Ranking of decision making units on interval data with application of AHP and DEA

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Abstract

Crisp comparison matrices lead to crisp weight vectors being generated. Accordingly, an interval comparison matrix should give an interval weight estimate. In this paper, a new method is proposed to obtain interval weights from an interval comparison matrix, which can be either consistent or inconsistent and ranking of decision making unit with interval data by using a new simulation method and an interval analytic hierarchy process (IAHP) method. The conventional data envelopment analysis (DEA) approach are not linear in case interval data are considered. They can be linearized by using the monte carlo simulation (MCS) with the interval data. The profit of the proposed method in comparison with the other methods of IDEA problem solution, is considering directly decision maker's judgments.

Keywords:
AHP, DEA, Interval Comparison matrix, Interval importance grades, Monte carlo simulation.
The authors declares that there is no conflict of interest regarding the publication of this paper.

1. Introduction

Data envelopment analysis (DEA) is a non parametric technique and one of the most popular tools for measuring and evaluating the relative efficiency of Decision Making Units (DMUs) which stand for decision making units with common input and output terms [1]. In DEA models, the maximum of relative ratio of multiple weighted outputs to multiple weighted inputs is regarded as the efficiency. Since, ranking of DMUs are very important in DEA, and it is truly felt the necessity of a strong technic for making correct decision, up to now many models regarding ranking of these DMUs have been presented. Analytic hierarchy process (AHP) is one of the most efficient techniques. AHP as a popular tool in the field of multiple criteria decision making (MCDM) and a weight estimation and ranking technique DMUs, was introduced by saaty [2]. this method has been extensively applied in numerous situation in real world problems such as computer science(e.g., software selection, evaluation of data base management systems, etc)political science ,banking ,economic, planning and development, forecasting and so on and give us impressive results. The traditional AHP requires crisp comparison matrices. however, due to the complexity and uncertainty involved in real world decision problems, it is sometimes unrealistic or impossible to acquire exact judgments. It is more natural or easier to provide interval judgments for part or all of the judgments in a pairwise comparison matrix. Saaty and Vargas [3] introduced a Monte Carlo simulation approach to find out weights from interval comparison matrices. They also pointed out difficulties in using this approach. Arbel [4] interprets interval judgments as linear constraints on weights and formulates the weight estimation problem as a linear programming (LP) model for find out weights from an interval comparison matrix $A=([a_{ij}^L,a_{ij}^U])_{n\times n}$ where a_{ij}^L and a_{ij}^U respectively denote the lower and upper bounds of a certain interval judgment. Kress [5] finds the infeasibility of Arbel's method in solving $n(n-1)$ number of linear programs to deal with inconsistent interval comparison matrices. Salo and Hmlinen [6] extend Arbel's LP approach to hierarchical structures. Their approach searches for the maximums and minimums for all interval weights and incorporates the resultant intervals into further synthesis of global interval weights. Arbel and Vargas [4] formulate a hierarchical problem as a nonlinear programming (NLP) model in which all local weights in a hierarchy are included as decision variables and also established a connection between Monte Carlo simulation and Arbel's LP approach. Islam et al. [7] used a Lexico graphic Goal Programming (LGP) to find out weights from inconsistent pair wise interval comparison matrices. Sugihara et al. [8] bring forward an interval regression analysis method, which involves the solution of lower and upper approximation models. Wang et al. [9] pro- pose a two-stage logarithmic goal programming (TLGP) method to generate weights from interval comparison matrices, which can be either consistent or inconsistent. Wang et al. [10] also suggested that For consistent interval comparison matrices, Arbel's linear programming (LP) model is useful to derive interval weights and for inconsistent interval comparison matrices, aneigen-vector method based nonlinear programming (NLP) approach is developed to generate interval weights.

