New Performance Evaluation Clustering Algorithm Based on Context-Dependent Data Envelopment Analysis

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Abstract.
Data envelopment analysis (DEA) is a popular approach for measuring the relative efficiency of homogenous units that utilize multiple inputs to produce multiple outputs. In spite of few researches on the relationship between clustering approach and DEA, this paper proposes an in-depth look at conceptual definition of the performance of clustering units. This study is different in a very significant way; specifically two kinds of approaches were integrated to develop the model. The first method is context-dependent DEA proposed by Seiford et al. (2003); which have formed the basis of many previous studies. The second method is obtained from finding degree-DMU, since finding degree-unit is always a concern. Andersen et al. (1993) have proposed a model for finding super-efficient DMU. The main reason for applying the super efficiency approach is that: (i) in a group of people consisting of president (CEO), the vice president, the manager and the general public, it is a rational way of putting each specific member in its relevant cluster, (ii) for each cluster, a cluster ranking orders the members, (iii) if we number the clusters from 1 up to r, then cluster 1istop priority, cluster 2 has the second highest priority, etc. This paper is intended to cluster all DMUs with the help of these two approaches. Additionally, we compared our approach with context dependent DEA, and finally, the proposed approach has been applied to classify 25 branches of an Iranian commercial bank.

Key words:
Data envelopment analysis; clustering; Degree-DMU; context-dependent; decision making units.

1. Introduction

Data Envelopment Analysis (DEA) has been a standard tool for evaluating the relative efficiencies of Decision Making Units (DMUs) since the study of Charnes et al. [4] based on the seminar work of Farrell [6]. The fundamental of DEA applies the non-parametric mathematical programming approach to approximate piecewise frontiers and envelop the DMU data sets. The DEA model constructs a relative efficiency score by transforming the multiple-input/multiple-output from a ratio of a single virtual output into a single virtual input. DEA is opening up as a result of many successful applications and case studies which appeared in its literature due to its possibilities for use in cases which have been resistant to other approaches because of some unknown nature of the relations between multiple inputs and multiple outputs required in many of these activities. The idea of this study is to employ the super efficiency approach to cluster DMUs. Andersen and Petersen [1] developed a modified version of DEA based on comparison of technical efficient DMUs relative to a reference technology spanned by all other DMUs due to weakness of both CCR and ADD in ranking technical efficient DMUs. The basic idea of their study was to compare the DMU under evaluation with a linear combination of all other DMUs in the sample, i.e., the DMU itself is deleted. The method supporting this idea is Context-dependent DEA, as initiated and developed by Seiford et al [16]. Clustering is a powerful data exploratory approach of grouping a set of items in such a way that items in the same group (i.e. clusters) are similar to each other (in some sense) than to those in other groups and to displaying the feature structure information of a given set of data. In general, we may roughly separate clustering methods into the following categories: hierarchical clustering [8,9], mixture-model clustering [12,13], learning network clustering [7,10,11,17], objective function-based clustering, and partition clustering [3,19]. Most clustering algorithms are procedures that minimize total distinction; samples of such algorithms are k-means [5,8], fuzzy c-means (FCM) [3,18,19]. The main goal of clustering is to maximize the homogeneity of items within the same group and to maximize the heterogeneity of items in different groups. Particularly, let $D=\{d_1, d_2, d_3, \ldots, d_n\}$ where each d_i is an N-dimensional feature vector. Clustering is to arrange data group of D, each group being a cluster for D,

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such that group in a cluster are more comparable to each other than to those in other clusters [14]. Furthermore, we find that the earlier research has some limitations and requires some extensions. To start with, different production functions were produced using the CCR model in Po et al.'s [15] study, which sometimes has inadequate discriminative power. In practice, this may have a consequence for multiple efficient DMUs being generated, as a result a large number of clusters being formed.

