Image De-noising using DMWT and Neural Network Thresholding

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Summary

Over the last few years, modern image processing technologies have spread all the aspects of our life. The observer can notice that information security, medical diagnosis, military communication, to name but a few, all depends on image processing techniques. However, a crucial part of this field is image filtering and denoising, being the first phase in image processing. On the other hand, there are many types of research proposed different techniques in the trial to return the noised image to its original status, which is commonly known as image denoising, most of them still lack the efficiency and accuracy of doing so. This proposed technique benefits from one of the highly efficient techniques, which is digital multi-wavelet thresholding technique, and enhanced its efficiency using feed forward neural network, which is used in order to reach the best thresholding values. After evaluating this novel technique by conducting several experiments, it proofed its efficiency and accuracy in defining the thresholding values accurately and restoring the noised image to its original status. Moreover, this technique proofed its efficiency and enhanced results over previous techniques..

Key words:

Image De-noising, Digital Multi-wavelet Thresholding, Neural Network, Thresholding, Bayes Shrinkage.

1. Introduction

Image denoising or image restoration can be considered as one of the most vital problem in the image-processing field. Where it aims for reduction or removal of the distortions produced when the image is being obtained. So far many studies have been done in this field, one of them is Multi-Wavelet Transform [1] which is a representation of a square integrable (real- or complex-valued) function by a certain orthonormal series generated by a wavelet. This transformation can be applied to both digital signals and image processing in order to get data denoising and compression.

In the case of noise removal, the success of the multi-wavelet theory based techniques is used to obtain a decorrelated image (noise and useful information signal being separated) [2]. For de-noising of image based on multi-wavelet, there are five steps usually used [3]; which are 1) image pre-filtering, 2) computation of discrete multi-wavelet transform, 3) the filtration of multi-wavelet domain, which uses a thresholding technique, 4) applying Inverse Multi-wavelet Transform (IMWT) and 5) post-filtering to reconstructed image to get back the de-noised image coefficients in scalar form.

The essential idea driving this work is to acquire a gauge uncorrupted picture from a debased low-quality uproarious picture, which is often referred to as “Image De-noising”. Various techniques are available for this purpose [4][5] but the selection of an appropriate technique plays an important role in extracting the desired clean image. However, in this proposed work, thresholding techniques [6] have been used for this purpose, in addition to the neural network. A combination of these two techniques resulted in an efficiently cleaned image, the details of this novel technique in addition to the experiments and the obtained results are discussed in later sections.

Besides this section, several related works have been discussed in the second section. The third section briefly explains the main tools used in this proposed technique. However, the design and implementation of the proposed technique are explained and discussed in the fourth section. Where the conducted experiments along with their results are analyzed in the fifth section. Finally, yet importantly, the entire work is summarized and conclusions are mentioned in the sixth section.

2. Previous Works

Concentrating on image denoising based on thresholding and neural network techniques. This section begins by reviewing the major techniques, used in image denoising, then focuses the review on the works that used similar tools, which are used in this proposed work.

Generally speaking, there are many studies have been conducted on image denoising, which consists usually of three phases [7-11]:

1- The transformation from domain to a certain domain, where noise components and data components can be separated. In this phase, transformations usually are based on wavelet and multi-wavelet [12].
2- Detect noise components and remove them, thresholding technique used so often to

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accomplish this phase. Usually, thresholding technique such as soft and hard thresholds is based on Donoho and Johnstone [4] works.

3- Apply inverse transformations.

With neural network field raises, the scientists use it as denoising technique, which we will present more details about it in technical section.

Thresholding of sub-bands one of many techniques used in multi-wavelet transform based image denoising. After multi-wavelet transform have been applied to noisy image with a Gaussian noise, resultant coefficients from MWT will be threshold by making a comparison with a threshold, if the threshold greater than the MWT coefficient then, the last mentioned set to zero or it is modified.

The thresholding theory was developed by Donoho and Johnstone [4].Their work gives two rules or types of thresholding of thresholding known as hard thresholding and soft thresholding, the soft threshold is used as the base for proposed technique that uses Bayes shrinkage for training sets production and neural network to get the thresholding.