Liu and Li [12] introduced a method for obtaining the integrated weights of decision makers with interval data in multiple attribute group decision making problems. Entani and Sugihara [13] presented an approach to obtain intervals of attributes from the given uncertain pairwise comparison matrices. Wang and Li [14] developed a method for achieving interval weights from interval fuzzy preferences relations using goal programming approaches. Yang et al. [15] suggested an alternative strategy for ranking DMUs by using interval DEA. A new method are introduced by Jahanshahloo et al. [16] for ranking DMUs with interval data in DEA using ideal points. Marbini et al. [17] proposed an interval DEA to mea- sure efficiency with positive and negative interval data. Hu et al. [18] presented some new linear programming models for generation of interval weights from an interval fuzzy preference relations. Liu et al. [19] developed a model for a group decision making problem with interval preference matrices.

It is obvious that comparison matrices lead to crisp weight vectors to be generated. It is more logical that an interval comparison matrix should give an interval weight estimate rather than an exact point estimate. Entani et al. [11] presented a new approach on based of interval regression analysis method that is proposed by sugihara et al. [8]. In this method, interval weights is obtained from interval comparison matrices by using DEA. Using this method, we develop in this paper an method for ranking of DMUs on interval data. An advantage of the proposed method is that it considers directly decision maker's judgments.The proposed method is also applicable to crisp comparison matrices.

The paper is organized as follow. Section 2 we explain the monte carlo simulation methods for can transform the interval indicators to their crisp form. In section 3 we present the Proposed method for obtain interval importance grades. Ranking of DMUs on interval data is put forward in section 4. Three numerical examples are examined in section 5 using the proposed method. The paper is concluded in section 6

2. Monte Carlo Simulation(MCS) Method

In general, the MC simulation steps is briefed as follows:

•Generate random numbers for each of intervals; that is uniformly distributed on (0,1). We can use the RAND() function to generate them.

•Compute r_i = a + RAND() × (b − a), i = 1, ..., N ; here N = 10000 and slightly interval is [a,b].This formula generates a random value between lower bound and upper bound

.

•We introduce average of obtained numbers as certain number in this interval.

for each experiment from 10000 simulation experiment ,the random variables for describing interval data and interval judgements can be generated randomly from uniform probability distribution over the intervals $[x_{ij}^L, x_{ij}^U]$ and $[y_{ij}^L, y_{ij}^U]$ randomly. So with this method and subject Monte carlo simulation, we can transform the interval indicators to their crisp form. Thus, we can solve the crisp model by the standard DEA/VRS models. Be attention that ,for reduce errors and increase delicacy, we do not use above method to transmuting interval comparison matrices into crisp matrices. Next we will explain in ext section proposed simulation method.

3. Proposed method for obtain interval importance grades

In general, interval pairwise comparison matrices is as follows:

$$A = \begin{pmatrix} 1 & \cdots & \cdots & [a_{1n}^L, a_{1n}^U] \\ \vdots & \cdots & [a_{ij}^L, a_{ij}^U] & \vdots \\ \vdots & [a_{ji}^L, a_{ji}^U] & \cdots & \vdots \\ [a_{n1}^L, a_{n1}^U] & \cdots & \cdots & 1 \end{pmatrix}$$

Since the elements of A (The interval comparison matrix for input and out- put items) are intervals, so we can not calculate local weights by using eigen-vector method. Therefore, we use fallow simulation method: Suppose we want estimate the importance grade of item i, as an interval denoted as c, that is determined by its center wc and its radius di. So we have

$$\omega_i = \left[\omega_i^L, \omega_i^U\right] = \left[\omega_i^c - d_i, \omega_i^c + d_i\right]$$

That w_i^L and w_i^U are the lower and upper bounds of interval. Since the elements of A(The interval comparison matrix for input and output items) are intervals, so we can not calculate local weights by using eigenvector method. Therefore, we use fallow simulation method:By Using Monte carlo simulation method that explained in previous section, we transform interval matrix to crisp matrix.

by using of eigenvector method, we account local weight vector for matrix in previous stage. that is: $A^{M_k}W = [\lambda_{max}W]$.

Where A^{M_K} is the crisp matrix in k iteration, $\ddot{u}_{\max}$ most eigenvalue and

W is the it's eigenvector. They are the decision variables of this problem.

Repeat 10000 times the last stage.

