Probabilistic Metric Spaces and Some Contraction Mappings

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Abstract:
There exist many types of contraction mappings in the case of single valued and multi- valued that researchers are interested in proving fixed point theorems. Now, I consider two types of contraction: $(\varphi - k) - B$ contraction and bn-contraction . In this paper, after presenting the definition of $(\varphi - k) - B$ contraction, I prove a lemma about continuity of $(\varphi - k) - B$ contraction and by using it I prove a unique single valued fixed point theorem for $(\varphi - k) - B$ contraction with t-norm which is φ-convergent in probabilistic metric space. Then multi-valued bn-contraction definition is illustrated. I obtain first multi-valued theorem with new assumptions. Finally, I prove another fixed point theorem for multi-valued case by the definition of a large class of mappings called weakly demicompact mappings.

Keywords:
Probabilistic metric space, bn-contraction, (φ − k) − B contraction, fixed point.

1. Introduction

The notion of a probabilistic metric space was introduced by Menger [1]. There exist many types of fixed point theorems in the field of contraction mappings [2]. One such theorem was formulated by Sehgal and Bharucha-Reid [3] who introduced the B-contraction mapping in probabilistic metric spaces. Hicks [4] established C-contractions in probabilistic metric spaces while Radu generalized C-contraction [5]. One of the contractions which is introduced by Mihet is (φ−k)−B contraction [6]. He demonstrated that every (φ−k)−B contraction is a B-contraction. The multi-valued contraction in probabilistic metric spaces was introduced by Hadzic and Pap [7]. Pap et. al generalized the C-contraction to multi-valued (ψ − C)- contraction [8]. They also obtained fixed point theorems for multi-valued cases in different settings [9]. Mihet in [10] introduced the notion of bn-contraction. He proved a fixed point theorem for multi-valued version of the strict probabilistic bn-contractions [10,11]. Previously, we also considered multi-valued (ψ , φ, ε, λ)-contraction in probabilistic metric spaces [12].

The sub-divisions of this paper as follows: In section 2, some notions and concepts in probabilistic metric spaces and probabilistic contractions are recalled. In section 3, some theorems for (φ − k) − B contraction and multi-valued bn-contraction will be illustrated.

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2 Preliminary Notes

We recall some concepts from probabilistic metric space, convergence and contraction. For more details, we refer the reader to [13, 14].

Let D+ be the set of all distribution of functions F such that F(0) = 0 (F is a non-decreasing, left continuous mapping from R into [0, 1] such that $\lim_{x\to\infty} F(x) = 1$).

The ordered pair (S, F) is said to be a probabilistic metric space if S is a nonempty set and F : S × S → D+ (F(p, q) written by Fpq for every (p, q) ∈ S × S) satisfies the following conditions:

1) Fuv (x) = 1 for every x > 0 ⇒ u = v (u, v ∈ S),
2) Fuv = Fvu for all u, v ∈ S,
3) Fuv (x) = 1 and Fvw(y) = 1) Fuw(x + y) = 1 for all u, v,w ∈ S, and all x, y ∈ R+.

A Menger space is a triple (S, F, T) where (S, F) is a probabilistic metric space , T is a triangular norm (abbreviated t-norm) and the following inequality holds
Fuv (x + y) ≥ T(Fuw (x), Fwv(y)) for all u, v,w ∈ S, and all x, y ∈ R+.
Recall that the mapping T : [0, 1] × [0, 1] → [0, 1] is called a triangular norm (a t-norm) if
the following conditions are satisfied:
T(a, 1) = a for every a ∈ [0, 1]; T(a, b) = T(b, a) for all a, b∈ [0, 1]; a ≥ b, c ≥ d)
T(a, c) ≥ T(b, d) a, b, c, d ∈ [0, 1]; T(T(a, b), c) = T(a, T(b, c)), a, b, c ∈ [0, 1].
Basic examples are t-norms TL (Lukasiewicz t-norm), TP and TM, defined by TL(a, b) =
max{a+b−1, 0}, TP (a, b) = ab and TM(a, b) = min{a, b}.
If T is a t-norm and (x1, x2, . . . , xn) ∈ [0, 1]n (n ∈ N*) in which N* = N∪{+∞}, one can define recurrently $x_i = T(T_{i=1}^{n-1} x_i, x_n)$
for all n ≥2. One can also extend T to a countable infinitary operation by defining $T_{i=1}^{n} x_i$
for any sequence $(x_i)_{i\in N^*}$ as $\lim_{n\to\infty} T_{i=1}^{n} x_i$.
If q ∈ (0, 1) is given, we say that the t-norm T is q-convergent if $\lim_{n\to\infty} T_{i=1}^{\infty}(1 - q^i) = 1$.
We remark that if T is q-convergent, then,
$\forall\lambda \in (0, 1)$ ∃s = s(λ) ∈ N ∀n ∈ N $T_{i=1}^{\infty}(1 - q^{s+i}) > 1 - \lambda$.
Also note that if the t-norm T is q-convergent, then sup 0≤ $t < 1$ T(t, t) = 1.

