Portfolio Performance Evaluation Using the DEA Based on That Variance is Placed Under Shadow of Mean or Vice Versa

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Abstract
Application of Data Envelopment Analysis (DEA) in selection of portfolio is for occasions when we are to consume rate of a capital in a financial market can be useful. In enforcement of this application, the most significant factors influencing on the performance evaluation of the investment companies are mean of the profit achieved from investment and it's variance. One of the discussed assumption in performance evaluation of portfolio can be in such a manner that the more the mean of the resulted profit and the less the variance, the better will be the performance. For this reasons mean as output and variance as input have been taken into consideration in the technique of DEA. But, since efficiency in DEA results from division of output by input, thus, variance may overshadow the mean, and a DMU which has a very low mean and little variance is to be placed better than a DMU which has a high mean and relatively-high variance. For this reason, in this article, a criterion is presented when variance as an input is supposed, but it is placed under shadow of mean. In other words, the first priority is evaluation via mean, but this evaluation must not be absolute priority. Supposing that there are n portfolios with equal inputs (purchase price) and various outputs (mean and variance) and considering variance as an undesirable output, we create a change in the mean with aid of the suggested relationship, and, then, technique of DEA will be enforced. Finally, this technique is to be employed on a real data set.

Keywords:

Management, Data envelopment analysis, Portfolio, Return, Risk.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric method for the purpose of determination of efficiency and estimation of production frontier of a set of Decision Making Units (DMU) with distinctive multiple inputs and multiple outputs. Farrell [6] began this method and, in continuation, it was developed by Charnes et al. [3]. In continuation, a lot of models were presented each of which had a specific technology from amongst which, as an example, CCR model with constant returns to scale and BCC model with variable returns to scale can be referred [2]. Using the inputs, each DMU produces the outputs. Therefore, a DMU is desirable that, by a low input, produces higher output; that is, we intend to decrease the input and increase the output, but this task may have not been conformed to the reality. If we consider the output as the factory's smoke or factory's losses, increase of these outputs known as undesirable outputs is not a right job and, regarding the inputs too, decrease of the undesirable inputs is not a right job as well. (In the trash recycling industry, trash is taken into consideration as undesirable input). In order to confront with this problem, a lot of approaches have been presented. Färe et al. [8] used a nonlinear model in order to increase desirable output and decrease undesirable output. Jahanshahloo et al. [11] employed a non-radial model for the purpose of simultaneous effectiveness on the undesirable inputs and outputs. Of other works, Hailu and Veeman [9], Färe and Grosskopf [7], Podinovski and Kuosmanen (weak disposability) [18] can be referred. One of the simplest suggested methods is to consider undesirable input as output and undesirable output as input.

Portfolio performance evaluation plays a remarkable role in the financial market decision makings. One of the ideas of portfolio evaluation is to use efficient portfolio frontier, namely criterion of portfolio evaluation is it's distance from efficiency frontier. In order to obtain efficiency frontier, Markowitz [16] presented a quadratic optimization model in the mean-variance framework. Due to problems of Markowitz method, Sharp [20] presented single-index method in which return market index is used, and calculation of the covariance material is not required. By development of these ideas, a lot of models known as diversification models (Nonlinear DEA models) were generated out of which, as an example , Mori and Mori [17] model can be referred that is a model inspired by DEA with nonlinear constraint. In fact, he considered the variance as input and the mean as an output. Of course, in some studies, semi-variance and value at risk [5] or conditional value at risk [19] have been used instead of variance, each of which has it's own related and specific advantages and defects. It is required to mention that the expressed models are often non-liner. Arditti [1], Kane [13] and Ho and Cheung [10] showed that positive skewness is a desirable criterion for the investors. Joro and Na [12] presented a model in the skewness-mean-variance framework in which variance is considered as input and mean and skewness as output. Lozano and Gutiérrez [15]

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presented a few diversification linear models (Adaptable to SSD) which differ from DEA classic models. Ding et al. [4] presented model in the mean-variance framework based on margin requirements and, then, used them for the purpose of portfolio performance evaluation through improvement of DEA models. Wenbin Liu et al. [14] presented a method in order to estimate the portfolio efficiency using DEA which we will express it in the next section briefly.