We put the average of obtained local weights vectors in each stage as the center of the interval importance grades of each input and output item and show by ω_i^{c*} $W^{c*} = \left(\omega_i^{c*}, \ldots, \omega_n^{c*}\right)^t$ Be attention that, center ω_i^{c*} is normalized to be

$$\sum_{i=1}^{n} \omega_i^{c*} = 1.$$

So, with this method for each interval importance grades that is belong to items,
we can obtain the center of intervals. In our problem, w_i, w_j is approximated as interval importance grades sjuch that the following relation holds.

$$\left[a_{ij}^{L}, a_{ij}^{U}\right] \subseteq \frac{W_i}{W_j} = \left[\frac{\omega_i^{c*} - d_i}{\omega_i^{c*} + d_j}, \frac{\omega_i^{c*} + d_i}{\omega_i^{c*} - d_j}\right]. \quad (1)$$

So, the interval importance grades are determined to include the interval comparison values. By using the obtained centers ω_{c*}^{i} by proposed simulation method, the radius should be minimized subject to the relation for all elements should be satisfied. that is:

$$\begin{aligned}
&\min\ \lambda \\
&s.t. \quad \frac{\omega_i^{c*} - d_i}{\omega_j^{c*} + d_j} \le a_{ij}^{\mathrm{L}} \quad i = 1, \ldots, \mathrm{n}-1,\ j = i+1, \ldots, \mathrm{n} \\
&\qquad \frac{\omega_i^{c*} + d_i}{\omega_j^{c*} - d_j} \ge a_{ij}^{U} \quad i = 1, \ldots, \mathrm{n}-1,\ j = i+1, \ldots, \mathrm{n} \\
&\qquad 0 \le d_i \le \lambda, \quad i = 1, \ldots, \mathrm{n}, \\
&\qquad \mathrm{d}_i \le \omega_i^{c*}, \quad i = 1, \ldots, n.
\end{aligned}$$

4. Ranking of DMUs on interval data

Suppose that our purpose is evaluating DMUo. We use a model which is defined as follows

$$\begin{aligned}
&\min\ \sum_{r=1}^{s} \mu_r \left[y_{ro}^{L}, y_{ro}^{U}\right] \\
&s.t.\ \sum_{r=1}^{s} \mu_r \left[y_{rj}^{L}, y_{rj}^{U}\right] - \sum_{i=1}^{m} \omega_i \left[x_{ij}^{L}, x_{ij}^{U}\right] \le 0,\ i = 1, \ldots, \mathrm{n}. \quad (3) \\
&\sum_{i=1}^{m} \omega_i \left[x_{io}^{L}, x_{io}^{U}\right] = 1, \\
&\mu_r \ge \in \rangle 0, \quad r = 1, \ldots, \mathrm{s}, \\
&\omega_i \ge \in \rangle 0, \quad i = 1, \ldots, \mathrm{m}.
\end{aligned}$$

Obviously, above model is non-linear. Now by considering the interval DEA model (3), we use of the following equations according to concept of convex intervals we have:

$$\begin{aligned}
&x_{ij}^{L} \le x_{ij} \le x_{ij}^{U} \Rightarrow x_{ij} = x_{ij}^{L} + s_{ij}\left(x_{ij}^{U} - x_{ij}^{L}\right), 0 \le s_{ij} \le 1, i = 1, \ldots, \mathrm{m};\ \mathrm{j} = 1, \ldots, \mathrm{n}, \\
&y_{rj}^{L} \le y_{rj} \le y_{rj}^{U} \Rightarrow y_{rj} = y_{rj}^{L} + t_{rj}\left(y_{rj}^{U} - y_{rj}^{L}\right), 0 \le t_{rj} \le 1, r = 1, \ldots, \mathrm{s};\ \mathrm{j} = 1, \ldots, \mathrm{n}, \\
&\bar{x}_{ij} = x_{ij}^{L} + \bar{s}_{ij}\left(x_{ij}^{U} - x_{ij}^{L}\right), 0 \le \bar{s}_{ij} \le 1, i = 1, \ldots, m;\ j = 1, \ldots, n, \\
&\bar{y}_{rj} = y_{rj}^{L} + \bar{t}_{ij}\left(y_{rj}^{U} - y_{rj}^{L}\right), 0 \le \bar{t}_{rj} \le 1, r = 1, \ldots, \mathrm{s};\ j = 1, \ldots, n.
\end{aligned}$$