Definition 2.1: Let (S, F, T) be a Menger space. If $\sup_{0\le t<1} T(t, t) = 1$, then the family $\{U_\varepsilon\}_{\varepsilon>0}$ where,

$$U_\varepsilon = \{(x, y) \in S\times S, | F_{x,y}(\varepsilon) > 1-\varepsilon\}$$

is a base for a metrizable uniformity on S, called the F-uniformity [2,13,14]. The F-uniformity naturally determines a metrizable topology on S, called the strong topology or F-topology [15], a subset O of S is F-open if for every p ∈ O there exists t > 0 such that $N_p = \{q \in S \mid F_{pq}(t) > 1 - t\} \subset O$.

Definition 2.2: Let φ : (0, 1) →(0, 1) be a mapping. We say that the t-norm T is φ-convergent if

$$\forall\delta \in (0,1)\ \forall\lambda \in (0,1)\ \ \exists s = s(\delta,\lambda) \in N\ \ \forall n \ge 1\ \ T_{i=1}^{\infty}(1-\varphi^{s+i}(\delta)) > 1-\lambda$$

Definition 2.3: A sequence $(x_n)_{n\in N}$ is called F-convergent sequence x ∈ S if for all ε > 0 and λ ∈ (0, 1) there exists $n_0 = n_0(\varepsilon,\lambda) \in N$ such that $\forall n \ge n_0\ \ F_{x_n,x}(\varepsilon) > 1-\lambda$

Definition 2.4: A sequence $(x_n)_{n\in N}$ is called a Cauchy sequence if for all ε > 0 and λ∈ (0, 1) there exists $n_0 = n_0(\varepsilon,\lambda) \in N$ such that $\forall n \ge n_0\ \ \forall m \in N\ \ F_{x_n,x}(\varepsilon) > 1-\lambda.$ We also have

$$\forall t > 0\ \ x_n \xrightarrow{F} x \ \Leftrightarrow\ \ F_{x_n,x}(t) \to 1.$$

A probabilistic metric space (S, F, T) is called sequentially complete if every Cauchy sequence is convergent. In the following, 2^S denotes the class of all nonempty subsets of the set S and C(S) is the class of all nonempty closed (in the F-topology) subsets of S. Now, we need a number of results about contraction mappings. We review them at below.

Definition 2.5 [16]: Let F be a probabilistic distance on S and M ∈ 2^S. A mapping f : S → 2^S is called continuous if for every ε> 0 there exists δ > 0, such that

$F_{uv}(\delta) > 1-\delta\ ) \Rightarrow\ \forall x \in fu\ \exists y \in fv : F_{xy}(\varepsilon) > 1-\varepsilon.$

Theorem 2.1 [16]: Let (S, F, T) be a complete Menger space such that $\sup_{0\le t<1} T(t, t) = 1$ and f : S →C(S) be a continuous mapping. If there exists a sequence$(t_n)_{n\in N} \subset (0,1)$ with $\sum_1^\infty t_n < \infty$ and a sequence $(x_n)_{n\in N} \subset$ S with the properties :

$$x_{n+1} \in fx_n\ for\ all\ n\ and\ \lim_{n\to\infty} T_{i=1}^{\infty} g_{n+i-1} = 1,$$

where $g_n := F_{x_n x_{n+1}}(t_n)$, then f has a fixed point.

