for predicting the shear behavior of reinforced concrete beams without any transverse reinforcement. In the design of R/C beams, this is usually considered as an initial condition that is supplemented by the strength provided by the transverse reinforcement in order to predict the total shear capacity of such a structural element. A comprehensive experimental program has been designed and carried out at the Laboratory of Experimental Structural Mechanics at the Aristotle University of Thessaloniki aimed to assess the structural efficiency and effectiveness of shear strength provided to R/C beams by externally attached open hoop FRP transverse shear reinforcement. Initially, a number of R/C beam specimens with only longitudinal reinforcement were tested. Identical R/C beam specimens were then tested which were provided with open hoop FRP strips external shear reinforcement in order to study in this way the additional shear strength that can be attained by such a strengthening scheme. Many researchers examined the effectiveness of this type of shear reinforcement in the form of closed hoop FRP strips attached externally on rectangular R/C beam specimens [50-53].

Teng et al. [54] tested nine FRP-strengthened RC beams: three as control specimens, three with bonded FRP full wraps, and three with FRP full wraps left unbonded to the beam sides. The test results showed that the unbonded FRP wraps had a slightly higher shear strength capacity than the bonded FRP wraps. In addition, Deifalla and Ghobarah [55] tested six half-scale beams externally bonded with FRP composites constructed using a specially designed test setup in order to investigate the strengthening techniques for beams subjected to combined shear and torsion. Test results indicated that externally bonded CFRP strengthening schemes enhanced the shear and torsion carrying capacities of RC beams. Grande et al. [56] tested 15 RC beams strengthened in shear by externally bonded fiber-reinforced plastics (FRPs) sheets to study the influence that the geometrical percentage of transverse steel reinforcement could had on the FRP resisting action. The experimental investigation indicated the variability of the FRP shear resisting action over the amount of the transverse steel reinforcement. In particular, the FRP shear resisting action was generally smaller in beams with closer stirrups. Fico et al. [57] studied the assessment of Eurocode design equations for the evaluation of the shear strength of FRP RC members, as proposed by the guidelines of the Italian Research Council. Li et al. [58] tested sixteen RC beams with or without FRP composites to study the shear strengthening effect. The experimental results indicated that the shear contribution of FRP composites was influenced by the applied FRP composite area, the spacing between the steel stirrups, and the longitudinal steel bar diameter of the RC beams. The contribution of FRP to the shear capacity has just been included in the design guides and recommendations (Concrete Society [59], fib [60], JSCE [31], ACI [61], CSA [33]).

Jung-Yoon Lee, Hyun-Bok Hwang, Jeung-Hwan Doh, [62] had an investigation and the results of an experimental and analytical study on the performance of reinforced concrete beams externally wrapped with FRP composites and internally reinforced with steel stirrups was presented. The effectiveness of FRP is maximized by bonding the external FRP reinforcement parallel to the direction of principal tensile stress. The shear failure modes of FRP strengthened concrete beams are quite different from those of the beams strengthened with steel stirrups. Ten RC beams strengthened with varying FRP reinforcement ratio, the type of fiber material (carbon or glass) and configuration (continuous sheets or strips) were tested and the strain of the FRP composites was measured at specified load intervals by the LVDTs attached to the beams and electrical strain gages installed on the surfaces of the FRP. Comparisons between the observed and calculated effective strains of the FRP in the tested beams failing in shear showed reasonable agreement. The test results indicated that the effective strain of the FRP at shear failure decreased as the amount of FRP increased and also as the spacing of the FRP strips decreased.

Daniel Baggio, Khaled Soudki, Martin Noël, [63], had a research to determine the structural behavior of shear-critical beams strengthened with externally bonded FRP sheets and FRP anchors used to delay or eliminate FRP debonding. The test variables included the use of three different FRPs (CFRP, GFRP, and FRCM), and the presence and type of FRP anchors (CFRP or GFRP). FRP anchors are of particular interest because they have the same material properties as the FRP sheets and can be installed simultaneously with the sheets using the same adhesives [64]. Their study is one of only a few studies which have used commercially manufactured FRP anchors as the anchoring mechanism to secure externally bonded FRP sheets [65-66]. The program comprised of one control (unstrengthened) beam and eight FRP strengthened beams. The mechanical properties of the FRP materials are given in Table 1.

