A New Method to Solve the Multi Traveling Salesman Problem with the Combination of Genetic Algorithm and Clustering Technique

Mersad Shabanpour[1], Mehdi Yadollahi[2], Mohammad Mehdi Hasani[3]

[1]Department of Computer Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran
[2,3]Department of Computer Engineering, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran

Abstract
The multi traveling salesman problem is the extension of the well-known problem of traveling salesman and considered as a NP-Complete problem having exponential solution space. MTSP is more complicated than TSP, because in MTSP first the cities must be divided between salesmen, and then the optimal order of cities for each traveling salesman should be determined. By looking at early studies, it is revealed that researchers had focused on MTSP less than TSP, and few studies have been done in this area so far. In this paper a new combinatorial algorithm named CGA-MTSP is proposed for solving MTSP problem which is a combination of Genetic Algorithm and Clustering Technique. The aim of this method is to reduce the travelled distance by salesmen. In the proposed method the Clustering Technique has been used intelligently to reduce the solution space of the mentioned problem. Experimental results show that the proposed algorithm (CGA-MTSP) leads to better results than other algorithms, but it requires more execution time.
Key-words:
Multi Traveling Salesman Problem, Clustering Technique, Genetic Algorithm, Optimization

1. Introduction

MTSP is an extension of the well-known TSP problem, which was introduced for modeling more real world problems. In fact, we can use MTSP for solving problems such as multi traveling robots [1, 2], vehicle routing problems [3] and job planning by adding few limitations based on the type of problem. Some of the other usages of MTSP mentioned in [4] are: print press scheduling, crew scheduling, school bus routing, mission planning and the design of global navigation satellites surveying system network.

The target of this problem is to find the shortest distance between cities randomly, such a way that each city visited only once [5].

Multiple traveling salesmen problem (MTSP) [6] is result of traveling salesman problem (TSP) [7], in this case the existence of one or more salesman is possible. Tt's obvious that MTSP is a NP-complete problem, since TSP is a kind of NP-complete problem. So its solutions are based on searching paths. The number of cities in MTSP is not clear, and salesmen must visit all cities. Here salesmen must start to travel from city 1 and return to that. In MTSP problem, n is the number of cities and m is the number of salesmen. Since the number of salesmen is not fixed, each salesman suffers from constant cost in each point of solution. The result of this situation is to reduce the number of salesmen and active solutions.

Several heuristic based methods have been presented to solve the TSP problem, such as Classical Search Maps [8], Simulated Annealing [9], Artificial Neural Networks (Kohonen-type self-organizing maps) [10], Hopfield–type NNs [11], Genetic Algorithms [12], Evolutionary Programming [13], Ant Colony Optimization [14], Tabu Search [15], Fine-Tuned Learning [16], etc.

Unlike the TSP, fewer studies have been done on MTSP. Bektas [4] introduced comprehensive formulations and solution procedures for MTSP, and indicated that exact algorithms are unable to solve MTSP problems, this way many heuristic algorithms have been proposed till now [17,18]. While Qu et al. have suggested competition – based neural network, Ryam et al. [19] have used tabu search for solving MTSP.

Up to now genetic algorithm has been used for a large range of practical fields such as MTSP. In [20] a genetic algorithm based tendency used to solve MTSP, and Liew et al. [21] have presented a search based combined genetic algorithm to solve MTSP. Carter et al. [22] have researched on chromosome structure and genetic operator to achieve proper solution for MTSP. IN [23, 24] ants algorithm and 2- opt method based algorithms have been used to solve MTSP respectively.

Hosseinabadi et al. [25] provided a new combined algorithm named GELS_GA for MTSP problem. The aim of the algorithm was to combine the public search ability of Genetic Algorithm and local search ability of Gravitational Emulation Local Search Algorithm to produce a stable algorithm to raise the possibility of reaching to a global optimum.

Manuscript received May 5, 2017
Manuscript revised May 20, 2017

In [26] MTSP_GELS was introduced for MTSP problem which uses Gravitational Emulation Local Search Algorithm. This algorithm uses two parameter of four main parameters velocity, gravitational force in physics. The advantages of this method are high velocity of converging and get the solution, execution time and low value of evaluation function.

In this paper a new method named CGA-MTSP has been proposed through the combination of Genetic Algorithm and Clustering Technique to solve the multiple traveling salesmen problem, the aim is to minimize the traveled distance by salesmen. In the proposed method the Clustering Technique has been used to efficiently reduce the solution space of problem.

The proposed algorithm particularly show the traveled distance by salesmen and other important parameters. As will seen in comparison table, CGA-MTSP is superior than other algorithms.

The remainder of the paper is organized as follows. First in section 2 the Problem description has been presented. Section 3 described the proposed algorithm. Then in section 4 the experimental results and in section 5 the conclusions have been presented respectively.

2. Problem description

In general MTSP is defined as follow: There exists a set of n cities and m traveling salesmen that in first step the cities should be divided between salesmen. Then the optimal order to visit the cities should be determined for each salesman so that each salesman just visits any city once and also the traveled distance by each salesman should be minimized. In addition to the mentioned definition, several other definitions exist for MTSP in [4]. In this we considered the following limitations for MTSP problem.