3. Technical Background

As mentioned earlier, the proposed work concentrates on image denoising based on thresholding, where Bayes shrinkage, feed-forward neural network, and multi-wavelet transformation techniques are combined together in order to obtain enhanced results. In this section, the mentioned tools, which are the major components of the proposed technique, are discussed briefly in addition to their values, which added to our work.

3.1 Bayes Shrinkage

Since thresholding technique became considerable interest (after De Vore and Lucier and Donoho and Kerkyacharian) [6], many thresholding rules have been developed, one of them is the Bayes Shrinkage rules.

Bayes shrinkage is a soft thresholding technique in which DWT coefficients whose absolute values smaller than a certain bound, are modified. Assume that the observed data are [4]:

$$X = S + N \qquad (1)$$

Where:

S: the original image.

N: Gaussian noise.

X: The noised image.

The de-noising process goes like this:

1- $Y = W(x)$ W is the MWT operator.

2- $Z = D(Y, \lambda)$ D is theshold operator with λ threshold.

3- $S^{`} = W^{-1}(Z), W^{-1}$ is Inverse Multi-Wavelet Transform (IMWT) and $S^{`}$ is an estimate of S.

Now, λ can be determined in Bayes shrinkage following the below steps:

Let us assume that (Y) is MWT of the noisy image, (S) being original image and (V) is the noise components. In considering the noise components follow Gaussian distribution$N(0, \sigma_{v^2})$, (S) and (V) are independent, σ_v σ_s and σ_y are stander deviation for V, S and Y respectively, and there relation with each other can be given by:

$$\sigma_y{}^2 = \sigma_s{}^2 + \sigma_v{}^2 \qquad (2)$$

Bayes shrinkage technique does a soft thresholding with adaptive data driven to obtain sub-band as well as level dependent near-optimal threshold. The thresholds given by:

If $\sigma_y \geq \sigma_v$ then $\lambda_s = \frac{\sigma_v^2}{\sigma_s}$ (3)

If $\sigma_y < \sigma_v$ then $\lambda_s = \max(|\mathrm{Am}|)$. where:

Where Am represent wavelet coefficients of the sub-bands that are being considered and λ_s: threshold for a certain sub band.

3.2 Artificial Neural Network

Artificial Neural Network (ANN) is a computational approach, which depends on a huge accumulation of neural units, considered as artificial neurons; freely displaying the way a natural cerebrum takes care of issues with extensive groups of organic neurons associated by axons. Each neural unit is associated with numerous neurons, and connections can implement or inhibitory in their impact on the actuation condition of associated neural units. Every single neural unit may have a summation function, which joins the estimations of every input together. There might be a thresholding function on every link and on the neural unit itself: with the end goal that the flag must outperform the breaking point before spreading to different neurons. These frameworks are self-learning and trained, instead of unequivocally modified, and exceed expectations in territories where the arrangement or highlight identification is hard to express in a customary computer program [5].

By investigating new applications for Neural Network, new applications in using the networks to find estimated optimal thresholds is found. Neural Network has already been used to de-noise images [5] but not as a threshold technique, which might be considered one of the contributions of this proposed technique. The network commonly used known as Convolutional Neural network (CNN) [5], or Multi-Layer Perceptron's (MLPs).

The common use of ANN in such problems is to map between noised and noise-free image [13]. In precisely, the common usage of neural networks is to map between noisy

image patches onto clean image patches where the noise is reduced or even removed.
A novel usage for ANN in this work is to give an accurate estimation for optimal thresholds. However, More details are explained in later sections.

3.2.1 Feed-Forward Neural Network

Feed-Forward Neural Network (FFNN) is a nonlinear function that maps vector-valued input via several hidden layers to vector-valued output. For instance, a Multi-Layer Perceptron's (MLP) with two hidden layers can be written as,

$$f(x) = b_3 + W_3 + \tanh\ (b_2 + W_2 \tanh\,(\,b_1 + W_1 x) \quad (4)$$

The weight matrices $W_1,\ W_2,\ W_3$ and the basis b_1, b_2, b_3, represent the parameters of the MLP. Moreover, the function $tanh$ performs in a component-wise. The architecture of an MLP is defined by the number of hidden layers and by the layer sizes.