Table 1. FRP and FRCM material properties [63]

| Material | Thickness (mm) | Elastic modulus (GPa) | Elongation at rupture (%) |
|---|---|---|---|
| CFRP – 230C | 0.381 | 67 | 1.33 |
| GFRP – 430G | 0.508 | 26 | 2.21 |
| FRCM – 350G | 1.17 | 75 | 2.80 |
| CFRP – Anchor C | 10 (diameter) | 215 | 0.74 |
| GFRP – Anchor G | 10 (diameter) | 70 | >4.0 |

The results are similar to the results by Jayaprakash et al. which reported that increasing the amount of internal and external FRP shear reinforcement may not proportionally increase the shear capacity but can change the mode of failure from shear to flexure [67]. As the test result, three modes of failure were observed: shear failure, shear failure with debonding of the FRP sheet and flexural failure with crushing of the concrete. A summary of the test results, ultimate load, deflection at ultimate load, percent increase over the control and mode of failure for all beams is provided in Table 2.

Table 2. Summary of test results [63]

| Beam description | Ultimate load (kN) | Deflection at ultimate load (mm) | Percent increase over control (%) | Failure mode |
|---|---|---|---|---|
| Beam 1 – Control | 223 | 8.80 | – | Shear |
| Beam 2 – CFRP – No anchors | 373 | 11.4 | 67.5 | Flexure |
| Beam 3 – CFRP – Anchors | 390 | 16.9 | 75.1 | Flexure |
| Beam 4 – FRCM – No anchors | 294 | 12.0 | 32.0 | Shear |
| Beam 5 – FRCM – Anchors | 300 | 10.7 | 34.7 | Shear |
| Beam 6 – GFRP – No anchors | 334 | 13.7 | 50.1 | Shear – debonding |
| Beam 7 – PD-GFRP – No anchors | 305 | 12.0 | 36.8 | Shear – debonding |
| Beam 8 – PD-GFRP – C anchors | 310 | 14.2 | 39.2 | Shear compression |
| Beam 9 – PD-GFRP – G anchors | 339 | 13.7 | 52.2 | Shear compression |

As conclusion, Daniel Baggio, Khaled Soudki, Martin Noël suggested, when the available bonded length is limited, the installation of FRP anchors is a viable option to prevent a brittle shear failure mode due to FRP debonding.

A total of ten rectangular beam specimens were subjected to four point bending as part of the study done by G.C. Manos, M. Theofanous, K. Katakalos [68], to assess the effectiveness of the proposed anchorage device. This anchorage was provided by the novel anchorage device [69] depicted in some detail in Fig. 4a and b. In this anchoring device a steel rod is used to wrap around it the FRP strip; this steel rod is secured by a steel plate of equal length which is bolted by two anchor bolts as shown in Fig. 4b.

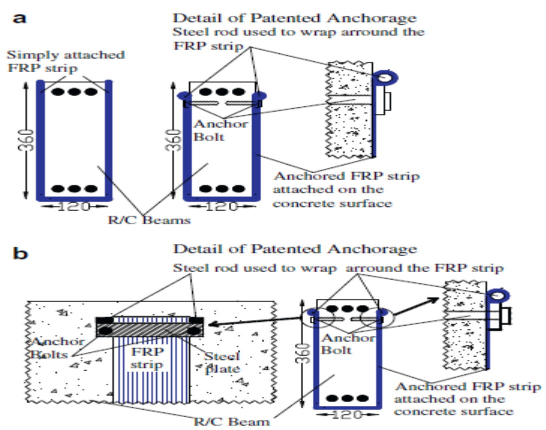



Fig. 4. Cross section of the tested rectangular beam specimens with the reinforcement details and the proposed anchoring device (dimensions in mm) [68]

Their paper deals with the objective of upgrading the shear capacity of R/C beams that are poorly reinforced against shear, as part of such a retrofitting scheme [70-72]. Various researchers have derived models for the prediction of the bond-slip response and required anchor lengths of FRP sheets attached to concrete [73-79]. It is evident that the maximum recorded shear capacity for all strengthened rectangular beams with CFRP/SFRP strips was significantly greater than the shear capacity of either the CRB or the CRBs specimens. The experimentally determined Young's modulus E, yield strength fy and ultimate strength fu are reported in Table 3.