Now, we are confronted with some problems in order to evaluate portfolio performance using CCR model, dealing with expression of them. Consider CCR model with one input and one output which has two DMUs on the efficiency frontier. From view point of the DEA, these two DMUs are not different from each other at all. But, perhaps, this subject is not true from managerial perspective because decision maker (Manager) may give priority over one of these DMUs according to the his/her thoughts and demands. This theme made up our mind to present a method in order to obviate this problem based on opinion and viewpoint of manager and to use a suggestive method for the purpose of evaluation of portfolio performance via expression of managerial interpretation.

This article has been classified as following. In section 2, we deal with estimation of the efficient portfolio frontier of the Markowitz model using DEA. In section 3, we present a method to eliminate the problem created while using CCR model in the portfolio performance evaluation. In section 4, we deal with portfolios performance evaluation using suggested methods via undesirable data and present a managerial interpretation for it. Section 5 includes the numerical examples presented to describe suggestive methods. The paper comes to an end through conclusion in section 6.

2. Portfolio efficiency estimation using dea

Supposing that we have n financial assets with expected return μ and standard deviation σ. In order to obtain efficiency frontier, Markowitz presented the following model:

$$
\begin{aligned}
& Min \ \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j\ \sigma_{ij}}\ , \\
& s.t.\ \sum_{j=1}^{n}\lambda_j\mu_j = R_{expected}\ , \\
& \sum_{j=1}^{n}\lambda_j = 1\ , \\
& \lambda_j \geq 0,\quad j=1,\dots,n\ .
\end{aligned}
$$

Where σ_{ij} is Covariance of the i th and j th financial assets. In order to obtain efficiency frontier, Mori and Mori used the following diversification model which is a DEA non-linear model:

$$
\begin{aligned}
& Min\ \ \theta \\
& s.t.\ \sum_{j=1}^{n}\lambda_j\mu_j \geq \mu_p\ , \\
& \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j\ \sigma_{ij}} \leq \theta\sigma_p\ , \\
& \sum_{j=1}^{n}\lambda_j = 1, \\
& \lambda_j \geq 0,\quad j=1,\dots,n\ .
\end{aligned}
$$

In order to estimate the portfolio efficiency frontier, Wenbin liu et al. [14] used BCC model. He showed that portfolio efficiency frontier of Markowitz model is concave function. Therefore, if we add L sample DMUs to the observed DMUs, BCC efficiency frontier will converge to portfolio efficiency frontier in probability when $L \to +\infty$.

$$
\begin{aligned}
& Min\ \ \theta \\
& s.t.\ \sum_{j=1}^{n+L}\lambda_j\mu_j \geq \mu_p\ , \\
& \sum_{j=1}^{n+L}\lambda_j\sigma_j \leq \theta\sigma_p\ , \\
& \sum_{j=1}^{n+L}\lambda_j = 1, \\
& \lambda_j \geq 0,\quad j=1,\dots,n+L\ .
\end{aligned}
$$

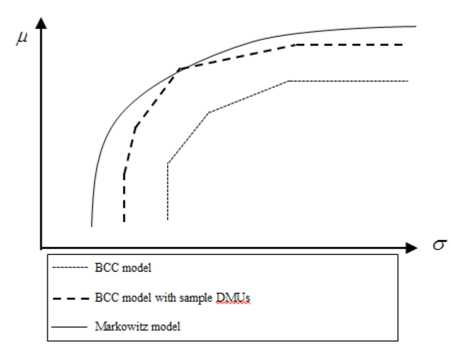

Figure 1. Efficient frontier.