1. Each traveling salesman starts from different cities.

2. For each traveling salesman a set of different cities are taken into account.

3. Each city should be visited once by each salesman.

The main objective in MTSP is to find a valid tour for any traveling salesman so that any salesman should cover the least distance to travel through all the cities dedicated to him/her, also the total cost (distance) to pass all cities by the salesmen should be minimized.

3. Proposed algorithm

The proposed algorithm (CGA-MTSP) has two steps. In first step cities are divided between traveling salesmen (Clustering Stage). In second step the order of visiting cities by each salesman is detected in a way that the distance traveled by each salesman and also the total traveling distance by all salesmen is minimized. In fact the proposed algorithm (CGA-MTSP), splits the MTSP problem to several TSP problems by using the Clustering Technique, in order to reduce the search space of problem.

The following subsections describe the steps of the proposed algorithm (CGA-MTSP).

3.1 Clustering

As it was mentioned, the proposed algorithm uses the Clustering Technique with the help of Genetic Algorithm to convert MTSP into several TSP problems in the first step.

To find a tour for small and large graphs (graphs with low and high numbers of nodes) in this paper, at first we make small size clusters from graph by using Clustering Technique, and then find a minimum cost tour for each cluster. We choose some nodes from each cluster with least distance to other clusters as cluster heads, and then we obtain a minimum cost tour between cluster heads by using Genetic Algorithm. Finally the tour cost for large graph is equal to the sum of internal tours in each cluster plus the cost tour of cluster heads. Then the optimal order of cities for each salesman is detected by using Genetic Algorithm in the second step. The structure of the Genetic Algorithm used in the first and second of the proposed algorithm which are similar together is described below.

3.2 Chromosome representations

The structure of chromosome in the proposed algorithm has been achieved from [22]. The first section of this structure shows the cities each salesman should visit respectively, and the second section determines the number of dedicated cities to each salesman. The sum of the positive integers in the second part of the chromosome must equal to the sum of all cities. Figure 1 shows a sample chromosome for 15 cities with 3 Clusters.

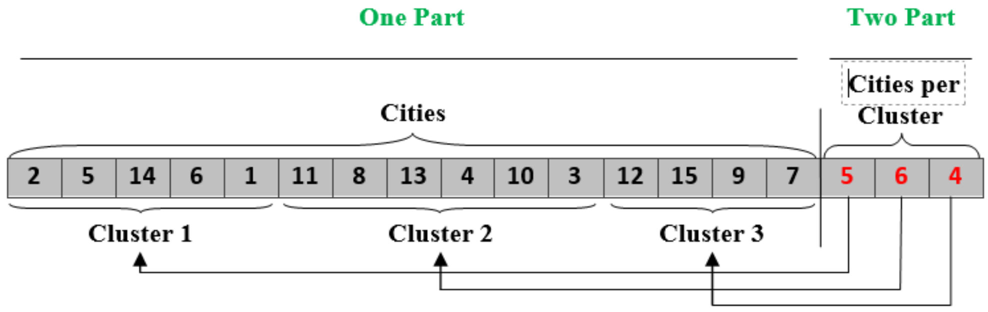

Fig. 1 Sample chromosome with 15 cities and 3 clusters.

3.3 Fitness function

Two kinds of fitness function are used in the proposed algorithm. The first fitness function is used for the clustering step. The amount of this function is equal to the sum of distances between the city placed at the head of each cluster and other cities of same cluster, added to the fitness amount of the maximum cluster (Max fit). This fitness function tries to dividing cities between the clusters in a balanced form.

$$f(x) = \sum_{i=1}^{m}\sum_{j=2}^{n}\sqrt{(x_1-x_j)^2+(y_1-y_j)^2} + MaxFit \quad (1)$$

The second fitness function is used in the second step to achieve the optimal order of cities covered by each salesman. The amount of the second fitness function is equal to the total traveled distances by each salesman and the maximum amount of a salesman traveled (Max fit).

$$f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sqrt{(x_{j+1}-x_j)^2+(y_{j+1}-y_j)^2} + MaxFit \quad (2)$$

3.4 Crossover operation

Crossover operator of the proposed algorithm is based on minor mapping technique. This method has been introduced by Goldberg & Lingel [27]. In this method two genes of a parent chromosome are selected randomly. Then the parent chromosomes are divided into three parts by using these two genes. Two outside parts of these two genes (the part starting from begin till the first gene and the part starting from the second gene to the end of chromosome) are copied in the similar places of the first child, and then the cities between these two genes of the second parent are copied to the similar places of the first child, if the cities in the middle of second parent don't exist in the first child, otherwise that places left empty. At last the empty places of the first child are filled with unused genes of the second father respectively. This procedure is repeated to produce the second child. Figure 2 shows a crossover operator based on minor mapping technique.