One of important concept in this paper is (φ−k)−B contraction. Mihet introduced this concept in [6].

Now, we define comparison functions from the class ϕ of all mapping φ: (0, 1) →(0, 1) with the properties:

1) φ is an increasing bijection;
2) φ(λ) < λ $\forall\lambda \in (0,1)$.

Since every such a comparison mapping is continuous, it is easy to see that if φ∈ ϕ then $\lim_{n\to\infty} \varphi^n(\lambda) = 0$ for every λ∈ (0, 1).

Definition 2.6 [6]: Let (S, F) be a probabilistic metric space, φ ∈ ϕ and k ∈ (0, 1) be given. A mapping f : S→ S is called a (φ − k) − B contraction on S if the following condition holds:

$$x,y \in S, \varepsilon \in (0,1)\ \lambda \in (0,1)\ F_{x,y}(\varepsilon) > 1-\lambda\ \Rightarrow\ F_{f(x),f(y)}(k\varepsilon) > 1-\varphi(\lambda)$$

Another contraction that we use is b_n-contraction.

Definition 2.7 [10]: Let (X, F) be a probabilistic metric space and $(b_n)_{n\in N}$ increasing sequence from (0, 1) such $\lim_{n\to\infty} b_n = 1$.

A mapping f : X → X is strict b_n-contraction if for every n ∈ N, there exists q_n ∈ (0, 1) and for all x_1, x_2 ∈ X, t > 0

$$F_{x_1,x_2}(t) > b_n\ \Rightarrow\ F_{f(x_1),f(x_2)}(q_n t) > b_n.$$

3 Main Results

This section consists of two parts. The first one is related to single valued fixed point, while the second one is about multi-valued theorems. In the first part, the definition of a continuous mapping is recalled and by using it, a Lemma and then a Theorem with new assumptions is proven.

Definition 3.1 : Let F be a probabilistic distance on S. A mapping f : S → S is called continuous if for every ε > 0 there exists δ> 0 such that

$$F_{u,v}(\delta) > 1-\delta\ \Rightarrow\ F_{fu,fv}(\varepsilon) > 1-\varepsilon.$$

The definition of the (φ−k)−B contraction was mentioned in Section 2. We proved two fixed point Theorems about (φ − k) − B contractions in [17]. These Theorems are recalled below.

Theorem 3.1: Let (S, F, T) be a complete Menger space, T be a t-norm such that $\sup_{0\le t<1} T(t, t) = 1$ and f : S → S a (φ − k) − B contraction. If $\lim_{t\to\infty} F_{x_0 f_{x_0}^m}(t) = 1$ for some x_0 ∈ S and m ∈ N, then there exists a unique fixed point x of the mapping f so that $x = \lim_{n\to\infty} f^n(x_0)$.

Theorem 3.2: Let (S, F, T) be a complete Menger space, T be a t-norm such that $\sup_{0\le t<1} T(t, t) = 1$ and f : S → S a (φ−k)−B contraction and suppose that for some p∈S and j > 0

$$sup_{x>j}x^j\left(1-F_{p,fp}(x)\right)<\infty$$

If t-norm T is φ-convergent, then there exists a unique fixed point z of mapping f and
z = $\lim_{l\to\infty} f^l p$.

Before starting the new Theorem, we need the following lemma and its proof.

Lemma 3.1: Every (φ − k) − B contraction is continuous.

Proof: Let ε> 0 then there exists δ∈ (0, 1) such that δ < ε. If $F_{u,v}(\delta) > 1-\delta$ then,
since f is a (φ− k) − B contraction we have $F_{fu,fv}(k\delta) > 1-\varphi(\delta)$, where we obtain that
$F_{fu,fv}(\varepsilon) \geq F_{fu,fv}(k\delta) > 1-\varphi(\delta) > 1-\delta > 1-\varepsilon$. Therefore; is continuous.