In order to obtain the sample DMUs, we use simulation. At first, we define set of W in the opposite form.

$$\Omega = \left\{ \left(\lambda_1, ..., \lambda_n \right) \mid \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\}$$

Then, using uniform distribution, we produce L weight vectors which are member of set W and, at the end, we add DMU_i to the observed DMUs in lieu of each produced vector.

$$DMU_i = \left\{ \left(\mu_i^{'}, \sigma_i^{'} \right) \mid \mu_i^{'} = \sum_{j=1}^{n} \lambda_j^i \mu_j \ , \sigma_i^{'} = \sqrt{\sum_{t=1}^{n} \sum_{j=1}^{n} \lambda_t^i \lambda_j^i \sigma_{tj}} \right\} \ , \ i = 1, ..., L \ .$$

According to experience, it is enough to consider L to be higher than, or to be equal to, fifty so that the obtained BCC model efficiency frontier is an appropriate estimation for Markowitz model portfolio efficiency frontier (Figure 1).

3. Establishment of suppositions in ccr in order to evaluate portfolio performance

Consider one input-one output CCR model of Figure (2). Since return to scale is constant in CCR model, DMU_1 and DMU_2 have thus an equal efficiency and are not different from each other, but it may not be so from managerial viewpoint; for example, if we consider input to be standard deviation and output to be mean return manager may consider DMU_2 to be better than DMU_1 because DMU_2 has a higher mean return (As an example, let's suppose that manager is a risk taking person and increase of risk is of a less degree of importance compared to increase of profit). Now, we want to change CCR model such as way in which efficiencies of, DMU_1 and DMU_2 are to be determined by manager's viewpoint. By creating some changes in the model or manipulating

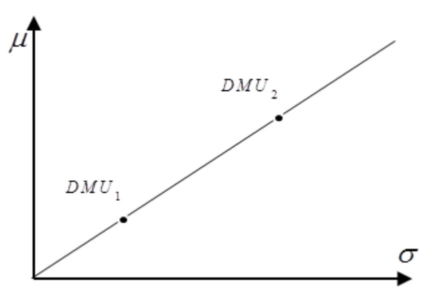

Figure 2. PPS of CCR model.

In the data, this task can be done, which we use manipulation in the data. For this purpose, we consider two following states:

The first state: manager is a risk-taking person

Let's suppose that PPS includes n observed DMUs and (observed) DMU_p placed on the efficiency frontier. We define X_N and M as following:

$$X_N = Max \left\{ x_j \middle| j = 1, ..., n \right\} \quad , \qquad M = \left(\frac{y_p}{x_p} \right) .$$

Where x_j is input of DMU_p . Now, we add $DMU_N \left(x_N , Mx_N \right)$ with x_N input and Mx_N output, as an observed, to PPS (Figure 3). It is evident that PPS doesn't change. Now, from perspective of manager, DMU_N is the most desirable unit because it is both efficient and has the maximum return among the efficient DMUs. On the efficiency frontier, if we move from DMU_N to downward, efficiency of the DMUs, on the efficient frontier, must be decreased by determined ratio. For this purpose, while calculating the efficiency of $DMU_p \left(x_p , y_p \right)$, we use $DMU_{p'} \left(x_{p'}, y_{p'} \right)$ whose input and output is to be obtained by the following method**.**

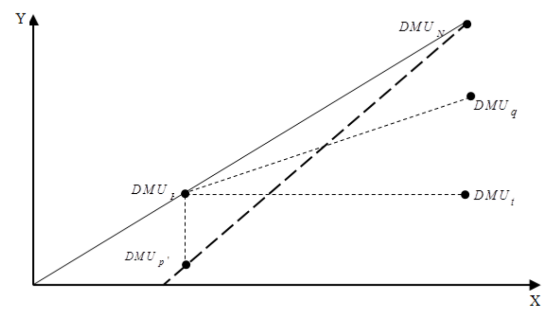

Figure 3. Manager is a risk-taking person.

Consider the equation of the line passed from DMU_N and $DMU_{p'}$ as following:

$$y = \frac{y_p \left(1 - \alpha \right) + y_N}{x_N} \left(x - x_N \right) + y_N \ .$$

Slope of the supposed line changes with coefficient α which is dependent on the manager's opinion for the purpose of output decrease proportion. As a result:

$$x_{p'} = x_p \quad , \qquad y_{p'} = \left(\frac{\alpha \left(x_N - x_p \right) + x_p}{x_N} \right) y_p \quad .$$