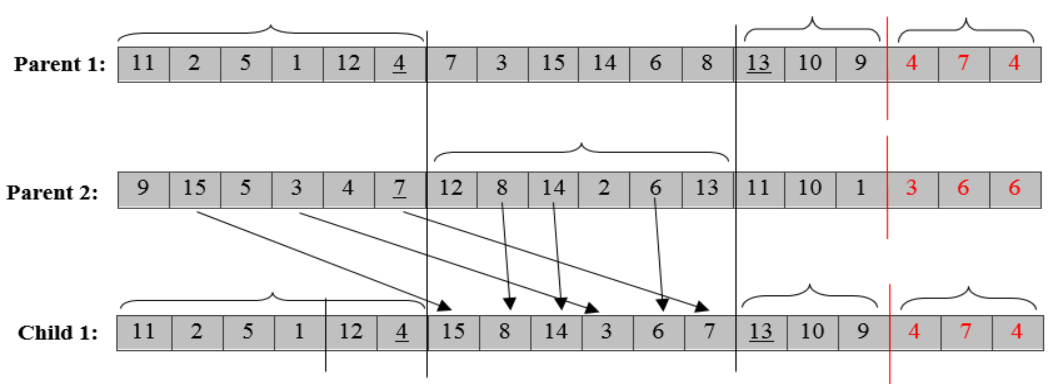

Fig. 2 Crossover operations based on minor mapping technique

3.5 Mutation operation

In the proposed algorithm a swap operation has been used for mutation. To perform this operation first one chromosome is chosen as the parent, then two random genes are exchanged.

4. Experiments results

To verify the efficiency of the proposed algorithm (CGA-MTSP), we compared it with GA algorithm in [22]. We implemented both of them in C# 2010 programing language on a Pentium (R) 4 CPU 3.00GH with 2.00 GB of RAM. Experimental results are discussed below. 6 test data and different number of cities are designed to evaluate the algorithms and cover the small, medium, and the large space problems. These test data have named TestX_n_m, where X means test number, n is the number of cities, and m indicate the number of salesmen required for multi traveling salesman problem. For each test data, algorithms have been performed 20 times independently. Table 1 shows the designed test data.

Table 1: The designed test data.

Test	SC_{avg}	Cross Rate	Mutation Rate	Pop.	No. of Generation
Test1_20_3	6.66	80%	10%	100	1000
Test2_50_3	16.66	80%	10%	100	1000
Test 3_100_5	20	80%	10%	100	5000
Test 4_300_5	60	80%	10%	100	15000
Test 5_500_7	71.42	80%	10%	100	30000
Test 6_1000_10	100	80%	10%	100	50000

Figure 3 shows the results of performing GA algorithm on Test2_50_3 and figure 4 shows the results of performing CGA-MTSP algorithm on Test 2_50_3. Figures 6, 7, 8, 9, and 10 show the results of performing 2 algorithms on the other test data.

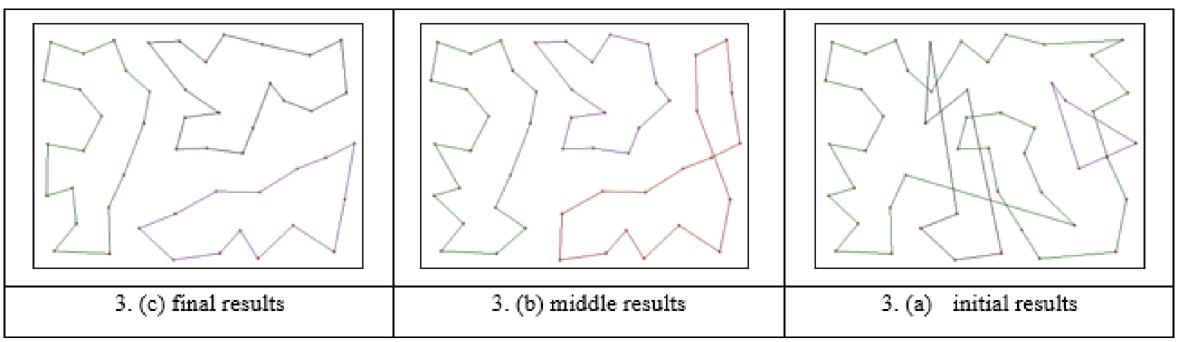

Fig. 3 The results of performing GA algorithm on data of Test 2_50_3.

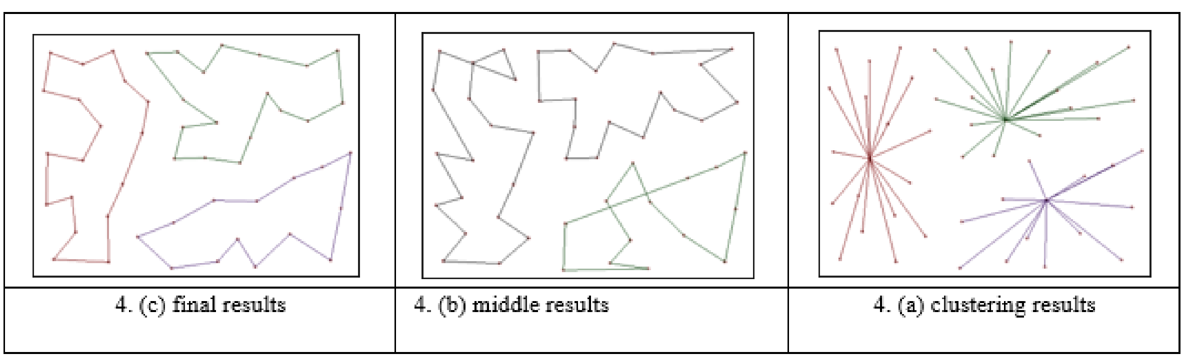

Fig. 4 The results of performing CGA-MTSP algorithm on data of Test 2_50_3.