3.3 Multi-wavelet Transform

The idea of Multi-wavelet originates from the generalization of scalar wavelets [14, 15]. Instead of one-scaling and one-wavelet function, multiple-scaling and multiple-wavelet functions are used. This lead to more degree of freedom in constructing Multi-wavelets. Therefore, opposed to scalar wavelets, properties such as orthogonally, symmetry, higher order of vanishing moments, compact sup-port, can be gathered simultaneously in Multi-wavelets. However, Multi-wavelets are constituted mainly of two types :

1. Orthogonal type such as Geronimo-Hardin-Massopust (GHM), Sym-metric Asymmetric (SA4), Chui-Lian (CL).
2. Bi-Orthogonal type such as Bi-Orthogonal Hermite (Bih52S).

The scaling functions θ_1 and θ_2 are symmetric (linear phase) and they have short support (two intervals or less). The coefficients of Multi-wavelets are a 2×2 matrix. It retains the orthogonality of the Multi-wavelets. The incoming signal is the scalar type and is converted to vector type by using pre-filter. The vector image is applied in discrete time to discrete Multi-wavelet transform for low-pass filtering, using low-pass filter coefficients and down sampled (decimated) by 2, to get c_k coefficients. On the other hand, high-pass filter coefficients are used for high-pass filtering and down sampled by 2 to get d_k coefficients. These coefficients are calculated following the below equations.

$$\phi(t) = \sum c_k\ (2t - k) \quad (5)$$

$$W(t) = \sum d_k\ (2t - k) \quad (6)$$

Where: $\phi(t)$ is a multi-scaling function, $W(t)$ is a Multi-wavelet function.

This is called the 2-band analysis bank. Which is perfect reconstruction synthesis bank recovers the image from the two down sampled outputs [16-18].
The sub-bands of a single-level Multi-wavelet decomposition is shown in **Figure 1**. It has 16 sub-bands of an image [19, 20]. Multi-wavelets are characterized by several scaling functions and associated wavelet functions as given in [21-23].

L1L1	L1L2	L1H1	L1H2
L2L1	L2L2	L2H1	L2H2
H1L1	H1L2	H1H1	H1H2
H2L1	H2L2	H2H1	H2H2

Figure 1: Image sub-bands after one level of Multi-wavelet decomposition

4. Proposed Technique

In this work, a new method to find optimal thresholds to be used alongside with multi-wavelet decomposition is proposed. The proposed threshold technique is based on Artificial Neural Network (ANN) approach in which the thresholds being estimated by a Feed-Forward Neural Network (FFNN), then these threshold values were used to threshold multi-wavelet coefficients. However, the proposed technique is constituted of six steps, which are as below:
Step 1: Original, noisy image is gray scaled.
Step 2: Pre-processing; implemented on noisy image, in order to transfer the multi-wavelet decomposition's scaler coefficients of an image into vector coefficients.
Step 3: It deals with the decomposition of pre-filtered image using various Multi-wavelets like GHM, CL, SA4, BiHermite52S, and their respective Multi-wavelet transforms.
Step 4: Thresholding methods are used to remove noise from decomposed image. By appling Multi-wavelet thresholds by Feed-Forward Neural network
Step 5: By applying Inverse Multi-Wavelet Transforms (IMWT), to thresholded coefficients, the de-noised vector output image can be obtained.
Step 6: Post filtering is done on the reconstructed image to get back the de-noised image coefficients in scalar form.
The main idea is to learn Multi-Layer Perceptron's (MLPs) network that can be used to find optimal thresholds. The parameters of MLPs network are estimated by training on thresholds extracted from Bayes shrinkage for a suitable number of images.
The computationally most intensive operations in the multi-wavelet and inverse multi-wavelet transform are the matrix-vectors manipulations, so well-equipped software

must be used. All experiments were performed on grey-scaled images obtained from grey-scaled originally color images.