By Appling above commutation on (3) model, we have:

$$\begin{aligned}
&\min\ \sum_{r=1}^{s} \mu_r \bar{x}_{ij} \\
&s.t.\ \sum_{r=1}^{s} \mu_r \bar{y}_{rj} - \sum_{r=1}^{s} \omega_i \bar{x}_{ij} \le 0, \quad j = 1, \ldots, n, \\
&\sum_{i=1}^{m} \omega_i \bar{x}_{io} = 1, \\
&\mu_r \ge \in \rangle 0, \quad r = 1, \ldots, \mathrm{s}, \\
&\omega_i \ge \in \rangle 0, \quad i = 1, \ldots, \mathrm{m}.
\end{aligned}$$

Above model is the IDEA model (3) that it was changed to linear and crisp with applying convex combinations that are used by the monte carlo simulation.
As mentioned before, in DEA models, the maximum of relative ratio of multiple weighted outputs to multiple weighted inputs is regarded as the efficiency which is calculated from the optimistic view point for each DMU. In the original DEA model it is troublesome to comparison with input and output items by using their weights. because these scales depend on the original data X, Y scales that are input and output matrices considering of all input and output vectors. For using weights concept in AHP, we normalize the given input and output data on *DMUo* so that the input and output weights submit the importance grades of items. The normalized input and output are as follows [11]:

$$\bar{x}_{jp} = x_{jp} / x_{op}, p = 1, \ldots, m,$$

$$\bar{y}_{jr} = y_{jr} / y_{or}, r = 1, \ldots, s.$$

The problem for obtaining efficiency with normalized inputs and output, as follows:

$$\begin{aligned}
&\theta_0^{E*} = \max\left(\hat{\mu}1 + \ldots + \hat{\mu}s\right) \\
&s.t.\ 1\hat{\omega}_1 + \ldots + 1\hat{\omega}_m = 1, \\
&\qquad \hat{\mu}^t \hat{Y} - \hat{\omega}^t \hat{X} \le 0, \\
&\qquad \hat{\mu} \ge 0, \\
&\qquad \hat{\omega} \ge 0.
\end{aligned}$$

Which $X^{\wedge}$ and $Y^{\wedge}$ are the normalized data. So explain in this way:

$$\hat{X} = \begin{pmatrix} \hat{x}_{11} & \ldots & 1 & \ldots & \hat{x}_{1n} \\ : & : & : & : & : \\ \hat{x}_{m1} & \ldots & 1 & \ldots & \hat{x}_{mn} \end{pmatrix}$$

$$\hat{Y} = \begin{pmatrix} \hat{Y}_{11} & \ldots & 1 & \ldots & \hat{y}_{1n} \\ : & : & : & : & : \\ \hat{y}_{s1} & \ldots & 1 & \ldots & \hat{y}_{sn} \end{pmatrix}$$

By using normalized input and output, the multiply input or output and its weight, equal to its weight. So the obtained weight submit importance grade itself. Thus we can use DEA with normalized data for choice the optimistic weight in the interval importance grade obtained by a Decision maker via interval AHP. By attention to represented model in [?] for obtaining the most optimistic weights for *DMUo*, we have:

$$\theta_0^{E*} = \max\left(\hat{\mu}1 + \ldots + \hat{\mu}s\right)$$

$$s.t.\ \hat{\omega}_1 + \ldots + \hat{\omega}_m = 1,$$

$$\hat{\mu}^t \hat{Y} - \hat{\omega}^t \hat{X} \le 0,$$

$$\hat{\mu}_r \ge \left(\hat{\mu}_1 + \ldots + \hat{\mu}_s\right)^L \omega_r^{out} \quad, r = 1, \ldots,$$

$$\hat{\omega}_i \ge {}^L\omega_i^{in} \quad, i = 1, \ldots, \mathrm{m},$$

$$\hat{\omega}_i \ge {}^U\omega_i^{in} \quad, i = 1, \ldots, \mathrm{m},$$

$$\hat{\mu} \ge 0,$$

$$\hat{\omega} \ge 0,$$

We should attend that any optimal solutions $\mu\hat{}$ and $\omega\hat{}$, are within the interval importance grades that are given by a decision maker based on his/her evaluation.