Conventional DEA models divide DMUs into two groups: efficient and inefficient. The efficiency score of all efficient DMUs are equivalent to one and these models are unsuccessful to differentiate the efficient DMUs. Efficient DMUs are only characterized by an efficiency score of one, even though the performance of inefficient DMUs depends on the efficient DMUs. It is well known that adding or removing an inefficient DMU or a set of inefficient DMUs does not change the efficiencies of the existing DMUs and the efficient frontier. The performance of efficient DMUs isn't affected by the presence of inefficient DMUs. The inefficiency scores are altered only if the efficient frontier is changed. It means that the performance of DMUs depends solely on the recognized efficient frontier characterized by the DMUs with the unity efficiency. If the performance of inefficient DMUs worsens or boosts, the efficient DMUs may possibly have a unity efficiency score. In spite of the original DEA method, context-dependent DEA can distinguish which efficient DMU is a better option, corresponding to the inefficient DMU. The reason is that all efficient DMUs have an efficiency score of one [16]. As a matter of fact, a set of DMUs can be separated into different levels of efficient frontiers.

In this study, members of each cluster are obtained from each context-dependent DEA layer. Therefore, we use all layers as a base to cluster data. That is, we stop trying traditional clustering approaches of feature dissimilarity and propose a new approach by integrating Andersen et al. and context-dependent DEA approaches to cluster all DMUs. To avoid the non-hierarchy problem, we utilized Andersen and Petersen approach [1] as the base of clustering approach instead of using the conventional models. As an extension of previous studies, we consider layers of context-dependent DEA in our super-efficient framework in this study. The assessment of a DMU's performance depends on different input/output measures. If we delete the (original) efficient frontier, then the remaining (inefficient) DMUs will construct a new second-level efficient frontier. If we delete this new second-level efficient frontier, a third-level efficient frontier is constructed, and so on, until the set of DMU becomes empty. In this study, each of such efficient frontier provides an evaluation context for measuring degree-DMU, e.g., the second-level efficient frontier serves as the evaluation context for measuring degree-DMU located on the second-level (original) efficient frontier. In this way, we obtain layers in context-dependent DEA as mentioned in Seiford et al. [16]. We consider that DMUs clustered into one group are not different from the DEA-contexture point of view and at the same time, we find a deeper meaning for managerial decision-making. The inefficient DMUs in one cluster may have different efficiency scores, and some DMUs may perform better than others. Therefore, the inefficient DMUs in a given cluster are in the same mode as the approach in Po et al [15]. In summary, based on the previous works of Seiford et al. and Andersen et al. [1, 16], we propose a new integrated DEA model. Degree-unit-based clustering approach has stronger discriminative power to decrease the number of clusters. The main rationale behind the clustering is driven by the recognition that three groups of approaches are different generalizations of the same elementary formulation. In this study, we express the features of these approaches and show how they relate to the basic formulation.

The rest of this paper is organized as follows: Section 2.1 looks into the BCC model. Section 2.2 discusses the super efficiency approach from which the proposed clustering approach is developed. In the following section, the algorithm of the integrated clustering approach is established (Sect. 3), and furthermore, the layers are identified to establish the new integrated clustering approach. Section 4 gives two numerical examples to illustrate the proposed DEA clustering approach. Discussion is made using these empirical examples with a comparison of the resultant clusters derived from integrated clustering approaches. We also illustrate our method by comparing the results in Section 4 with results obtained by Context-dependent DEA approach (Sect. 5). Finally, in Section 6 conclusions are drawn. Throughout the paper, we suppose that the reader is familiar with at least the key works on DEA (see, e.g., Charnes et al. [4]), as we will not define basic concepts such as, e.g., production set, virtual inputs and outputs, return to scale, technical efficiencies.

2. Literature Review

2.1 BCC model

We focus on technical aspects of efficiency so that no price or cost data are necessary throughout this study. Banker et al. [2] extended the earlier work of Charnes et al. [4] by proposing the model for variable RTS (VRS or BCC). Suppose that we have n DMUs (decision making units) where each $\mathrm{DMU_j}$ j=1,…, n uses the same inputs as m, $\mathrm{X_{ij}}$ (i = 1,…,m), also in (possibly) different amounts to produce the same s outputs in (possibly) different amounts,

y_{rj} (r = 1,…,s). The BCC and CCR models vary only in consisting of an additional convexity constraint, $\sum_{j=1}^{n}\lambda_j = 1$, in the primal BCC model and an additional variable, u_o, in the dual BCC model as shown in Equation (2).