Theorem 3.3: Let (S, F, T) be a complete Menger space such that $\sup_{0\leq t<1} T(t, t) = 1$and let f : S →S be a (φ− k) − B contraction. If T is φ-convergent, i.e.,

$$\forall\delta>0\ \lim_{n\to\infty} T_{i=1}^{\infty}(1-\varphi(\delta)^i)=1 \qquad (1)$$

and $F_{p,fp} \in D^+$ for every p ∈ S, then there exists a unique fixed point x of the mapping f
and x = $\lim_{n\to\infty} f^n(p)$ for every p ∈ S.

Proof: Let p ∈ S and δ > 0 be such that $F_{p,fp}(\delta) > 0$. Since $F_{p,fp} \in D^+$, such a δ exists.
Let $\lambda_1 \in (0, 1)$ be such that $F_{p,fp}(\delta) > 1-\lambda_1$; by assumption we have:

$$F_{p,f^2p}(k\delta)>1-\varphi(\lambda_1)$$

and generally for every n ∈ N

$$F_{f^np,f^{n+1}p}(k^n\delta)>1-\varphi^n(\lambda_1) \qquad (2)$$

We will prove that $(f^np)_{n\in N}$ is a Cauchy sequence, i.e., for all ε > 0 and λ ∈(0, 1) there
exists $n_0(\varepsilon,\lambda)\in N$ such that $F_{f^np,f^{n+m}p}(\varepsilon)>1-\lambda$ for all n≥ $n_0(\varepsilon,\lambda)$ and m ∈ N. Let ε > 0
and λ∈(0, 1) be given. Since the series $\sum_{i=1}^{\infty} k^i\delta$ converges, there exists $n_0=n_0(\varepsilon)$ such that
$\sum_{i=1}^{\infty} k^i\delta<\varepsilon$. Then for every $n\geq n_0$.

$$F_{f^np,f^{n+m}p}(\varepsilon)\geq F_{f^np,f^{n+m}p}\left(\sum_{i=n_0}^{\infty}k^i\delta\right)$$
$$\geq F_{f^np,f^{n+m}p}\left(\sum_{i=n}^{n+m-1}k^i\delta\right)$$
$$\geq T(\dots(T(F_{f^np,f^{n+1}p}(k^n\delta),\ F_{f^{n+1}p,f^{n+2}p}(k^{n+1}\delta)\ ))),$$
$$\dots,\ F_{f^{n+m-1}p,f^{n+m}p}(k^{n+m-1}\delta)).$$

Let $n_1 = n_1(\lambda)$∈N be such that $T_{i=n_1}^{\infty}(1-\varphi(\lambda_1)^i)>1-\lambda$. Since relation (1) holds, the
number n_1 exists. Using relation (2) we obtain for all n ≥ max{n_0, n_1} and m ∈ M,

$$F_{f^np,f^{n+m}p}(\varepsilon)\geq T_{i=n}^{n+m-1}(1-\varphi^i(\lambda_1))$$
$$T_{i=n}^{\infty}(1-\varphi^i(\lambda_1))$$
$$\geq 1-\lambda.$$

By assumption the Menger space (S, F, T) is complete, so the sequence $(f^np)_{n\in N}$ is convergent to a value like x.
By lemma 3.1 f is continuous, so the relation x = $\lim_{n\to\infty} f^n(p)$implies that:

$$fx=f(\lim_{n\to\infty}f^np)=\lim_{n\to\infty}f^{n+1}p=x.$$

It remains to prove the uniqueness of the fixed point x. Suppose that y = fy, y ≠x. If
ε> 0 be such that $F_{x,y}(\varepsilon)>0$ and $F_{x,y}(\varepsilon)>1-\lambda$ we have

$$F_{fx,fy}(k\varepsilon)>1-\varphi(\lambda)$$

and similarly

$$F_{x,y}(k^n\varepsilon)=F_{f^nx,f^ny}(k^n\varepsilon)>1-\varphi^n(\lambda) for\ every\ n\in N$$

Therefore, $F_{x,y}(u)$ = 1 , for every u > 0 (since $\lim_{n\to\infty}\varphi^n(\lambda)=0$) which contradicts to x≠ y.

In the second part, it is the turn of multi-valued case. Mihet in [10,11] introduced multi-valued b_n-contraction.