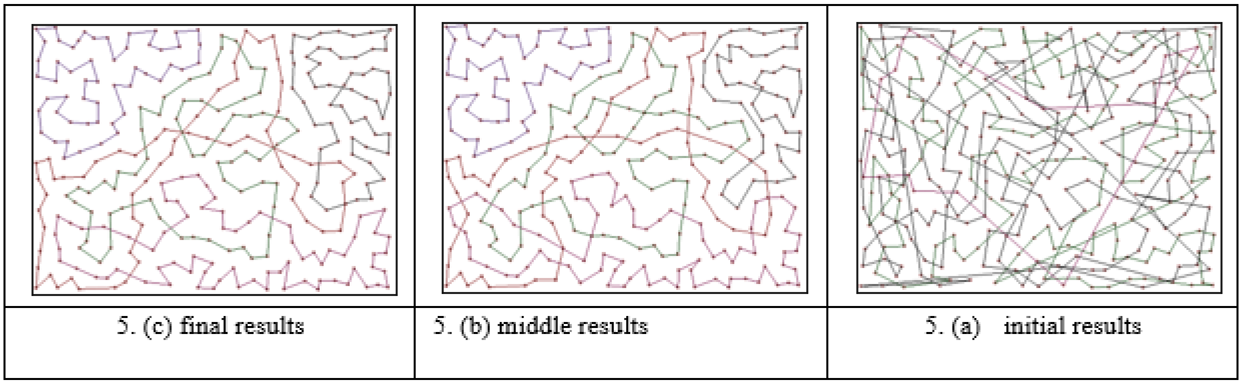

Fig. 5 The results of performing GA algorithm on data of Test 4_300_5.

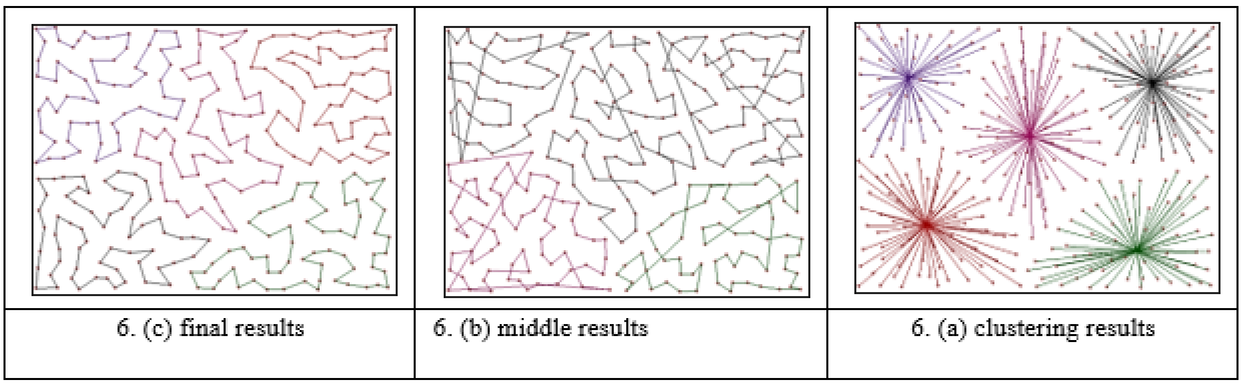

Fig. 6 The results of performing CGA-MTSP algorithm on data of Test 4_300_5.

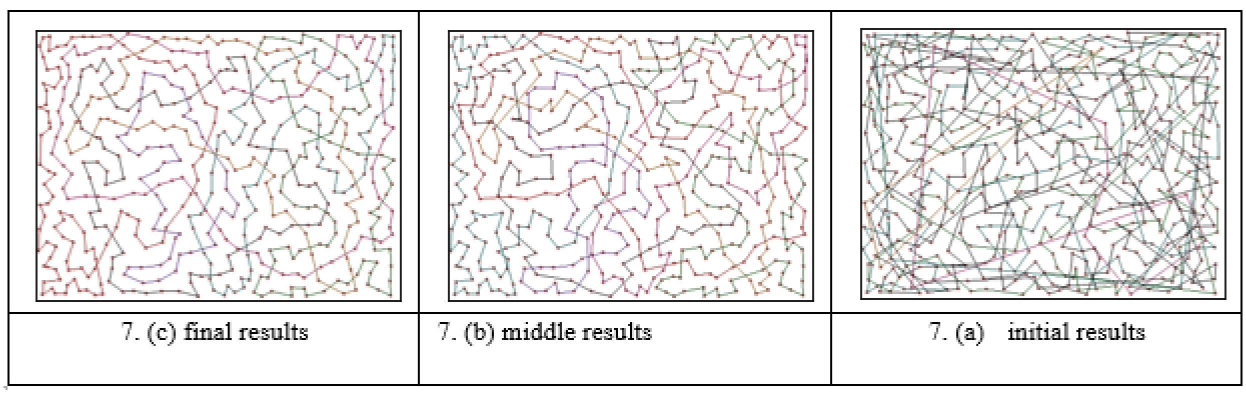

Fig. 7 The results of performing GA algorithm on data of Test 5_500_7.