Primal BCC model (input oriented) Dual BCC model (input oriented)

$$\begin{aligned} &\text{Min} \quad \theta \\ &\text{s.t.} \quad \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{ip}, i = 1,\dots,m \\ &\qquad \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{rp}, r = 1,\dots,s \qquad (1) \\ &\qquad \sum_{j=1}^{n}\lambda_j = 1 \\ &\qquad \lambda_j \ge 0,\ j = 1,\dots,n \end{aligned}$$

$$\begin{aligned} &\text{Max} \quad \sum_{r=1}^{s} u_r y_{rp} - u_o \\ &\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ip} = 1, \qquad (2) \\ &\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_o \le 0; \quad j = 1,\dots,n \\ &\qquad u_r \ge 0, r = 1,\dots,s; v_i \ge 0, i = 1,\dots,m, \end{aligned}$$

The u_r and v_i in Equation (2) are the weights assigned to the rth output and the ith input, respectively. The primal and dual models are referred to as the envelopment and the multiplier forms, respectively. The multiplier DEA models can be explained as the ratio of the weighted sum of outputs to the weighted sum of inputs for every DMU, and with the assigning of weights to the inputs and outputs of DMUs, this ratio is maximized. The optimal value of the objective function in the CCR and BCC models is unity, however, DMU_p can be inefficient even if the optimal value of the objective function is less than unity.

2.2 Super efficiency model

To rank the relative efficiency of DMUs with unit efficiency, Andersen et al. [1] propose that evaluated unit be excluded from the mathematical program, leading to the following input oriented super efficiency model, depending on the unit p to be evaluated:

The AP - model Dual of the AP - model (input oriented)

$$\begin{aligned} &\alpha_p^* = \min \alpha_p \qquad (3) \\ &\text{s.t.} \quad \sum_{\substack{j=1\\ j\ne p}}^{n}\lambda_j x_{ij} \le \alpha_p x_{ip} \quad i = 1,\dots,m \\ &\qquad \sum_{\substack{j=1\\ j\ne p}}^{n}\lambda_j y_{rj} \ge y_{rp} \qquad r =,\dots,s \\ &\qquad \lambda_j \ge 0,\ j = 1,\dots,n \end{aligned}$$

$$\begin{aligned} &\max \sum_{r=1}^{s} u_r y_{rp} \qquad (4) \\ &\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} v_i x_{ij} \le 0; \quad j = 1,\dots,n \ \&\ j \ne p \\ &\qquad \sum_{r=1}^{s} v_i x_{ip} = 1 \\ &\qquad u_r \ge 0, r = 1,\dots,s; v_i \ge 0, i = 1,\dots,m, \end{aligned}$$

If the optimal objective value of the super efficiency model is greater than 1, DMU_p that is efficient in the BCC model is super-efficient. Otherwise, $DMU_p$is not super-efficient. Hence the super efficiency model can be resolved for ranking efficient units without solving the BCC model (1).

Remark 1: To assess the VRS-AP model, we extend the original CRS-AP model (1) by adding a convexity constraint $\sum_{j=1}^{n}\lambda_j = 1$ to it.

Definition 1. The DMU that has the highest rank in the model (3) is called the degree-DMU.

3. The proposed clustering algorithm

Suppose that there are n decision making units; DMU_j (j=1,…,n), that uses inputs $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ to produce outputs.

The main purpose of this study is to introduce a new clustering algorithm which make all decision making units (DMUs) to cluster in reality overview. In this algorithm, we are trying to use the concept of the super efficiency and context-dependent DEA through some steps.

Supposing: $J_1^1 = \{DMU_j,\ j = 1,\dots,n\}$ and $C(k = 0) = \{\}$. Note that $C(k)$ shows the kth cluster of DMUs set.

Step 1: Put L=1 and K=1 (L counters the layers of context-dependent DEA).

Step 2: Find the efficient points of set J_L^K (using model (1)) and define:

T_L^K ={The efficient DMUs of Lth layer which one of them will belong to C(k)}

Step 3: By solving model (4), find the degree-DMUs of set J_L^K and put it in the cluster C(k).

Step 4: Put $J_{L+1}^K = J_L^K - T_L^K$. If $J_{L+1}^K \neq \emptyset$, then put L=L+1 and go to step 2, otherwise go to step 5

Step 5: put L=1.

Step 6: Let $J_L^{K+1} = J_1^K - C(K)$. If $J_L^{K+1} \neq \emptyset$, , then put K=K+1 and go to step 2. Otherwise, go to step 7.