Definition 3.2: Let (X, F) be a probabilistic metric space and $(b_n)_{n\in N}$ an increasing
sequence from (0, 1) such that $\lim_{n\to\infty} b_n$ = 1, a mapping f : X → X is multi-valued
b_n-contraction if for every n ∈ N, there exists q_n ∈ (0, 1) and for all x, y ∈ X, ε> 0

$$F_{x,y}(\varepsilon)>b_n\Rightarrow\forall p\in fx\quad\exists q\in fy:\ F_{p,q}(q_n\varepsilon)>b_n.$$

Now, we will prove two new Theorems about multi-valued b_n-contraction by applying
new conditions.

Theorem 3.4: Let (S, F, T) be a complete Menger space with t-norm T such that
$\sup_{0\leq t<1} T(t, t) = 1$and f : S → C(S) be a multi-valued b_n- contraction. If there exist
p_0 ∈ S and $p_1 \in fp_0$ such that for all ε> 0 and n ∈ N, $F_{p_0,p_1}(\varepsilon)>b_n$ and $\sum_1^{\infty} q_n^n<\infty$
and $\lim_{n\to\infty} T_{i=1}^{\infty}b_{n+i-1}=1$ then f has a fixed point.

Proof: Let ε > 0 be given and δ ∈ (0, 1) be such that for every δ≤min{ε, 1 − b_n}. If

$F_{uv}(\delta) > b_n$, since f is a multi-valued b_n-contraction for each $x \in fu$ we can find $y \in fv$ such that $F_{xy}(q_n\delta) > b_n$. We can now obtain that for every $n \in N$, $F_{xy}(\varepsilon) > b_n$ holds. On the other hand, for enough large n, $b_n > 1 - \varepsilon$ so that $F_{xy}(\varepsilon) > 1 - \varepsilon$. This means that f is continuous.

Next, by the assumption, there exist $p_0 \in S$ and $p_1 \in fp_0$ such that for all $\varepsilon > 0$ and $n_0 \in N$, $F_{p_0,p_1}(\varepsilon) > b_{n_0}$. By using the contraction relation, we can find $p_2 \in fp_1$ such that $F_{p_0,p_1}(q_{n_0}\varepsilon) > b_{n_0}$ and by induction, we can find $p_{n+1} \in fp_n$ such that $F_{p_n,p_{n+1}}({q_{n_0}}^n\varepsilon) > b_{n_0}$ every $n_0 \in N$, especially for $n_0 = n$. Defining $t_n = q_n^n\varepsilon$, we have $g_j = F_{p_j,p_{j+1}}(t_j) \ge b_j$. On the other hand, $\sum q_n^n < \varepsilon$, so $\lim_{n\to\infty} T_{i=1}^{\infty} g_{n+i-1} \ge \lim_{n\to\infty} T_{i=1}^{\infty} b_{n+i-1} = 1$. Now we can apply Theorem 2.1 to find a fixed point of f.

We need the definition of a large class of mappings called weakly demicompact mappings.
This definition is necessary for the next theorem.

Definition 3.3 [17]: Let (S, F) be a probabilistic metric space, M a nonempty subset of S and $f : M \to 2^S - \{\emptyset\}$, a mapping f is weakly demicompact if for every sequence $(p_n)_{n\in N}$ from M such that $p_{n+1} \in fp_n$, for every $n \in N$ and $\lim F_{p_{n+1},p_n}(\varepsilon)$= 1, for every $\varepsilon > 0$, there exists a convergent subsequence $(p_{n_j})_{j\in N}$.

In the next theorem, we will not use the conditions of Theorem 3.4. We use the condition of weakly demicompact for multi-valued b_n-contraction.

Theorem 3.5: Let (S, F, T) be a complete Menger space, T a t-norm such that $\sup_{0\le t<1} T(t, t) = 1$, M a non-empty and closed subset of S, $f : M \to C(M)$ be a multi valued b_n-contraction that is also weakly demicompact. If there exist $p_0 \in M$ and $p_1 \in fp_0$ such that for all $\varepsilon > 0$ and $n \in N$. $F_{p_0,p_1}(\varepsilon) > b_n$ and $\lim_{n\to\infty} q_n^n = 0$, then f has a fixed point.