Image manipulations during the process are done according to these equations:

$$\grave{X} = X + Z \quad (7)$$

Where $\grave{X}$: is the noised image, X: is the clean image, Z: is the Gaussian noise components.

$$Y = \phi(\grave{X}) \quad (8)$$

Where Y: is pre-scaled image, and ϕ expressed by:

$$\phi(t) = \sum ck\,(2t - k) \quad (9)$$

$$\grave{Y} = W(Y) \quad (10)$$

Where $\grave{Y}$: is the multi-wavelet transform result and W can be obtained by:

$$W(t) = \sum dk\,(2t - k) \quad (11)$$

$$f(x) = b_3 + W_3 + \tanh\,(b_2 + W_2 \tanh\,(b_1 + W_1 x))$$

(12)

Refer to neural network section above for variables descriptions. Then the soft shrinkage thresholding is done by finding applying the below equation, which represents the MWT Coefficients $F_s(x)$.

$$F_s(x) = \begin{Bmatrix} x - \lambda & x \geq \lambda \\ 0 & |x| < \lambda \\ x + \lambda & x \leq \lambda \end{Bmatrix} \quad (13)$$

λ is the threshold at certain sub-band.

4.1 Proposed Technique Evaluation

In the proposed technique, clean, colorful and standard images were used. However, these images were noised intentionally, for simplicity and to be able to trace the consequence of each step on them. Moreover, to be able to measure the performance of the proposed technique i.e. the resulted output image, versus the original image. Thus, the input and output of the proposed technique are as follow:

Input: original image without noise, which is be corrupted by Gaussian noise.

Output: de-noised image.

As summarized in the Flow-chart bellow, which shows the main stages of the proposed technique, the original image is initially converted to a noisy image that is decomposed using the disc

rete multi-wavelet transform with single-level decomposition, which yields to 16 sub-bands. Then yielded MWT coefficient to be thresholded, using thresholds obtained from the proposed neural network. Now the noise components removed and the image data need to be reconstructed by inverse multi-wavelet transform. The output from IMWT is a vector, so post-processing needed to convert this vector values to scalar values, witch is the de-noised image.

The results obtained are compared with the noisy image parameters, Bayes shrinkage, and NightSure shrinkage techniques, in order to prove significant of the proposed technique in image de-noising.

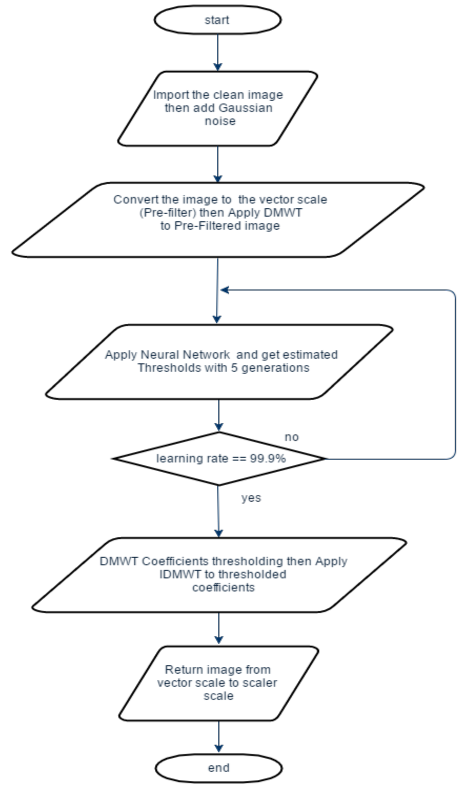

Figure 2: Proposed Technique used Flow-Chart

5. Experiments and Results Analysis

In order to evaluate the proposed technique, several experiments were conducted, by implementing the proposed technique on MATLAB 2015Ra. The results are obtained by applying the proposed technique on several standard images, which are commonly used for testing filters, and image processing techniques. In this section, the results for Lena, Barbara and Boat 512x512 images obtained from [24] are shown. These results were obtained using different de-noising techniques. The results when the de-noising process is applied onto the images are shown in **figures 3**, **4**, and **5** along with **table 1** showing the evaluated quality assessment metrics PSNR, SSIM, SD, and MSE, of the obtained results for the proposed algorithm and the other two techniques.