Therefore, we reach the following model through implementation of the above change in CCR model within the input oriented:

$$\text{Min} \quad \theta$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_j \le \theta x_p , \qquad (1)$$
$$\sum_{j=1}^{n} \lambda_j y_j \ge \left(\frac{\alpha (x_N - x_p) + x_p}{x_N} \right) y_p ,$$
$$\lambda_j \ge 0 \quad , \quad j = 1,...,n \; .$$

Now, we find one upper bound and one lower bound for α . Coefficient y_p in model (1) must be led to reduction of output. Therefore:

$$\frac{\alpha (x_N - x_p) + x_p}{x_N} \le 1 \Rightarrow \alpha (x_N - x_p) \le x_N - x_p \Rightarrow \alpha \le 1 \; .$$

On one hand, in figure (3), DMU_p is better than DMU_t because it has produced an output being equal to DMU_t by a less input. Thus, model (1) must allocate higher efficiency to DMU_p . In order to evaluate performance of DMU_p , model (1) uses input of x_p and output of $\frac{\alpha (x_N - x_p) + x_p}{x_N} y_p$. Now, by multiplication of input and output of DMU_p by expression of $\frac{x_N}{x_p}$, it's input and output are re-written in form of x_N and $\frac{\alpha (x_N - x_p) + x_p}{x_p} y_p$, respectively. On the other hand, in order to evaluate performance of DMU_t , model (1) uses x_N input and y_p output. Therefore, due to equality of the inputs, the following condition must be established for the outputs:

$$\frac{\alpha (x_N - x_p) + x_p}{x_P} y_p > y_p \Rightarrow \alpha (x_N - x_p) > 0 \Rightarrow \alpha > 0 \; .$$

In continuation, we will study influence of various quantities of α on the model (1).
Let's suppose to consider: $\alpha = 0$ (Figure 3). In this state, all DMUs on the $\overline{pt}$ segment have equal efficiency. Now, if we consider $\alpha = 1$, all DMUs on the $\overline{pN}$ segment will have equal efficiency as well. Therefore, there exists $\alpha = c$ $\left(0 < c = \frac{(y_q - y_p) x_p}{(x_N - x_p) y_p} < 1 \right)$, that all the DMUs on the $\overline{pq}$ segment have equal efficiency. Now, consider DMU_p and DMU_q . The former one has a low mean and variance and the latter one has a relatively high mean and variance. Now, if manager wants to consider DMU_p to be better than DMU_q , it is sufficient to select $\alpha \ge c$, and if he/she considers DMU_q to be better than DMU_p , he/she must consider $\alpha \le c$.

These second state: manager is not a risk- taker person

From view point of manager, DMU_N is the most desirable unit in the figure (4) because both it is efficient and has the least risk among the efficient DMUs. Now, if we move from DMU_N , on the efficiency frontier, upward, efficiency of DMUs, on the efficient frontier, must become less with a determined ratio. For this purpose, we use $DMU_{p'}(x_{p'}, y_{p'})$ while calculating the relative efficiency of $DMU_p (x_p, y_p)$. If we act like previous part, we will have as following:

$$\text{Max} \quad \varphi$$
$$s.t. \; \sum_{j=1}^{n} \lambda_j x_j \le \left(\frac{y_p}{\alpha (y_p - y_N) + y_N} \right) x_p , \qquad (2)$$
$$\sum_{j=1}^{n} \lambda_j y_j \ge \varphi y_p ,$$
$$\lambda_j \ge 0 \quad , \quad j = 1,...,n \; .$$

Quantity of y_N and upper bound and lower bound of α is to be determined as follows:

$$y_N = min \{ y_j | j = 1,....,n \} , \qquad 0 < \alpha \le 1.$$

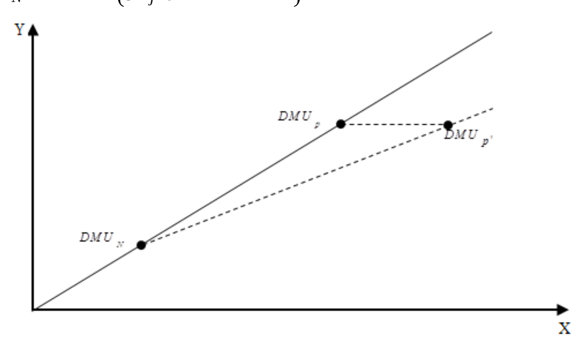

Figure 4. Manager is not a risk-taking person.