Proof: We can construct a sequence $(p_n)_{n\in N}$ from M, such that $p_1 \in fp_0$, $p_{n+1} \in fp_n$.
Given $t > 0$ and $\lambda \in (0, 1)$, we will show that

$$\lim_{n\to\infty} F_{p_{n+1},p_n}(t) = 1 \qquad (3)$$

Indeed, by assumption there exist $p_0 \in M$ and $p_1 \in fp_0$ such that for all $\varepsilon > 0$ and $n_0 \in N$, $F_{p_0,p_1}(\varepsilon) > b_{n_0}$. By using the contraction relation we can find $p_2 \in fp_1$ such that $F_{p_1,p_2}(q_{n_0}\varepsilon) > b_{n_0}$ and by induction $p_{n+1} \in fp_n$ such that $F_{p_n,p_{n+1}}(q_{n_0}\varepsilon) > b_{n_0}$ for every $n_0 \in N$, especially for $n_0 = n$. Since $\lim_{n\to\infty} q_n^n = 0$ and $\lim_{n\to\infty} b_n = 1$, for all $t > 0$ and $\lambda \in (0, 1)$ by choosing enough large n, $q_n^n \varepsilon < t$ and $b_n > 1 - \lambda$, so $F_{p_{n+1},p_n}(t) > 1 - \lambda$, the proof of (3) is complete. By definition 3.3, there exist a subsequence $(p_{n_j})_{j\in N}$. such that $\lim_{j\to\infty} p_{n_j}$ exists. We shall prove that $x = \lim_{j\to\infty} p_{n_j}$ is a fixed point of f. Since fx is closed, $fx = \overline{fx}$ and therefore, it remains to prove that $x \in \overline{fx}$, i.e. , for all $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exists $b'(\varepsilon,\lambda)$, such that $F_{x,b'(\varepsilon,\lambda)}(\varepsilon) > 1 - \lambda$. From the condition $\sup_{0\le t<1} T(t, t) = 1$, it follows that there exists $\eta(\lambda) \in (0, 1)$ such that

$$u > 1 - \eta(\lambda) \Rightarrow T(u,u) > 1 - \lambda.$$

Let $j_1(\varepsilon, \lambda) \in N$ be such that $F_{p_{n_j},x}\left(\frac{\varepsilon}{2q_n}\right) > b_n$ for every $j \ge j_1(\varepsilon, \lambda)$ and enough large n.
Since $x = \lim_{j\to\infty} p_{n_j}$ such a number $j_1(\varepsilon, \lambda)$ exists. As f is multi-valued (b_n)-contraction, for $p_{n_j+1} \in fp_{n_j}$ there exists $b'_j(\varepsilon) \in fx$ such that $F_{p_{n_j+1},b'_j(\varepsilon)}\left(\frac{\varepsilon}{2}\right) > b_n > 1 - \eta(\lambda)$ for all $j \ge j_1(\varepsilon, \lambda)$ and enough large n. From (3) it follows that $\lim_{j\to\infty} p_{n_j+1} = x$ and therefore there exists $j_2(\varepsilon, \lambda) \in N$ such that $F_{p_{n_j+1},x}\left(\frac{\varepsilon}{2}\right) > 1 - \eta(\lambda)$ for every $j \ge j_2(\varepsilon, \lambda)$. Let $j_3(\varepsilon, \lambda) = \max\{ j_1(\varepsilon, \lambda), j_2(\varepsilon, \lambda) \}$, then for every $j \ge j_3(\varepsilon, \lambda)$ we have $F_{x,b'_j(\varepsilon)} \ge T(F_{x,p_{n_j+1}}\left(\frac{\varepsilon}{2}\right), F_{p_{n_j+1},b'_j(\varepsilon)}\left(\frac{\varepsilon}{2}\right)) > 1 - \lambda$. Hence, if $j > j3(\varepsilon, \lambda)$, then we can choose $b'(\varepsilon,\lambda) = b'_j(\varepsilon) \in fx$. The proof is complete.

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