5. Numerical Examples

In this section, we provide 3 numerical examples to illustrate potential ap- plications of the proposed methodology and show its advantage over ranking approaches that represented for interval data in DEA.

Example1

Consider the Table1 Which was investigated by entani et al.[11].

As for proposed method and by using matlab software ,we can obtain the centers of importance grades of output item as follow:

$$AW^{c*} = \lambda_{\max} W^{c*} \Rightarrow W^{c*} = (0.1353, 0.5227, 0.0496, 0.2923).$$

Table 1: Importance grades of the output item.

	$y1$	$y2$	$y3$	$y4$
$y1\ y2\ y3\ y4$	1	[1/6,1/3] 1	[3,7]	[1/6,1/3]
	[3,6]	[3,7]	[6,8]	[2,4]
	[1/7,1/3]	[1/4,1/2]	1	[1/9,1/3] 1
	[2,6]		[3,9]	

As for the model (2), we obtain centers of intervals as follow:

$$d^* = (0.0651, 0.1293, 0.021, 0.1293).$$

Thus, we can obtain importance grades of output item that are shown in Table 2. In proposal method, weight vector W^{c*} obtained with average of 10000 weight vector, so it can be a good indicator for evaluating weights. Nevertheless we ranking, the interval weights with proposal method that offered by Wang et al.[?]:

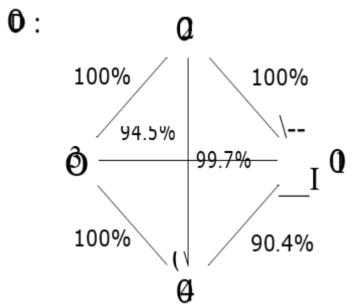
Step1: Calculate the matrix of degree of preference

$$P_D = \begin{pmatrix} - & 0 & 0.997 & 0.096 \\ 1 & - & 1 & 0.945 \\ 0.003 & 0 & - & 0 \\ 0.904 & 0.055 & 1 & - \end{pmatrix}$$

Step2: Calculate the matrix of preference relation

$$P_D = \begin{pmatrix} - & 0 & 1 & 0 \\ 1 & - & 1 & 1 \\ 0 & 0 & - & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

Step3: Draw a corresponded diagram with the matrix of preference relation



Step4: Overall ranking for intervals

$W_2^* \succ W_4^* \succ W_1^* \succ W_3^*.$

Table 2: Interval weights of output item.

Interval weights	Entani et al.	Proposed Method
$W1\ \ W2\ \ W3$	[0.071,0.200]	[0.070,0.200]
$W4$	[0.391,0.652]	[0.393,0.652]
	[0.029,0.070]	[0.028,0.070]
	[0.163,0.424]	[0.163,0.421]

As an example, consider Table 3, which consist of 1-input and 2-output data and the same importance grades with the example1.Using the MCS that explained in section2, we transform all the intervals inputs-outputs to their crisp state. Thus, get the crisp values represented in the Table4.Suppose that our purpose is evaluating DMUA. We calculate the normalized outputs by using illustrated method in section4. thus, we will reach to the bottom normalized outputs.

$$\hat{Y}_A = \begin{pmatrix} 1 & 1.99 & 1.995 & 2.981 & 2.980 & 3.980 & 3.967 & 4.976 & 5.955 & 6.963 \\ 1 & 0.375 & 0.777 & 0.376 & 0.873 & 0.259 & 0.259 & 0.625 & 0.251 & 0.13 \end{pmatrix}$$

Where, $\hat{Y}_A$ indicates normalized outputs for DMUA. So, we find out that $w_1 \in [.143, 0.24]$ and $w_2 \in [.759, .857]$ (first and second interval importance grade outputs respectively). Therefore we gain efficiency of DMUA by using model(6). Hence

$\theta_A^{E*} = .0852$ Do this process for the other DMUs and show in the Table5.