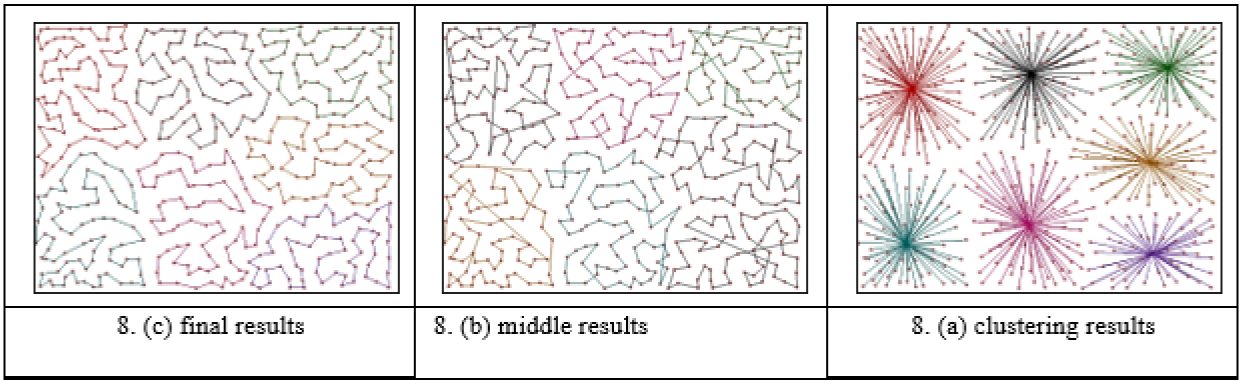

Fig. 8 The results of performing CGA-MTSP algorithm on data of Test 5_500_7.

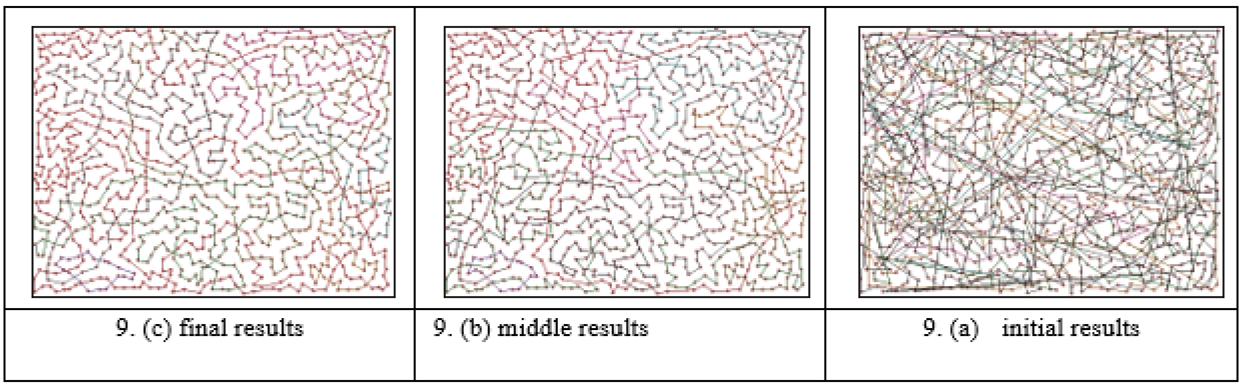

Fig. 9 The results of performing GA algorithm on data of Test 6_1000_10.

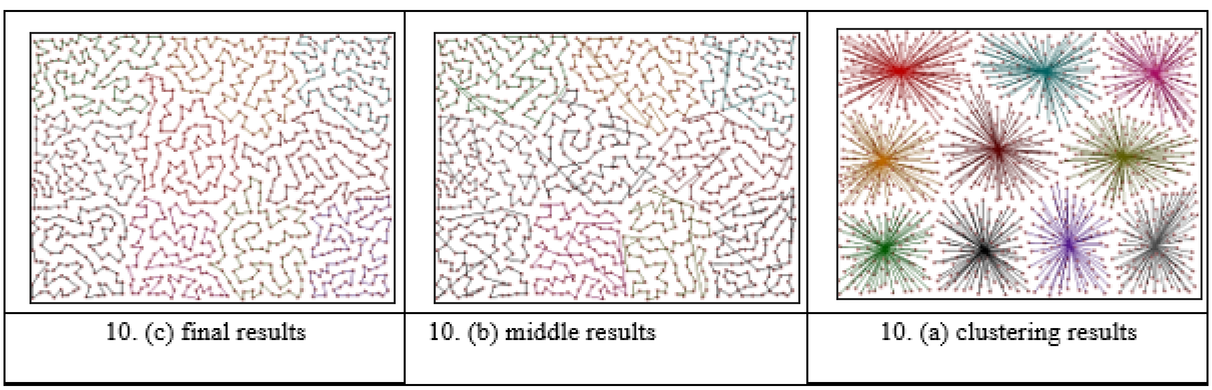

Fig. 10 The results of performing CGA-MTSP algorithm on data of Test 6_1000_10.