Step 7: Stop and introduce $C(1), C(2), \ldots, C(K)$ that are the set of DMUs clusters.

Remark 2: Put all DMUs in the cluster C(K) in case of finding single efficient level.

Remark 3: In case of existing multiple optimal solutions of model (4) in layer L, the proposed algorithm puts all degree-DMUs in corresponding cluster.

Clearly, after removing degree-DMU from each efficient layer, the number of next efficient layers will be less than or at most equal to the current number of efficient layers.

In fact C(1) is the set of super-efficient DMUs of each layer. The action removes C(1) from the whole. The rational way for clustering a group of people consisting of president (CEO), the vice president, the managers and the general public is to put each specific member in its relevant cluster. DMUs set will be similar to delete the president (CEO) of each efficient layer. In addition, C(2) will be evaluated while finding degree-unit of each layer after removing C(1) from the whole set of DMUs. We cluster DMUs not only for ordering the members in each cluster (the member preference ranking is from left to right), but for ranking clusters, that is, if i<j then C(i) $\succ$C(j). The notation $\succ$represents the higher priority of ith cluster than jth cluster that is called layer hierarchy priority. The same explanation is applied to obtain other clusters.

4. Illustrative Examples

In this section, two numerical examples are presented. In the first case, a single input/output example with 14 DMUs provides a complete explanation of the presented method; the second one surveys a set of 25 DMUS with the multiple-input/multiple-output that clarify each step in greater detail. All of the related Models were solved using GAMS software.

4.1 Case 1.

Table 1 shows 14 DMUs evaluated on a single input/output. At the beginning, we will find the efficient point of set J_1^1 ={DMU_j; j=1,…,14} [using model(1)] and define T_1^1={1,10,11,12,14}. By solving the output-oriented model (4), degree-DMU is DMU12. Put Put $J_2^1 = J_1^1 - T_1^1$= {2, 3, 4, 5, 6, 7, 8, 9, 13}$\neq \emptyset$. The rest of the algorithm is as follows:

Table1. Sample DMUs

DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Input1	1.5	2	4	5.5	7	8	6	3	2.5	2	4.2	5	8	7
Output1	1	2	1.3	3	1.8	4	5	3	3.5	2.8	6	7	7.5	8

$$\left.\begin{array}{ll} & T_2^1=\{2,9,13\} \\ J_3^1=\{3,4,5,6,7,8\} & T_3^1=\{7,8\} \\ J_4^1=\{3,4,5,6\} & T_4^1=\{3,4,6\} \\ J_5^1=\{5\} & T_5^1=\{5\} \end{array}\right\} \Rightarrow C(1)=\{12,9,8,4,5\},$$

$$\left.\begin{array}{ll} J_1^2=\{1,2,3,6,7,10,11,13,14\} & T_1^2=\{1,10,11,14\} \\ J_2^2=\{2,3,6,7,13\} & T_2^2=\{2,13\} \\ J_3^2=\{3,6,7\} & T_3^2=\{3,7\} \end{array}\right\} \Rightarrow C(2)=\{11,2,7,6\},$$

$$\left.\begin{array}{ll} J_1^3=\{1,3,10,13,14\} & T_1^3=\{1,10,14\} \\ J_2^3=\{3,13\} & T_2^3=\{3,13\} \end{array}\right\} \Rightarrow C(3)=\{10,13\}$$

$$\left.\begin{array}{ll} J_1^4=\{1,3,14\} & T_1^4=\{1,14\} \\ J_2^4=\{3\} & T_2^4=\{3\} \end{array}\right\} \Rightarrow C(4)=\{14,3\}.$$

Due to $J_L^5 = J_1^4 - C(4)$ = {1}, so C (5) = {1}.