Table 3. Material properties of employed steel plates, rebars and bolts [68]

| Material property | Longitudinal rebars of $d = 20$ mm diameter | Transverse rebars (stirrups) $d = 8$ mm | Steel plates of anchoring device | Anchor bolts Hilti HUS |
|---|---|---|---|---|
| $E$ (MPa) | 202,700 | 203,600 | 210,000 | 210,000 |
| $f_y$ (MPa) | 527 | 570 | 255 | 800 |
| $f_u$ (MPa) | 645 | 710 | 390 | 917 |

Both the experimentally and the analytically determined material properties are reported in Table 4.

Table 4. Material properties of CFRP and SFRP [68]

| Material property | $E_1$ (MPa) | $v_{12}$ | $f_{u1}$ (MPa) | $E_2$ (MPa) | $G_{12}$ (MPa) |
|---|---|---|---|---|---|
| CFRP | 41,784 | 0.305 | 376 | 4476 | 1935 |
| SFRP | 36,350 | 0.312 | 330 | 4400 | 1900 |

The general purpose FE software ABAQUS [80] was employed to generate FE models to simulate numerically the structural response of the previously described concrete beams strengthened with externally attached FRP sheets. Fig. 5.

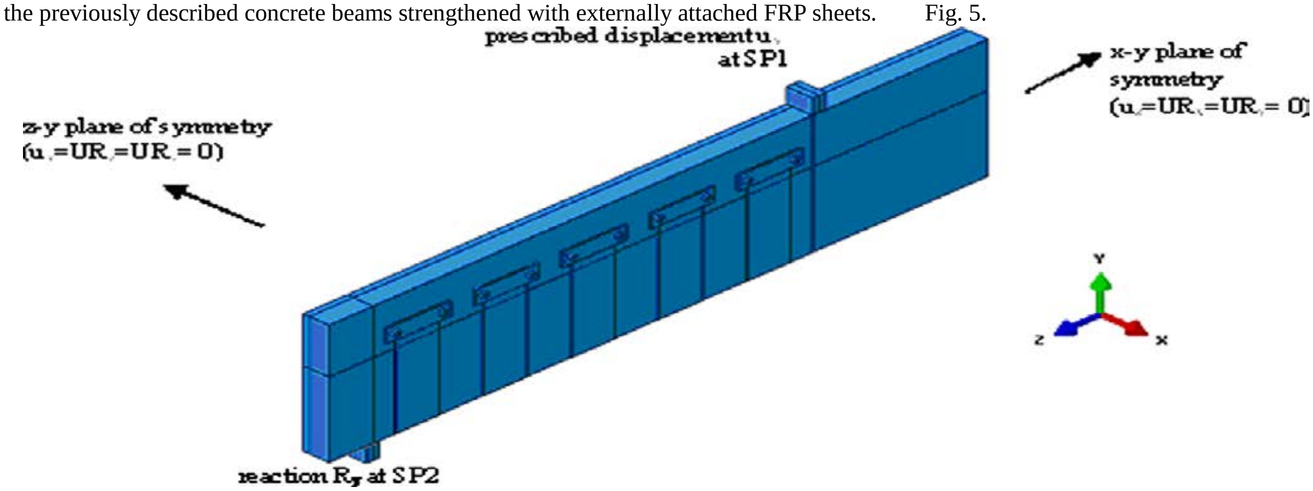



Fig. 5. Modelled geometry and applied boundary conditions [68]

This successful numerical simulation predicts with a very good degree of approximation the observed load-deformation behaviour and the ultimate shear capacity of all these specimens as well as the observed modes of failure including diagonal concrete cracking, debonding of the FRP strips in the case of no anchoring, or the plastification of parts of the anchoring devices plus the adjacent crushing of the concrete.