Figure 11, shows the fitness diagram of performing both algorithms on some test data. It is clear that, in comparison with GA algorithm, the proposed algorithm CGA-MTSP could obtain better results in most cases. This improvement becomes more obvious while the problem becomes larger. This improvement is because of using clustering method in the proposed algorithm to reduce the solution space.

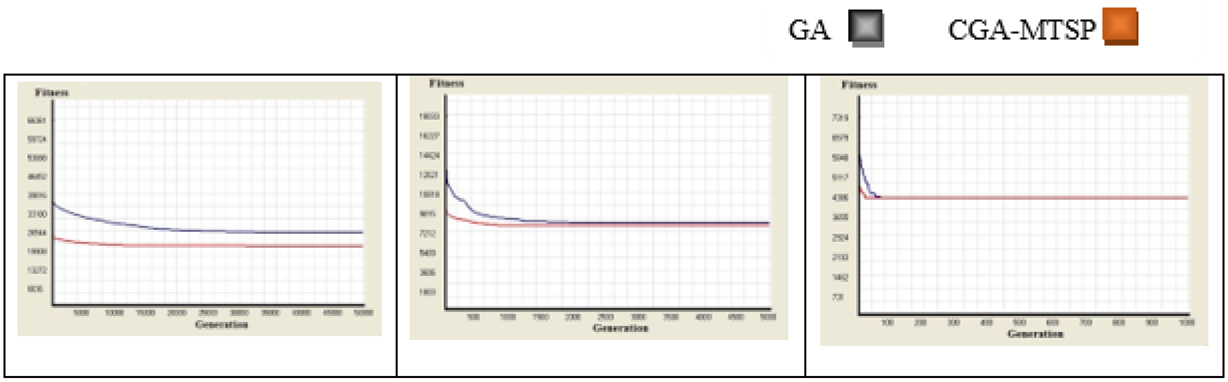

Fig. 11 The fitness diagram of performing both algorithms on some test data

Figure 12 shows the balance diagram of salesmen in the proposed algorithm and GA algorithm. This figure shows that the proposed algorithm creates better balance among salesmen than GA algorithm in most test data. In other words, unlike GA algorithm where salesmen travel different distances, in the proposed algorithm CGA-MTSP almost all salesmen travel the same distances.

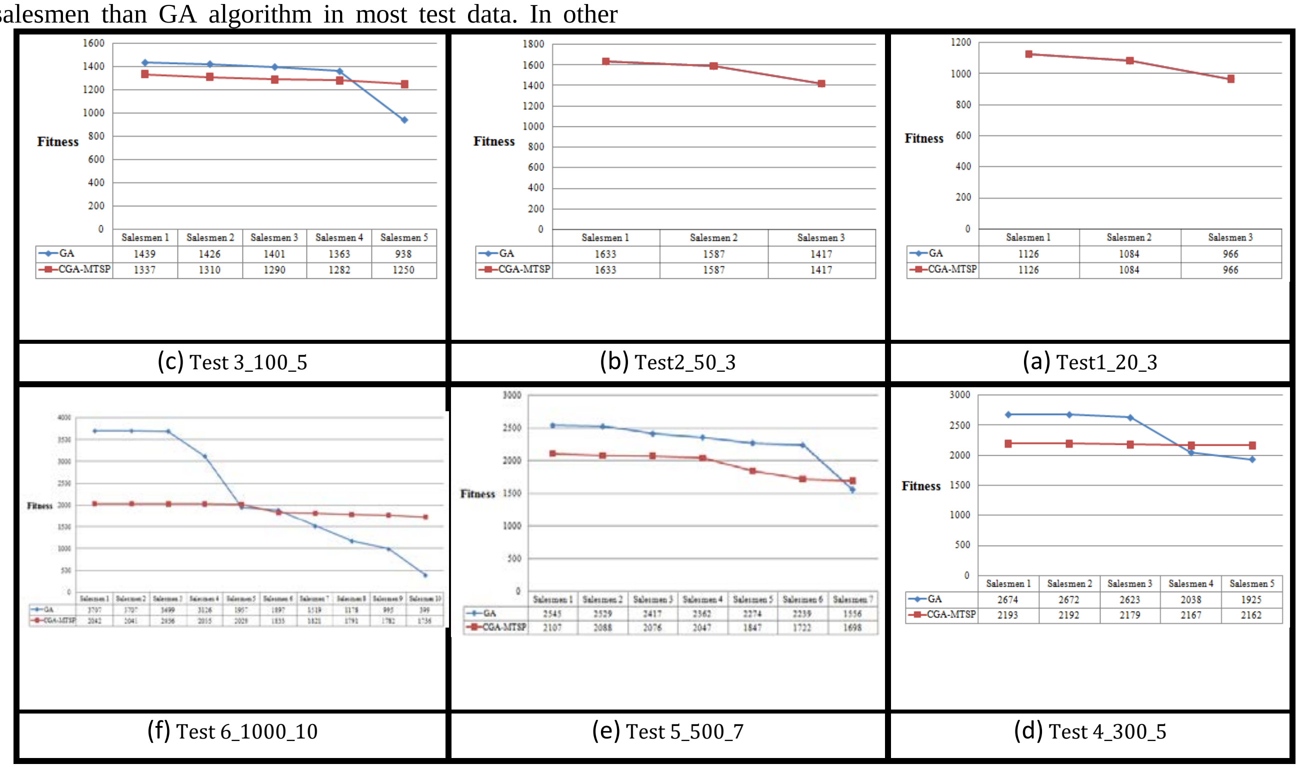

Fig. 12 The diagram of the balance of salesmen in CGA-MTSP and GA algorithms for the test data