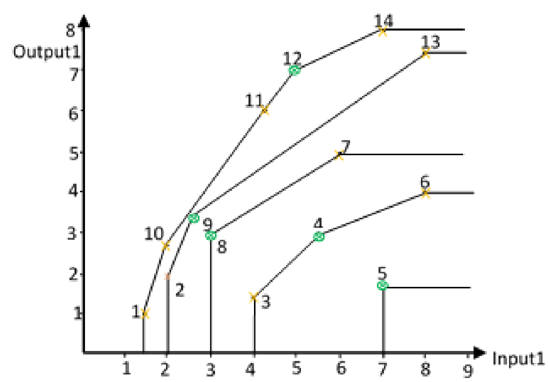

Fig. 1. Efficient frontiers from different levels - clustering DMUs into C (1) = {12, 9, 8, 4, 5}

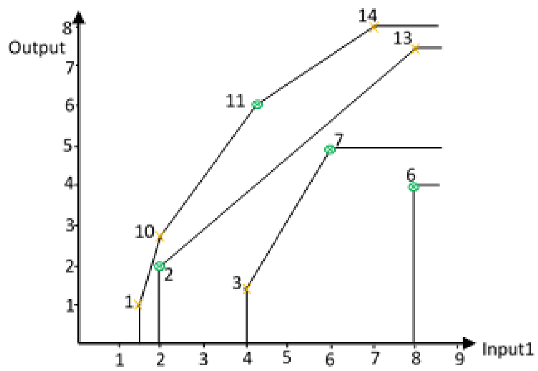

Fig. 2. Efficient frontiers from different levels - clustering DMUs into C (2) = {11, 2, 7, 6}

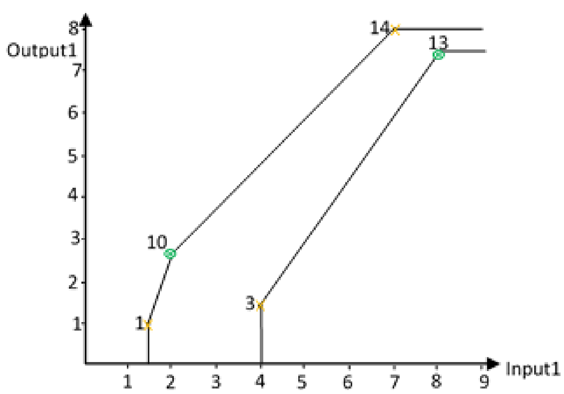

Fig. 3. Efficient frontiers from different levels - clustering DMUs into C (3) = {10, 13}

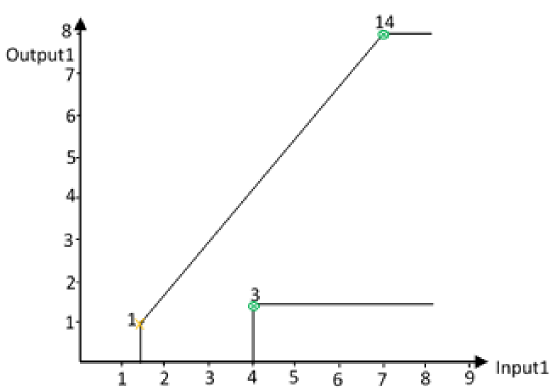

Fig. 4. Efficient frontiers from different levels - clustering DMUs into C (4) ={14, 3}

Note that the notation × shows that DMU belongs to the column while the notation $\otimes$ shows that unit belongs to C(k).

4.2 Case 2.

This case is related to the data set of 25 bank branches of a major Iranian Commercial Bank for 2012–2013. Table 2 shows that each of the 25 branches in the bank consumes three inputs which are 'Interest payable' (Interest payable is the amount of interest on its debt and capital leases that a company owes to its lenders and lease providers as of the balance sheet date.), 'Personnel' and 'Non-performing loans' (A nonperforming loan (NPL) is the sum of borrowed money upon which the debtor has not made his or her scheduled payments for at least 90 days.) to produce five outputs which are 'The total sum of four main deposits', 'Other deposits', 'Loans granted', 'Received interest' and 'Fee'. By using the proposed clustering algorithm, we have:

Step 1: Put K=L=1; J_1^1= {the set of 25 DMUs under evaluation} = **{**$DMU_1, \dots, DMU_{25}$**}.**
Step 2: Find the efficient units of J_1^1. By applying the original BCC model (1), the set of efficient DMUs is determined as follows:
T_1^1={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_8, DMU_9, DMU_{10} , DMU_{11} , DMU_{15} , DMU_{17} , DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}, DMU_{24}**}.**
Step 3: Let $J_2^1 = J_1^1 - T_1^1$& L=1+1=2.
Once again, the BCC model (1) will be utilized for detecting the efficient units in J_2^1 , set T_2^1= {The efficient DMUs of the second layer will determine which one of them will belong to C(1)}
T_2^1={DMU_2, DMU_{13}, DMU_{14} , DMU_{16} , DMU_{20}, DMU_{21}, DMU_{25}}.
Therefore, we set $J_3^1 = J_2^1 - T_2^1$ & L=2+1=3.
However, it is important to note that J_3^1 is a single set and there is no need to solve the BCC model (1). In other words, $DMU_{12}$is efficient and T_3^1= {DMU_{12}}. Hence the whole set of DMUs is cleaved into three efficient levels.
Step 4: To find degree-DMUs in three efficient levels, Andersen and Petersen (3) is used. This shows that DMU_{11}, $DMU_2$and $DMU_{12}$are degree-DMU members in the sets T_1^1, T_2^1 and T_3^1, respectively. These units are the members in the cluster C(1) which are ordered more significant from left to right:
C(1) = {DMU_{11}, DMU_2, DMU_{12}}.
Note that the priority order of DMUs in each cluster is from left to right. In terms of organizational hierarchy, DMUs which are in the left side of the cluster have higher position than the ones in the right side. According to this sequential arrangement, $DMU_{11}$is in a higher position than DMU_2 and DMU_2 is in a higher position than DMU_{12}.
Step 5: Set L=1, $J_1^2 = J_1^1$ −C(1), K=1+1=2, then go to step 2. The complete procedure is once again performed to the

efficient layers of the set J_1^2 and the following sets are sorted:
T_1^2={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_8, DMU_9, DMU_{10}, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}, DMU_{24}}.
T_2^2= {DMU_{13}, DMU_{14}, DMU_{16} , DMU_{20} , DMU_{21}, DMU_{25}}.
Therefore, degree-units in C(2), respectively, are: C(2) = { DMU_{10}, DMU_{16}}.
Accordingly, for the third time, put L=1, $J_1^3 = J_1^2 -$C(2) and k=2+1=3. Afterward, go to the step 2. Determine the efficient levels in the set J_1^3, which are as follows:
J_1^3={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_8, DMU_9, DMU_{13}, DMU_{14}, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{20}, DMU_{21}, DMU_{22}, DMU_{23}, DMU_{24}, DMU_{25}}.
T_1^3={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_8, DMU_9, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{22}, , DMU_{23}, DMU_{24}}.
T_2^3=J_2^3={DMU_{13}, DMU_{14}, DMU_{20} , DMU_{21} , DMU_{25}}.
The set of DMUs in the third cluster is: C(3) = {DMU_8 , DMU_{21}}. Then set L=1, $J_1^4 = J_1^3 -$C(3) and k=4. Then go the step 2. Find the efficient levels in J_1^4, so:
J_1^4={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_9, DMU_{13}, DMU_{14}, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{20}, DMU_{22}, DMU_{23}, DMU_{24}, DMU_{25}}.
T_1^4={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_9, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{22} , DMU_{23}, DMU_{24}}.
T_2^4=J_2^4={DMU_{13}, DMU_{14}, DMU_{20}, DMU_{25}}.
C(4) = {DMU_{24} , DMU_{20}}
In this example, the efficient units of each layer are the branches from different degrees, that is, the units in the first layer are 1st degree cluster, units in the second cluster are 2nd degree cluster and so on.

Similarly, we get the following results:
J_1^5={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_9, DMU_{13}, DMU_{14}, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}, DMU_{25}}.
T_1^5={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_9, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}}.
T_2^5=J_2^5={ DMU_{13}, DMU_{14}, DMU_{25}}.
C(5)= {DMU_9, DMU_{13}}.
J_1^6={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_{14}, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}, DMU_{25}}.
T_1^6={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_{15}, DMU_{17}, DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}}.
T_2^6=J_2^6={DMU_{14}, DMU_{25}}.
C(6)= {DMU_{17}, DMU_{14}}.
J_1^7={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_{15}, DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}, DMU_{25}}.
T_1^7={DMU_1, DMU_3, DMU_4 , DMU_5 , DMU_6, DMU_7, DMU_{15}, DMU_{18}, DMU_{19}, DMU_{22}, DMU_{23}}.
T_2^7=J_2^7={DMU_{25}}.
C(7)= {DMU_{19}, DMU_{25}}.
Remark 3: Due to finding single efficient level in this stage, all DMUs are put in the cluster C(8).
To find the preferences of DMUs in this cluster, we use the super efficiency model (1). The set of DMUs in the fourth cluster is:
J_1^8=T_1^8=C(8)={DMU_{22}, DMU_4, DMU_7 , DMU_6 , DMU_1, DMU_3, DMU_5, DMU_{15}, DMU_{23}, DMU_{18}}.
In this way, degree-units are obtained and excluded from J_1^8. When the set of J_1^8 becomes empty ($|J_1^8|$=1), this process is stopped.
Therefore, the 25 DMUs are clustered into the following 8 clusters C(1), C(2), C(3) , C(4) , C(5) , C(6) , C(7) and C(8). The results of the proposed algorithm are illustrated in the Table 3.