Hee Sun Kim, Yeong Soo Shin [81], presented an experimental work and their paper reports empirical studies of reinforced concrete (RC) beams retrofitted with new hybrid fiber reinforced polymer (FRP) system consisting carbon FRP (CFRP) and glass FRP (GFRP). From the FRP retrofitted beams, rip-off type of failure modes A and B are observed as illustrated in Figs. 6 and 7, respectively. Failure mode A is failure from a horizontal crack. When FRPs begin to experience tensile stresses, the shear stresses along the beam length begin to flow onto the internal longitudinal reinforcements and initiate horizontal cracks, causing failure of the weak covering and rupture of the FRPs with concrete. Failure mode B is caused by the development of vertical cracks. Vertical cracks begin to develop from both sides of the beam, which leads to rupture of FRPs with concrete.

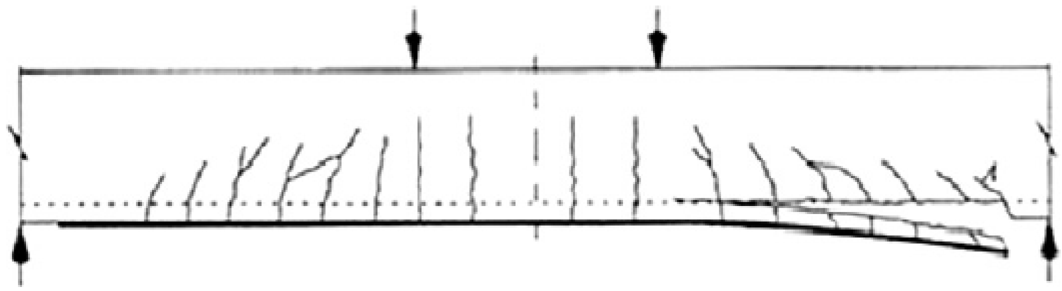


Fig. 6. Failure mode A. [81]

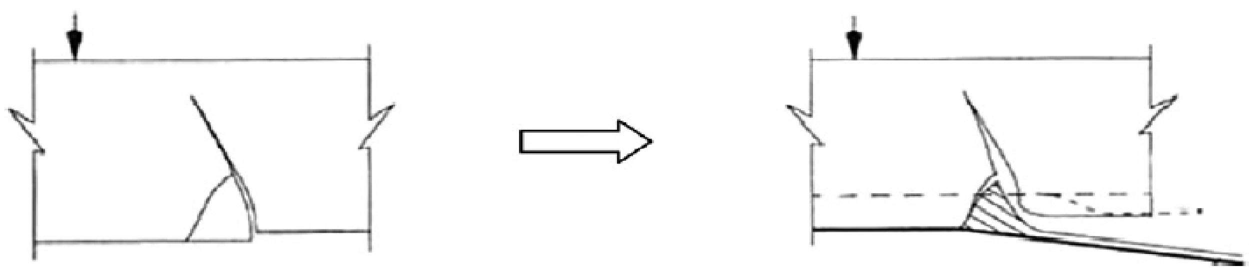


Fig. 7. Failure mode B. [81]

As the applied load increased, cracks propagated from the beam center and loaded points. Using hybrid FRPs is effective in improving the ultimate strength and stiffness of a strengthened beam. If different types of FRPs are attached to the surface of RC beam, the order of attaching affects the strength, stiffness and ductility of the RC beams retrofitted with hybrid FRPs. From the tests, the beams with glass fiber attached prior to carbon fiber show the most improved strength and ductility.