4. Managerial interpretation of the suggestive methods in order to evaluate portfolio performance

Supposing that we have n DMUs (portfolio) with equal prices. Each portfolio has a specified return and risk. Thus, each DMUs has one input (purchasing price) and two outputs (return and risk).we consider the following CCR model:

$$\begin{aligned} &\text{Max} \quad \varphi \\ s.t.\ &\sum_{j=1}^{n} \lambda_j x_j \le x_p \ , \qquad (3) \\ &\sum_{j=1}^{n} \lambda_j \mu_j \ge \varphi \mu_p \ , \\ &\sum_{j=1}^{n} \lambda_j \sigma_j \ge \varphi \sigma_p \ , \\ &\lambda_j \ge 0 \quad , \qquad j = 1,\dots,n \ . \end{aligned}$$

In model (3), considering constancy of the inputs and banding of the input constraint in the optimum response, the first constraint of the above model is re-written in the opposite form: $\sum_{j=1}^{n} \lambda_j = 1$.

Since output related to risk is a part of undesirable outputs, it, thus, can be considered as input. Now, by addition of the sample DMUs to PPS and usage of the technique applied in the suggestive methods, we present the following models in order to estimate portfolio efficiency frontier.

The first state: manager is to be risk-taker

$$\begin{aligned} &\text{Min} \quad \theta \\ s.t.\ &\sum_{j=1}^{n+L} \lambda_j \sigma_j \le \theta \sigma_p \ , \\ &\sum_{j=1}^{n+L} \lambda_j \mu_j \ge \left(\frac{\alpha(\sigma_N - \sigma_p) + \sigma_p}{\sigma_N} \right) \mu_p \ , \\ &\sum_{j=1}^{n+L} \lambda_j = 1 \ , \qquad (4) \\ &\lambda_j \ge 0 \quad , \qquad j = 1,\dots,n+L \ . \end{aligned}$$

Where

$$\sigma_N = \max\left\{ \sigma_j \middle| j = 1,\dots,n+L \right\} \ , \qquad 0 < \alpha \le 1 \ .$$

The second state: manager is not to be risk-taker

$$\begin{aligned} &\text{Max} \quad \varphi \\ s.t.\ &\sum_{j=1}^{n+L} \lambda_j \sigma_j \le \left(\frac{\mu_y}{\alpha(\mu_y - \mu_N) + \mu_N} \right) \sigma_p \ , \\ &\sum_{j=1}^{n+L} \lambda_j \mu_j \ge \varphi \mu_p \ , \qquad (5) \\ &\sum_{j=1}^{n+L} \lambda_j = 1 \ , \\ &\lambda_j \ge 0 \quad , \qquad j = 1,\dots,n+L \ . \end{aligned}$$

Where

$$\mu_N = \min\left\{ \mu_j \middle| j = 1,\dots,n+L \right\} \ , \qquad 0 < \alpha \le 1 \ .$$

5. Numerical examples

In this section, we deal with explanation of the models suggested in the previous sections by presentation of two numerical example.

Example 1

Supposing that we have ten DMUs with input and output presented in table 1. PPs which is analogous to these data has been presented in figure (5). We obtain the relative efficiency of the DMUs for various quantities of α via the suggestive methods. The results have been reflected in Table 2.

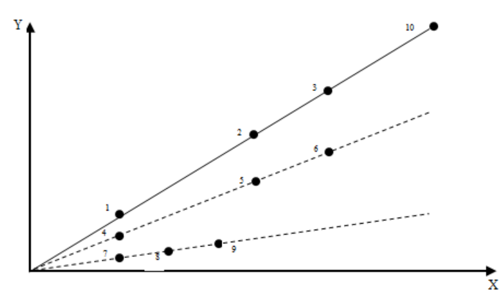

Figure 5. PPS for Example 1.

Table 1. Data for Example 1.