.Figure 13 shows the comparison of the execution time of GA algorithm and CGA-MTSP algorithm. As expected, while the number of cities increases, the problem gets larger and so the required time to obtain an optimal tour increases progressively. It shows that the proposed algorithm (CGA-MTSP) requires more time for execution. It's because of using clustering technique to efficiently reduce the solution space of problem.

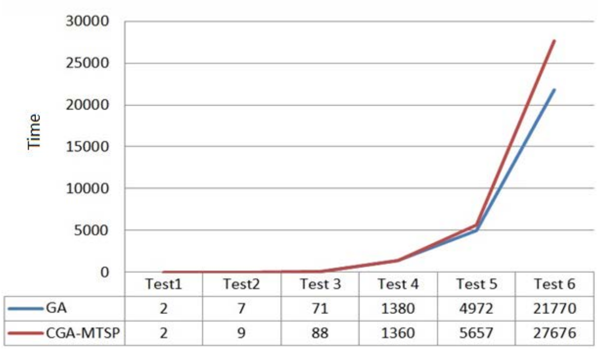

Fig. 13 The comparison of execution time of GA and CGA-MTSP algorithms

Table 2 shows the overall results of performing algorithms. In this table the endurance rate (Err) is computed through the difference between the average fitness (Favg) and the best fitness (Fbest) divided by amount of best fitness multiplied by 100 for each test data which represents the reliability of the algorithm. Equation 3 shows how the endurance rate is computed.

$$\text{Err} = \frac{(F_{avg} - F_{best})}{F_{best}} * 100 \qquad (3)$$

Low endurance rate (Err) indicates the more reliability. As table 2 shows, the reliability rate of the proposed algorithm (CGA-MTSP) is less than GA algorithm in all test data, which shows that CGA-MTSP is more reliable than GA algorithm.

Table2: The results achieved through performing the proposed algorithm and GA algorithm on test data.

Test	GA [22]					CGA-MTSP					
	F_{best}	F_{avg}	G_{best}	T_{run}(S)	Err(%)	F_{best}	F_{avg}	G_{best}	CT_{run}(S)	GAT_{run}(S)	Err(%)
Test1_20_3	4302	4447.2	70	2	3.37	4302	4302	21	0	2	0.0
Test2_50_3	6270	6637.6	665	7	5.86	6270	6298	227	2	7	0.44
Test 3_100_5	8006	8655.8	2925	71	8.11	7806	7851.83	1065	21	67	0.58
Test 4_300_5	14606	14861.85	8813	1380	1.75	13086	13220.2	10011	151	1209	1.02
Test 5_500_7	18467	19120.2	24594	4972	3.53	15692	15853	25621	788	4869	1.02
Test 6_1000_10	25891	26863	48189	21770	3.75	21188	21557.33	43247	6148	21528	1.74

F_{best}: best fitness T_{run}(S): run time of the algorithm
F_{avg}:average fitness CT_{run}(S): run time of clustering
G_{best} : number of generation GAT_{run}(S): run time of the proposed algorithm after clustering
Err(%): endurance rate

The results of performing both algorithms on several test data demonstrated that the proposed algorithm (CGA-MTSP) always lead to equal or better results in comparison with other algorithms. The improvement of the obtained results by proposed algorithm (CGA-MTSP) is more obvious in larger test data. That is because of using Clustering Technique to efficiently reduce the solution space of problem. On the other hand the use of Clustering Technique has caused to increase the required time for executing the proposed algorithm, but it is not considerable.

5. Conclusion and Future Work

In this paper a new method has been proposed for solving MTSP problem. This is a combination of Genetic Algorithm and Clustering Technique. The aim of the proposed algorithm is to minimize the distance traveled by the salesmen. The main idea of the proposed algorithm (CGA-MTSP) is the use of Clustering Technique to efficiently reduce the solution space of problem. Experimental results showed that because of combining Clustering Technique with Genetic Algorithm, the proposed algorithm has high performance and leads to better results. The amount of this improvement is more obvious and considerable in larger test data. Experimental results also demonstrated that the proposed algorithm (CGA-MTSP) is more reliable than GA algorithm, but it needs more execution time that can be the disadvantage of the proposed algorithm. We would like to explore the effects of different types of operators and parameters on the performance and quality of solutions.

References

[1] Qu, H., et al., Real-time robot path planning based on a modified pulse-coupled neural network model. Neural Networks, IEEE Transactions on, 2009. 20(11): p. 1724-1739. Sariel-Talay, S., T.R. Balch, and N. Erdogan, Multiple traveling robot problem: A solution based on dynamic task selection and robust execution. Mechatronics, IEEE/ASME Transactions on, 2009. 14(2): p. 198-206.