## 3. Flexure

Experimental studies have been performed and reported by Swamy and Roberts [82], Saadatmanesh and Eshani [83], Ritchie [84], Chajes et al. [85], and Sharif et al. [86] to investigate flexural, shear, and failure modes of FRP retrofitted RC beams. These studies indicated failure modes that can limit the strengthening effect of FRP retrofitted structures. The observed failures include delamination of the FRP, debonding of concrete layers, and peeling due to shear crack, which occurs at significantly lower loads than the theoretical strength of the retrofitted beams. More recently, Kachlakev and McCurry [87] performed four-point bending experiments on FRP strengthened RC beams and compared structural behavior of the beams retrofitted by CFRP (carbon FRP) for flexural strengthening and by GFRP (glass FRP) for shear strengthening. Wu et al. [88] studied hybrid FRP retrofitted RC beams using CFRP and GFRP for tensile and shear strengthening, respectively. In their experiments, the tested beams were wrapped with GFRP sheet after the CFRP sheets were attached to the bottom side of the tested beams. The FRP retrofitting technique for bridge and continuous RC beam applications has been investigated by Pham and Al-Mahaidi [89] and Ashour et al.[90], respectively. Both studies used CFRP sheets and observed the failure mechanism of the retrofitted beams. Ashour et al. [90], tested 16 reinforced concrete (RC) continuous beams with different arrangements of internal steel bars and external CFRP laminates. All test specimens had the same geometrical dimensions and were classified into three groups according to the amount of internal steel reinforcement. Each group included one non-strengthened control beam designed to fail in flexure. Three failure modes were observed, namely laminate rupture, laminate separation and peeling failure of the concrete cover attached to the composite laminate. The ductility of all strengthened beams was reduced in comparison with their respective reference beam. Additionally, simplified methods for estimating the flexural load capacity and the interface shear stresses between the adhesive and the concrete material were presented. As in previous studies, they observed that increasing the CFRP sheet length in order to cover the entire negative or positive moment zones did not prevent peeling failure of the CFRP laminates. The effect of initial load on the structural behavior of FRP retrofitted RC beams was studied by Shin and Lee [91] and Wenwei and Guo [92], who performed experiments on CFRP retrofitted RC beams under different sustaining loads. The former study showed that the different sustain load levels have influence on the deflection of the beam. The latter study investigated the effect of initial load on the ultimate strength of the CFRP retrofitted RC beams and a theoretical model was proposed that can predict ultimate strength of the beams retrofitted under sustaining loads. Recent studies of shear capacity of FRP retrofitted beams were presented by Bencardino et al. [93], Jayaprakash et al. [94], Bousselham and Chaallal [95]. The first study tested CFRP retrofitted RC beams cast without shear reinforcements and observed failure modes in shear. It was concluded that the FRP retrofitted RC beams can avoid shear failure if a carefully designed anchorage system is installed in the beams. Bousselham and Chaallal [95] presented the effects of FRP thickness, transverse steel ratio, and beam size on shear capacity of CFRP retrofitted RC beams and investigated the shear mechanism of the beams by analyzing load sharing between concrete, CFRP, and transverse steel bars. Jayaprakash et al.[67] showed that bi-directionally attached CFRP strips significantly increased the shear capacity of the RC beams. They also investigated the effect of orientation of CFRP strips on ultimate shear capacity as well as crack propagation of the retrofitted beams. The prediction of premature failures such as delamination and debonding has been studied by Roberts [96], Quantrill et al. [97], Malek et al. [98], Buyukozturk and Hearing [99], and Nguyen et al. [100]. Experimental and theoretical studies showed that the debonding load was dependent on strengthening materials and the thickness of the FRP layers. Compared to GFRP, CFRP with the same fiber volume fraction as GFRP shows higher strength per layer and higher stress concentration at the end of curtailment of the FRPs when attached to beams. In order to have the same strength as CFRP, the larger thickness of GFRP sheet was needed, which tended to cause early debonding failures. Therefore, FRP with low elastic modulus and high strength was needed to minimize the premature failure. Leung [101] studied debonding failure of the FRP retrofitted beams by examining stresses at the end of FRP plates and the effect of FRP plate thickness on FRP debonding strain. This study also recommended, under some practical situations, using U-shape FRP stirrups with a certain distance from the FRP plate end in order to prevent FRP debonding failure of the FRP retrofitted beams. Cracking behavior of FRP retrofitted beams has been reported by Ceroni and Pecce [102] and Tan and Saha [103]. The former examined crack spacing and width of the beams from experiments then compared these to the analytical model in order to evaluate the effectiveness of the existing code formulations. The latter studied cracking behavior of retrofitted RC beams under long term and short term loading and proposed formulations for predicting crack width of the beams as a function of applied loads.

## 4. Combined Torsion and Shear and Flexure

The 1998 report by the ASCE-ACI Committee 445 on shear and torsion outlined the challenges of reviewing RC beams under combined shear and torsion and integrating and designating a physical significance for current torsion design provisions [104]. Very valuable contributions concerning the behavior of RC beams under combined shear and torsion were made by several researchers [105-107,45].