To have an accurate evaluation of the proposed technique and obtain a fair comparison with the other three techniques. Quantitative measurement tools were used. The way based on which they were used are briefly explained below.

5.1 Mean Square Error (MSE)

An estimator measures the average squares of the errors. The error being the difference between the original and the denoised images, which is given by [25]:

$$Error = (x(i,j) - n(i,j)) \quad (14)$$

Here $x(i,j)$ is the de-noised image and $n(i,j)$ is the noisy image. Thus, the mean square error can be expressed as:

$$MSE = \sum_{ij} \frac{error^2}{M * N} \quad (15)$$

Where $i, j = 1,2 \ldots. M$.

Both images should be of the same order, represented as $M \times N$ matrices. An MSE of zero means that the de-noised image obtained is perfect in accuracy same as the original image. This is the ideal situation, which is not practically possible.

5.2 Peak Signal to Noise Ratio (PSNR)

The ratio between the maximum signal power, to the power of the corrupting noise. The PSNR of an image is given by [26]:

$$PSNR = 10 \log_{10}\left(\frac{MAX_I^2}{MSE}\right) \quad (16)$$

Where Max $_I$ is the maximum possible pixel value of the image. When the pixels are represented using 8 bits per sample, which is 255, as in our case.

5.3 Standard Deviation (SD)

The measurement of the amount of variation or dispersion from the average. A low value of standard deviation shows that the data points tend to be very close to the mean also called the expected value while a high value of standard deviation shows that the data points are spread out over a large range of values [27]:

5.4 Structural Similarity Index Metric (SSIM)

A method used for measurement of the similarity between two images. The SSIM index is a measure of the quality of one image being compared to a reference original image of perfect quality. The SSIM can be expressed as [19]:

$$SSIM_{x,y} = \frac{(2u_x u_y + C_1)(2\sigma_{xy} + C_2)}{(u_x^2 + u_y^2 + C_1)(2\sigma_x^2 + \sigma_y^2 + C_2)} \quad (17)$$

Where x and y are two signals with μ being their respective mean and σ being their respective standard deviation and $C_1$and C_2 are constants. The resultant SSIM index, which has a range between -1 and 1, where the value of 1 is considered as ideal value and happen only when two images are identical. These measuring parameters, mentioned above, have been applied to the same original image and the de-noised version by Proposed, Bayes and Neigh sure techniques. The results of this experiment are shown below.

Figure 3: LENA image for all de-noised techniques

From top right and counter-clock-wise **Figure 2** shows noised image and the de-noised image for the proposed, Bayes shrinkage with wavelet transform and NightSure shrinkage with wavelet transform. It is clear from the figure that the Gaussian noise has done its effect on corrupting the original image. However, comparing the three techniques together it appears that the proposed technique has almost recovered the original image, while NightSure shrinkage technique, failed to obtain this recovery. Knowing that the same noised image was used. On the other hand, Bayes shrinkage has done a good correction to the image but it is still far away from the original one. While the proposed technique has almost recovered the original image, it has darkened it a little bit, but without affecting its features.

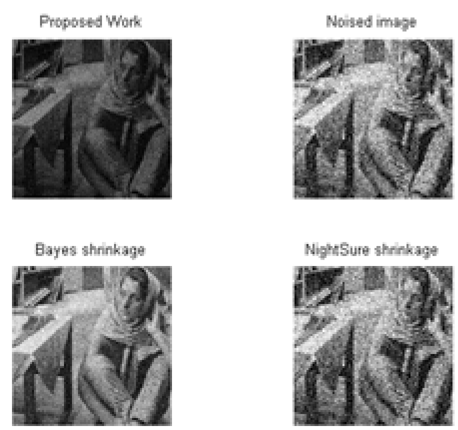

Figure 4: BARBARA Picture for all de-noised techniques

Moving to Barbara image, which is shown in **figure 4**, and following the same direction. From top right and moving counter-clock-wise, it shows the noised image and the resultant images from proposed, Bayes and NightSure techniques being applied on Barbara image. It is clearly noticed, as in Lena image, the proposed technique has proofed its efficiency over the other ones, while still showing a little darkness but without affecting the image features.