Table 2 The data for 25 Commercial bank branches

DMU (bank branches)	Inputs			Outputs				
	Interest payable	Personnel	Non-performing loans	The total sum of four main deposits	Other deposits	Loans granted	Received interest	Fee
1	5007.37	36.29	87243	3126798	382545	1853365	125740.28	6957.33
2	2926.81	18.8	9945	440355	117659	390203	37836.56	749.4
3	8732.7	25.74	47575	1061260	503089	1822028	108080.01	3174
4	945.93	20.81	19292	1213541	268460	542101	39273.37	510.93
5	8487.07	14.16	3428	395241	12136	142873	14165.44	92.3
6	13759.35	19.46	13929	1087392	111324	574355	72257.28	869.52
7	587.69	27.29	27827	165818	180617	323721	45847.48	370.81
8	4646.39	24.52	9070	416416	486431	1071812	73948.09	5882.53
9	1554.29	20.47	412036	410427	449336	1802942	189006.12	2506.67
10	17528.31	14.84	8638	768593	15192	2573512	791463.08	86.86
11	2444.34	20.42	500	696338	241081	2285079	20773.91	2283.08
12	7303.27	22.87	16148	481943	29553	275717	42790.14	559.85
13	9852.15	18.47	17163	574989	23043	431815	50255.75	836.82
14	4540.75	22.83	17918	342598	26172	126930	11948.04	1468.45
15	3039.58	39.32	51582	317186	270708	810088	111962.3	4335.24
16	6585.81	25.57	20975	347848	80453	379488	165524.22	399.8
17	4209.18	27.59	41960	835839	404579	9136507	41826.51	4555.42
18	1015.52	13.63	18641	320974	6330	29173	10877.78	274.7
19	5800.38	27.12	19500	679916	684372	3985900	95329.87	1914.25
20	1445.68	28.96	31700	120208	17495	308012	27934.19	471.22
21	4555.32	34.87	41521	358220	202321	359520	10875.21	2158
22	5665.87	16.24	21555	321586	34287	546825	28903.17	6257.22
23	8798.14	11.24	9522	516222	12933	225861	29852.11	552.12
24	2545.08	22.54	45588	411258	512364	125684	255951.54	3399.01
25	8552.54	18.98	25695	522351	10356	132555	26571.97	2221.87

Table 3 The result of the proposed algorithm on 25 Commercial bank branches

DMU (bank branches)	K=1			K=2		K=3		K=4		K=5		K=6		K=7		k=8
	T_1^1	T_2^1	T_3^1	T_1^2	T_2^2	T_1^3	T_2^3	T_1^4	T_2^4	T_1^5	T_2^5	T_1^6	T_2^6	T_1^7	T_2^7	T_1^8
DMU_1	×			×		×		×		×		×		×		⊗
DMU_2		⊗														
DMU_3	×			×		×		×		×		×		×		⊗
DMU_4	×			×		×		×		×		×		×		⊗
DMU_5	×			×		×		×		×		×		×		⊗
DMU_6	×			×		×		×		×		×		×		⊗
DMU_7	×			×		×		×		×		×		×		⊗
DMU_8	×			×		⊗										
DMU_9	×			×		×		×		⊗						
DMU_{10}	×			⊗												
DMU_{11}	⊗															
DMU_{12}			⊗													
DMU_{13}		×			×		×		×		⊗					
DMU_{14}		×			×		×		×		×		⊗			
DMU_{15}	×			×		×		×		×		×		×		⊗
DMU_{16}		×			⊗											
DMU_{17}	×			×		×		×		×		⊗				
DMU_{18}	×			×		×		×		×		×		×		⊗
DMU_{19}	×			×		×		×		×		×		⊗		
DMU_{20}		×			×		×		⊗							
DMU_{21}		×			×		⊗									
DMU_{22}	×			×		×		×		×		×		×		⊗
DMU_{23}	×			×		×		×		×		×		×		⊗
DMU_{24}	×			×		×		⊗								
DMU_{25}		×			×		×		×		×		×		⊗	