A milestone point in the analysis of RC beams under combined shear and torsion was the work presented by both Hsu, and Rahal and Collins [27,45]. Hsu presented a unified theory for combined shear and torsion ‘‘Softened Truss Model’’ that was based on: (1) equilibrium equations; (2) compatibility equations; (3) the softened constitutive laws of concrete [27]. Rahal and Collins [45] updated the existing space truss model to include; (1) concrete softening; (2) tension stiffening; (3) improved modeling for the cover spalling; and (4) an equivalent uniform stress distribution block for the concrete strut. Another key point in the history of RC beams under combined actions was the work by Greene and Belarbi [108]. They presented a ‘‘Combined-Action Softened Truss Model,’’ which was based on the ‘‘Softened Truss Model’’ by Hsu and Mo for pure torsion with improvements over existing models [45,109].

Grace et al.,1999 [110] investigated the behaviour of RC beams strengthened with CFRP and GFRP sheets and laminates. They studied the influence of the number of layers, epoxy types, and strengthening pattern on the response of the beams. They found that all beams experienced brittle failure, with appreciable enhancement in strength, thus requiring a higher factor of safety in design.

Experimental investigations, theoretical calculations and numerical simulations showed that strengthening the reinforced concrete beams with externally bonded CFRP sheets in the tension zone considerably increased the strength at bending, reduced deflections as well as cracks width (Ross et al., 1999 [111]; Sebastian, 2001 [112]; Smith & Teng, 2002 [75]; Yang et al., 2003 [113]; Aiello & Ombres, 2004 [114]). It also changed the behaviour of these beams under load and failure pattern. Most often the strengthened beams failed in a brittle way, mainly due to the loss of connection between the composite material and the concrete. The influence of the surface preparation of the concrete, adhesive type, and concrete strength on the overall bond strength is studied as well as characteristics of force transfer from the plate to concrete. They concluded that the surface preparation along with along with soundness of concrete could influence the ultimate bond strength. Thereafter, Study on de-bonding problems in concrete beams externally strengthened with FRP composites are carried out by many researchers. Many investigators used externally bonded FRP composites to improve the flexural strength of reinforced concrete members. To evaluate the flexural performance of the strengthened members, it is necessary to study flexural stiffness of FRP strengthened members at different stages, such as pre-cracking, post-cracking and post-yielding. However, only few studied are focused on the reinforced concrete members strengthened under pre-loading or pre-cracking (Arduni & Nanni, 1997) [115]. F.Ceroni(2010) [116] investigated the experimental program on Reinforced Concrete (RC) beams externally strengthened with carbon Fibre Reinforced Plastic (FRP) laminates and Near Surface Mounted (NSM) bars under monotonic and cyclic loads, the latter ones characterized by a low number of cycles in the elastic and post-elastic range.

Fiber Reinforced Polymer (FRP) composites can be effectively used as an external reinforcement for upgrading such structurally deficient RC structures [117]. Santhakumar et al. [118] presented the numerical study on un-retrofitted and retrofitted reinforced concrete beams subjected to combined bending and torsion using finite element method. They reported improvement in behavior by the addition of FRP laminate being effective only after initial cracking of the beam but no significant effect on the initial stiffness of beams. The laminates with ±450 fiber orientation were more effective for higher values of twisting to bending moment ratios. Analytical method for evaluating torsional capacity of FRP strengthened RC beams was presented by Ameli and Ronagh [119] by considering interaction of concrete, steel and FRP. The method was in close agreement with experimental results of fully wrapped beams. Grace et al., (2001) [120] investigated the experimental performance of CFRP strips used for flexural strengthening in the negative moment region of a full-scale reinforced concrete beam. They considered two categories of beams (I and II) for flexural strengthening. Category I beams were designed to fail in shear and Category II beams were designed to fail in flexure. Five full scale concrete beams of each category were tested. It was found that Category I beams failed by diagonal cracking with local debonding at the top of the beams, meanwhile Category II beams failed by delamination at the interface of the CFRP strips and the concrete surface, both with and without concrete-cover failure by means shear/tension delamination. When the beams failed, the CFRP strips were not stressed to their maximum capacity, which led to ductile failures in all the beams. The maximum increase of load-carrying capacity due to strengthening was observed to be 29% for Category I beams, and 40% for Category II beams with respect to corresponding control beams. On the other hand, Grace et al. [121] performed another research work where three continuous beams were tested. One of those beam was considered as the reference beam and conventional ductile flexural failure occurred. They strengthened the other two beams along their negative and positive moment regions around the top and bottom face on both sides as a U-wrap. It was concluded that the strengthened beams with the triaxial fabric showed greater ductility than those strengthened with CFRP sheets.