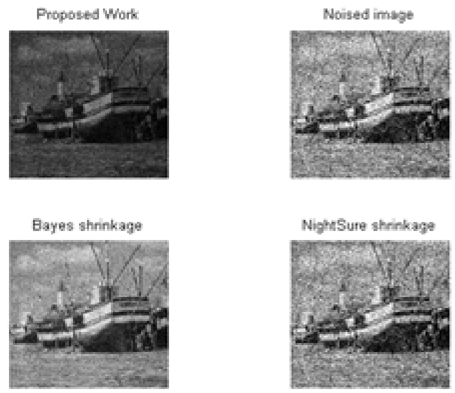

Figure 5: BOAT Picture for all de-noised techniques

On the other hand, **Figure 5**, shows from top right and moving counter-clock-wise, the noised image and the resultant images from proposed, Bayes and NightSure techniques after applying them on Boat image. Knowing that this image is more complex that the previous ones, as it is not a human face image, rather it contains several components of them are tiny objects. Even though, the proposed technique has shown its efficiency over the other techniques, by nearly restoring the original image, but with little bit darkness. While the other techniques almost failed to do so.
The below table summarizes the results of the evaluation tools after applying them on the above original and denoised images resulted from the proposed, Bayes shrinkage and NightSure shrinkage techniques.

Table 1: Measuring Parameters Values for three images and de-noised versions

Technique	*PSNR*	*SSIM*	*SD*	*MSE*
Boat image				
Noise value	28.2145	0.2353	57.3709	98.0920
Proposed	52.7416	0.3339	28.3901	0.3459
Bayes Shrink	29.4780	0.4277	48.0661	73.3298
Neigh Sure	28.2145	0.5075	46.1605	98.0920
Lena image				
Noise value	28.2136	0.1767	58.8001	98.1124
Proposed	65.4302	0.3194	31.4523	0.0186
Bayes Shrink	29.6738	0.3817	50.0883	70.0970
Neight Sure	28.2136	0.4796	48.1373	98.1124
Barbara image				
Noise value	28.2138	0.2901	63.6412	98.1073
Proposed	55.2757	0.332	33.4999	0.1930
Bayes Shrink	29.3618	0.4531	55.0077	75.3186
Neight Sure	28.2138	0.5838	53.7925	98.1073

6. Conclusions and Future Works

As mentioned earlier in this work, the proposed technique focuses on image denoising based on multiwavelet coefficients thresholding technique. However, the main contribution of this work is the use of Feed Forward Neural Network (FFNN), in finding the optimal threshold. After training the FFNN on real thresholding values obtained from previous authentic techniques i.e. Bayes shrinkage. The FFNN was able to reach better thresholding values, which in consequence, fulfilled enhanced and more accurate denoised images.

The proposed technique was evaluated by applying it to several standard images, where it proofed its efficiency over the other techniques that are previously proposed for this case. On the other hand, having reached enhanced and accurate results, the proposed technique added a little bit darkness to the image, but without affecting its efficiency and its features. However, it is worthy to concentrate on the image illumination issue in order to enhance the obtained results or at least obtain a denoised image with the same illumination level. This might be done by applying illumination level enhancement techniques on the resulted denoised image. Moreover, it is worthy to enhance this technique, which proofed its efficiency by expanding its scope to color images, in order to input color image noised mage and output colour-denoised image. As an overall conclusion, benefiting from the capabilities of FFNN enhanced the capabilities of thresholding techniques and was able to nearly restore the noised image to its original one.

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