Table 3: Data with 1-input and 2-output.

	x1y1	y2
A1	[0.8,1.2]	[7.5,8.5]
B1	[1.8,2.2]	[2.3,3.7]
C1	[1.7,2.3]	[5.7,6.7]
D1	[2.5,3.5]	[2.7,3.3]
E1	[2.8,3.2]	[6.7,7.3]
F1	[3.8,4.2]	[1.8,2.2]
G1	[3.4,4.6]	[4.6,5.4]
H1	[4.7,5.3]	[1.5,2.5]
I 1	[5.6,6.4]	[1.7,2.3]
J 1	[6.7,7.3]	[3.0,9.0]

Table 4: Fixed interval input-outputs data.

	x1y1	y2
A1	1.005	8.004
B1	1.999	3.001
C1	2.005	6.217
D1	2.996	3.009
E1	2.995	6.988
F1	4.000	2.002
G1	3.987	5.007
H1	5.001	2.007
I 1	5.985	2.000
J 1	6.998	1.041

According to Table5, DMUE is one efficient unit and has maximum degree among other units.

Table 5: Efficiency and ranking of DMUs

	Efficiency	Ranking
A	0.852	2
B	0.466	6
C	0.849	3
D	0.476	5
E	1.000	1
F	0.326	7
G	0.784	4
H	0.324	9
I	0.325	8
J	0.160	10

Example3

Consider the following interval comparison matrix, which is borrowed from Kress[6].

$$A = \begin{pmatrix} 1 & [1,2] & [1,2] & [2,3] \\ [1/2,1] & 1 & [3,5] & [4,5] \\ [1/2,1] & [1/5,1/3] & 1 & [6,8] \\ [1/3,1/2] & [1/5,1/4] & [1/8,1/6] & 1 \end{pmatrix}$$

Kress[5] showed that this interval comparison matrix is inconsistent and hence we want to show that the proposed method in compare with one of the most efficient technique that was proposed by Wang et al.[?], what obtain the interval priorities.
By do the proposed approach ,we submit the interval priorities for inconsistent matrix of A and represent in Table6. It is obvious that obtained intervals by proposal method are contain of obtained intervals by Wang et al.[?].

Table 6: Interval priorities for Example3.

	Efficiency	Ranking
W_1	[0.2282,0.3830]	[0.1411,0.4695]
W_2	[0.3264,0.4758]	[0.2347,0.5631]
W_2	[0.1792,0.2819]	[0.1126,0.3406]
W_4	[0.0562,0.0843]	[0.0426,0.0958]

Thus, proposal method in this paper can be a good approach for consistent and inconsistent matrices for obtaining interval priority from a comparison matrix.

6. Conclusion

The main target of this paper was to propose an approach for ranking of DMUs on interval data. The advantage of the proffered method in comparison with the other methods, is considering importance grades of input and output items. so we can consider decision maker's judgments in our evaluation and can be a good implement for make wise and logical decision. Now, if we don't consider decision maker' judgments, can obtain the several DMUs that are efficiency and should we rank those by using of the represented models for ranking efficient units. Because our judgments is interval , it is better that obtained weights for alternatives be interval too. Because interval numbers have more informations than crisp numbers. Our proffer method in this paper can be suitable way for ranking of DMUs with interval data. Another of preference of this method compared with others methods is that we use a method to transforming interval into certain values, which is very simple and simulation occurs by short time consuming length as a few seconds. Also we know before, if data are as interval in IDEA models, as model (3)the efficiency value will be interval. Hence, we deal with interval ranking. But in the our proffer method, obtained efficiency value is a crisp value, and so it is not necessary to ranking based on lower and upper bounds. So with this method we can solve ranking problem for interval efficiency.

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