In another research, El-Refaie et al. [122] examined 11 reinforced concrete (RC) two-span beams strengthened in flexure with external bonded CFRP sheets. According to the arrangement of the internal steel reinforcement, the beams were classified into two groups. Each group included one non-strengthened reference beam. It was noted that, all strengthened beams exhibited less ductility compared with the non-strengthened control beams. An optimum number of CFRP layers were found beyond which there was no further enhancement in the beam capacity. It was also investigated that extending the CFRP sheet length to cover the entire hogging or sagging zones did not prevent peeling failure of the CFRP sheets, which was the dominant failure mode of tested beams.

More recently, El-Refaie et al. [123] tested five reinforced concrete continuous beams strengthened in flexure with external CFRP laminates. All beams had the same geometrical dimensions and internal steel reinforcement. The main parameters examined were the position and form of the CFRP laminates. Three of the beams were strengthened using different lay-up arrangements of CFRP reinforcement, and one was strengthened using CFRP sheets. The performance of the CFRP strengthened beams was compared with a non-strengthened reference beam. It was found that, peeling failure was the principal failure mode for all the strengthened tested beams. It was found that the longitudinal elastic shear stresses at the adhesive/concrete interface calculated at beam failure were close to the limiting value recommended in (Concrete Society Technical Report 55, 2000) [59]. They also found that, strengthened beams at both sagging and hogging zone produced the highest load capacity.

Aiello et al. [124] compared the behaviour between continuous RC beams strengthened with of CFRP sheets at negative or positive moment regions and RC beams strengthened at both negative and positive moment regions. All the beams were strengthened with one CFRP sheet layer and with the remark that the beams were not loaded at the middle of span. The control beams underwent a typical flexural and failure of the strengthened beams occurred by debonding of the CFRP sheets, together with concrete crushing. It was found out that when the strengthening was applied to both hogging and sagging regions, the ultimate load capacity of the beams was the highest and about 20% of moment redistribution could be achieved by CFRP sheets externally glued in the sagging region. Maghsoudi et al. [125] examined the flexural behaviour and moment redistribution of reinforced high strength concrete (RHSC) continuous beams strengthened with carbon fibre. They observed that by increasing the number of CFRP layers, the ultimate strength increases, meanwhile ductility, moment redistribution, and ultimate strain of CFRP sheet decrease. Test results also showed that by increasing the number of CFRP sheet layers, there was a change in the failure mode from tensile rupture to IC debonding. End U-straps were effective in limiting end debonding, but not intermediate span debonding. Again, Akbarzadeh et al. [126] conducted an experimental program to study the flexural behaviour and moment redistribution of reinforced high strength concrete (RHSC) continuous beams strengthened with CFRP and GFRP sheets. As the previous work, test results showed that by increasing the number of CFRP sheet layers, the ultimate strength increases, while ductility, moment redistribution, and ultimate strain of CFRP sheet decrease. However, by using the GFRP sheets in strengthening the continuous beams, it is possible to reduce the loss in ductility and moment redistribution but a significant increase in the ultimate strength cannot be achieved. The moment enhancement ratio of the strengthened continuous beams was significantly higher than the ultimate load enhancement ratio for the same beam. They also developed an analytical model for moment–curvature and load capacity which they used for the tested continuous beams in this current study and in other similar researches.

Majid Mohammed Ali Kadhim [127] focused on the behavior of the high strength concrete continuous beam eams using engineered cementitious composites (ECC) and found that the ECC could improve the fatigue life of the beam in controlling the growth of small cracks. Manfredi and Pecce [144] studied the failure modes and strengthened with carbon fibre-reinforced polymer (CFRP) sheet with different CFRP sheet lengths. Three full-scale continuous beams are analyzed under two points load, and the data of analysis are compared with the experimental data provided by other researchers. ANSYS program is used and the results obtained from analysis give good agreement with experimental data with respect to load–deflection curve, ultimate strength, and the crack patterns. The length of CFRP sheet is changed in the negative and positive regions and the results showed that the ultimate strength of the beam was reached when the value of Lsheet/Lspan reaches 1.0, and when the value decreases, the ultimate strength of beam also decreases a little (1.4%), but when it decreases less than 0.6, the ultimate strength also decreases a lot (15%). From the above information, it is, thus, clear that there lies a vast scope of research in the field of retrofitting of RC continuous beam. Although a great deal of research has been carried out on simply supported reinforced concrete (RC) beams strengthened with Fibre Reinforced Polymer composites (FRP), a few works has been focused on continuous beams.