DMU	1	2	3	4	5	6	7	8	9	10
Input	1	4	6	1	4	6	1	2	3	10
Output	3	12	18	2	8	12	1	2	3	30

For example, in model (1), for $\alpha = 0.5$, we use the following ranking:

$DMU_7 \prec DMU_8 \prec DMU_9 \prec DMU_4 \prec DMU_1 \prec DMU_5 \prec DMU_6 \prec DMU_2 \prec DMU_3 \prec DMU_{10}$

But for $\alpha = 0.7$, we use the following ranking:

$DMU_7 \prec DMU_8 \prec DMU_9 \prec DMU_4 \prec DMU_5 \prec DMU_6 \prec DMU_1 \prec DMU_2 \prec DMU_3 \prec DMU_{10}$

Considering the obtained results in table 2, various rankings are obtained in lieu of various α .

Table 2. Obtained efficiency in lieu of $\alpha = 0.3,\ 0.7,\ 1.$

model	Efficiency Of Model(1)			Efficiency Of Model(2)		
DMUs	$\alpha = 0.3$	$\alpha = 0.7$	$\alpha = 1$	$\alpha = 0.3$	$\alpha = 0.7$	$\alpha = 1$
1	0.37	0.73	1	0.533	0.8	1
2	0.58	0.82	1	0.358	0.725	1
3	0.72	0.88	1	0.339	0.717	1
4	0.246	0.487	0.667	0.433	0.567	0.667
5	0.387	0.547	0.667	0.258	0.492	0.667
6	0.48	0.587	0.667	0.239	0.483	0.667
7	0.123	0.243	0.333	0.333	0.333	0.333
8	0.147	0.253	0.333	0.217	0.283	0.333
9	0.17	0.263	0.333	0.178	0.267	0.333
10	1	1	1	0.323	0.71	1

EXAMPLE 2

Consider the data related to five portfolios with given mean return and matrix of covariance in table 3 [14]. Using the method presented in section 2, we produce a number of sample DMU and add to PPS. Then, we concern with estimation of the portfolios performance evaluation of Markowitz model using the suggested methods. As much as coefficient α approaches to one, it suggests low sensitivity of manager, and as much as coefficient α approaches to zero, it suggest high sensitivity of manager to mean return and standard deviation. Considering the obtained results in table 4, various rankings are obtained in lieu of various α .

Table 3. Data for example 2.

	1	2	3	4	5
Expected return	2.57	1.36	1.83	0.49	0.85
Covariance	1	2	3	4	5
1	174.4	97.14	75.5	74.11	91.28
2	97.14	280.52	125.87	143.97	133.02
3	75.5	125.87	265.62	107.28	72.04
4	74.11	143.97	107.28	172.83	88.83
5	91.28	133.02	72.04	88.83	168.25

For example, if manager is not a risk-taker person, he/she uses the following ranking for $\alpha = 0.5$:

$$DMU_4 \prec DMU_5 \prec DMU_2 \prec DMU_3 \prec DMU_1$$

Now, if manager is a risk-taker person, he/she uses the following ranking for $\alpha = 0.5$:

$$DMU_2 \prec DMU_3 \prec DMU_4 \prec DMU_5 \prec DMU_1$$

With regard to the obtained data, portfolio No .1 has the best efficiency in both two states.

Table 4. Obtained efficiency in lieu of $\alpha = 0.5$.

DMUs	Efficiency of model (4)	Efficiency of model (5)
1	0.921	1
2	0.461	0.529
3	0.675	0.712
4	0.814	0.192
5	0.833	0.331

6. Conclusion

In this article, we, at first, expressed created weakness while evaluation of the portfolio performance using CCR model and, then, presented some methods in order to solve this problem using coefficient α , which is dependent of opinion of manager regarding this subject that whether he/she is a risk-taker person or not and how much he/she is risk-taker. In this methods, the DMUs on the efficiency frontier of CCR model will not have relative equal efficiency anymore and, instead, one another group of DMUs will have equal efficiency originated from the manager's view point. In continuation, a managerial interpretation for presented method was expressed by DEA using undesirable data and estimation of Markowitz portfolio efficiency frontier and it was shown that selection of portfolio based on opinion and viewpoint of manager changes.

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