[2] Sh. Shamshirband, M. Shojafar, A. R. Hosseinabadi, A. Abraham, "A Solution for Multi-objective Commodity Vehicle Routing Problem by NSGA-II", International Conference on Hybrid Intelligent Systems (HIS), 12-17, 2014.

[3] Bektas, T., The multiple traveling salesman problem: an overview of formulations and solution procedures. Omega, 2006. 34(3): p. 209-219.

[4] Plebe, A. Marcello Anile, A Neural Network Based Approach to the Double Traveling Salesman Problem, Department of Mathematics and Informatics, University of Catania V.le Andrea Doria, 8 I-95125 Catania, Italy, (2001) 1-23.

[5] Bektas, The multiple traveling salesman problem: an overview of formulations and solution procedures, Omega, Vol. 34, (2006) 209–219.

[6] G. Gutin, A. P. Punnen, The Traveling Salesman Problem and Its Variations, Combinatorial Optimization. Kluwer Academic Publishers, Dordrecht, The Nederlnds, (2002) 1-40.

[7] Gu, J. and X. Huang, Efficient local search with search space smoothing: A case study of the traveling salesman problem (TSP). Systems, Man and Cybernetics, IEEE Transactions on, 1994. 24(5): p. 728-735.

[8] Chen, Y.-W., Y.-Z. Lu, and P. Chen, Optimization with extremal dynamics for the traveling salesman problem. Physica A: Statistical Mechanics and its Applications, 2007. 385(1): p. 115-123.

[9] Saadatmand-Tarzjan, M., M. Khademi, and H. Moghaddam, A novel constructive-optimizer neural network for the traveling salesman problem. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, 2007. 37(4): p. 754-770.

[10] Qu, H., Z. Yi, and H. Tang, Improving local minima of columnar competitive model for TSPs. Circuits and Systems I: Regular Papers, IEEE Transactions on, 2006. 53(6): p. 1353-1362.

[11] Katayama, K. and H. Narihisa, An efficient hybrid genetic algorithm for the traveling salesman problem. Electronics and Communications in Japan (Part III: Fundamental Electronic Science), 2001. 84(2): p. 76-83.

[12] Fogel, D.B., Applying evolutionary programming to selected traveling salesman problems. Cybernetics and systems, 1993. 24(1): p. 27-36.

[13] Dorigo, M., V. Maniezzo, and A. Colorni, Ant system: optimization by a colony of cooperating agents. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, 1996. 26(1): p. 29-41.

[14] Misevičius, A., Using iterated tabu search for the traveling salesman problem. Information technology and control, 2004. 32(3): p. 29-40.

[15] Coy, S.P., et al., See the forest before the trees: Fine-tuned learning and its application to the traveling salesman problem. Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, 1998. 28(4): p. 454-464.

[16] Laporte, G. and Y. Nobert, A cutting planes algorithm for the m-salesmen problem. Journal of the Operational Research society, 1980: p. 1017-1023.

[17] Iqbal Ali, A. and J.L. Kennington, The asymmetric< i> M</i>-travelling salesmen problem: A duality based branch-and-bound algorithm. Discrete Applied Mathematics, 1986. 13(2): p. 259-276.

[18] Qu, H., Z. Yi, and H. Tang, A columnar competitive model for solving multi-traveling salesman problem. Chaos, Solitons & Fractals, 2007. 31(4): p. 1009-1019.

[19] Király, A. and J. Abonyi, Optimization of multiple traveling salesmen problem by a novel representation based genetic algorithm, in Intelligent Computational Optimization in Engineering. 2011, Springer. p. 241-269.

[20] Liaw, C.-F., A hybrid genetic algorithm for the open shop scheduling problem. European Journal of Operational Research, 2000. 124(1): p. 28-42.

[21] Carter, A.E. and C.T. Ragsdale, A new approach to solving the multiple traveling salesperson problem using genetic algorithms. European journal of operational research, 2006. 175(1): p. 246-257.

[22] Liu, W., et al. An ant colony optimization algorithm for the multiple traveling salesmen problem. in Industrial Electronics and Applications, 2009. ICIEA 2009. 4th IEEE Conference on. 2009. IEEE.

[23] Hou, M. and D. Liu, A novel method for solving the multiple traveling salesmen problem with multiple depots. Chinese Science Bulletin, 2012. 57(15): p. 1886-1892.

[24] R. Hosseinabadi, M. Kardgar, M. Shojafar, Sh. Shamshirband, A. Abraham, "GELS-GA: Hybrid Metaheuristic Algorithm for Solving Multiple Travelling Salesman Problem", International Conference on Intelligent Systems Design and Applications (ISDA), 76-81, 2014.

[25] S. Rostami, F. Mohanna, H. Keshavarz, A. R. Hosseinabadi, "Solving Multiple Traveling Salesman Problem using the Gravitational Emulation Local Search Algorithm", Applied Mathematics & Information Sciences, Vol. 9, No. 2, 1-11, 2015.

[26] Kellegöz, T., B. Toklu, and J. Wilson, Comparing efficiencies of genetic crossover operators for one machine total weighted tardiness problem. Applied Mathematics and Computation, 2008. 199(2): p. 590-598.