More recently, through either experimental, the finite element or analytical approaches [128-133], extensive researches have been undertaken on behaviour of reinforced concrete beams strengthened by externally bonded FRP sheets for enhancing their flexural and shear performance. However, those studies primarily considered static behaviour of the FRP strengthened beams under monotonic loading. In fact, many structures such as bridges and marine structures are subjected to repeated cyclic loadings rather than static ones, and this is often overlooked in the analysis and design of RC beams strengthened with FRP sheets. It has been well established that externally bonding of the FRP on RC beams is an effective strengthening technique to increase their static strength and ductility, as well as fatigue resistance with high energy dissipation [134-139]. However, the scarcity of experimental data on fatigue behaviour of RC beams strengthened by the FRP sheet is unanimously recognized [140], which does not satisfy the design need. At the present, the main research work increasingly focuses on experimental flexural fatigue and shear fatigue. Nanni [141] showed that steel fibre reinforced concrete (SFRC) could enhance the fatigue performance and the fibre content was an effective parameter to influence the fatigue characteristics of beams tested. Chang and Chai [142] developed a test methodology to investigate the flexural fracture and fatigue of the SFRC beams. Leung et al. [143] studied the flexural fatigue performance of concrete b

the relationship between the damage function and the cyclic degradations of the normal and high strength concrete beams under monotonic and cyclic loading. Research on concrete beams strengthened with carbon fibre reinforced polymer (CFRP) was carried out to investigate the influence of loading history on the fatigue life and crack width [145], and to analyze the relationship between the fatigue performance and the electrical property of CFRP under flexural loading [8]. Although a reasonable amount of research has been undertaken on the flexural fatigue [142-149], research on the shear fatigue performance of RC beams strengthened by CFRP and glass fibre reinforced polymer (GFRP) sheets however is limited up to date. Kwak and Kim [150] focused on the shear fatigue loading on the fatigue behaviour and strength of the polymer reinforced concrete (PRC) beams. Czaderski and Motavalli [138] studied RC beams strengthened by the CFRP Lshaped plates under shear fatigue loading. Moreover, some experimental research has also been conducted on the bonding behaviour between the FRP sheet and concrete under cyclic fatigue loading [15,131,134,135,140], since such the bonding behaviour influences the failure mode of the beam strengthened [148]. It was observed in these studies that if there was no obvious interfacial debonding between the FRP and concrete occurred fatigue behaviour of the FRP-strengthened beams would be improved. However, once debonding occurred, the deflection would be increased significantly and the tension cracks appeared near the position of the point loads applied [149]. Carloni et al. [151] investigated the role of the FRP-concrete interface debonding under fatigue loading and found that debonding occurred during fatigue, which was related to the load range applied.

## 5. Conclusions

Based on the experimental and numerical works carried out in previous studies for improving resistance of RC beams with FRPs, following conclusions are drawn:

1. All specimens wrapped with FRPs show better resistance in torsion, shear and flexure compared to the ones which are not retrofitted.
2. When the available bonded length is limited, the installation of FRP anchors is a viable option to prevent a brittle shear failure mode due to FRP debonding.
3. The CFRP strengthened beam has the highest ultimate strength but the lowest deflection, and the diagonal GFRP reinforcing arrangement is more effective than the vertical arrangement in enhancing the shear strength and stiffness.
4. The test results have shown that externally bonded CFRP or GFRP to the lateral and bottom face of a beam can increase the first crack load and ultimate strength greatly, arrest concrete crack extension, and enhance the rigidity of strengthened beams.
5. Strengthening of beams should not be seen as strengthening for torsion or shear or flexure but as a unified approach of strengthening for torsion and shear and flexure.

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