5. General Comparison

We provide performances comparison between the proposed method and the context-dependent DEA approach to show the features of our method and how it will be compared under standard DEA in terms of the quality of results. We note that the performance of the clusters in the proposed approach is measured by using model 1 and model 2 to get the most suitable view of DMUs in the society.

Context-dependent DEA is accounted a type of clustering approach. In this method, the DMUs which are located on Ith level belongs to the Ith cluster. The general distinction between the proposed method and the context-dependent DEA approach is can be stated as follows:

1. In our proposed algorithm, the members of each cluster have been sorted in cluster based on their preferences. However, in Context-dependent DEA clustering approach, the members of each cluster have the same priority. As an example for the first clusters of two approach in Case 1., we have:

First cluster in the proposed Method: C(1) = {12, 9, 8, 4, 5}.

First cluster in Context-dependent DEA approach (See Fig. 5): C(I) = {1, 10, 11, 12, 14}

To illustrate the difference between these two approaches, consider C(1):

DMU12≺ DMU09≺ DMU08≺ DMU04≺ DMU05

However, one could claim that using Context-dependent DEA clustering approach does not determine the member priority in C(I) (i.e. this results inclusion non-related DMUs in the cluster and produces unfair comparisons among the clusters). The main advantage compared to Context-dependent DEA clustering approach from society point of view is that the proposed method is better adapted to the specific occasions in organizations (for example some sessions and task bonus, …), namely each member in C(1) is the president (CEO) of each efficient layer.

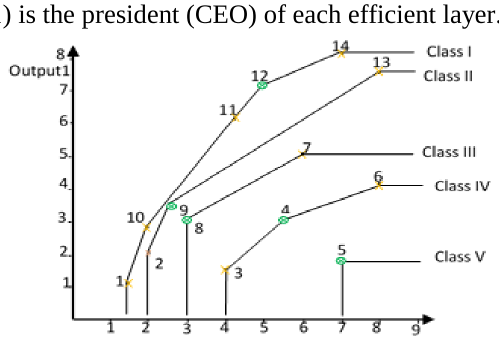

Fig. 5. Clusters obtained from Context-dependent DEA approach.

2. Indeed, the results from both of these clustering methods indicates that for i <j we have C(j) ≺ C(i) (C(i) is in the higher priority than C(j)).

Additionally, the clustering priority concept in our proposed method is different from Context-dependent DEA approach, the priority in Context-dependent DEA approach, is based on the performance efficiency value of the members in each cluster towards other cluster. But the priority concept in ours is according to the layers hierarchy preferences.

6. Summary and Conclusion

In this paper, with the help of degree-unit and the performance layers concepts, an algorithm was proposed to cluster DMUs. The proposed approach was derived from the DEA method to cluster the data with input and output items. Without loss of generality, while this approach has been followed through BCC model, the proposed approach can be plainly extended to other DEA models. Perhaps these clusters at first glance looks intangible as there exist DMUs from each efficient layers, but with a little more attention to definition of the clusters, the readers evidently perceive that such a clustering exists in everyday life of human society since every society includes different clusters and each cluster consists of the president (CEO), the vice president, manager and the general public. For instance, in all societies, some sessions are held to improve and solve the society's biggest problems (example 2 bank issues). These sessions, sometimes need to be held among managers or vice presidents or presidents (CEO) of each cluster. It is very important for managerial decision-making where decision-makers are interested in knowing the people required for being involved in these sessions so that it can be re-clustered into a different and desired cluster. In this paper, we have attempted to cluster some of DMUs following this principle. In summary, in view of the merits of the integrated clustering approach, it is peculiarly adjustable for clustering issues. Future researches need to identify the full and actual potential of this integrated clustering approach to be used for various clustering problems. Finally, we need to point out that the proposed integrated clustering algorithm is robust to analyze communities in hierarchical categories, also to same categories. Our future research will consider developing a robust-type DEA-based clustering algorithm.

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