Investigating the efficiency and promotion of watermarking methods based on decomposition of singular values

Moein Sadeghi Askari† , Amir Hossein Javanshir††,

† Department of Electrical Engineering, Besat Institute of Higher Education, iran
†† Department of Electrical Engineering, Najafabad Branch, Islamic Azad University, iran

Abstract
In this paper, a watermarking algorithm based on SVD transformation (decomposition of singular values) is presented which is less complex than other methods in this field. In suggested method, first a discrete cosines transform is gotten from the image of host and after division of image in transformation domain to not spanning blocks, the SVD transform is applied for each of them. Finally, watermark is inserted in singular values. The results of scientific comparing experiments indicates the efficiency of suggested methods in comparison with basic method based on SVD in terms of quality and resistance.
Keywords:
watermarking blind watermarking, resistant watermarking, SVD

1. Introduction

In recent years, by growth of technology the need to take care of multimedia contents from illegal copying is being sensed. The watermarking is one of the usual methods against the illegal changes of data, which is used, for copyright. The watermarking methods hide the secret information inside the multimedia content like video, image, and voice and when it is needed, the watermark added to multimedia and digital content can be unhidden or remade. Form the copyright point of view the purpose of watermarking is distinguishing the owner of work.

The methods of watermarking can be divided to two place and time domains. Some examples of watermarking in place domain which usually by changing of a subset of pixels of host image is done, is investigated in [1] [2] [3]. In transformation domain, after applying the transformation to host image, inserting the watermark is done. As an instance of a watermark in frequency domain, the watermarking by DCT (discrete Fourier transform) [5], watermarking by DWT (discrete wave transform) [6], or methods based on SVD transform can be addressed [16].

One of the most important requirements of copyright domain is reaching to a balance between clarity of watermark and its resistance against the intentionally and unintentionally attacks. From another point of view, the watermark algorithms can be divided to three category: fragile, semi-fragile, and resistant. The resistant algorithms are highly resistant against the different attacks [7]. This kind of algorithms in watermarking are widely used in copyright domain. In fragile algorithms by the smallest change in the watermarked image, the watermark gets disturbed. One of the applications of this algorithm is authentication and verification of image [7]. Semi-fragile algorithms are fragile against the intentional attacks and act like resistant algorithms against the unintentional attacks [8].

The other categorization that can be addressed is the three blind, semi-blind, and unblinded algorithm. In unblinded algorithm the main image and sometimes main watermark is required in the process of detection. In Semi-blinded only main watermark and some of marginal information is used. In blind watermarking in extracting process not host image and nor main watermark is used [9]. The blinded watermarking can be categorized to two reversible and irreversible types. If we suppose that the main image is I_0 and the watermark is W_0, the insert process by E function will be as following equation:

$$I_W = E\{I_0, W\} \quad (1)$$

In which the I_W is watermarked image. A method is irreversible if an attacker not be able to create the fake W_{0F} watermark and fake I_{0F} host image using watermarked I_W image therefore:

$$\mathrm{E}\{\mathrm{I}_0, \mathrm{W}\} = \mathrm{E}\{\mathrm{I}_{0\mathrm{F}}, \mathrm{W}_\mathrm{F}\} \quad (2)$$

In applying the retain of copyright by the time of detecting process the main images must not be used directly. Thereby, the unblinded methods are not appropriate for retaining copyright [10].

The calculation complexity of SVD transformation for a matrix with mxn dimensions is equal to [38]:

$$O(\min\{m \times n^2, m^2 \times n\}) \quad (3)$$

The important advantage of the methods based on SVD is the resistance of singular values of images and low changes on images do not have intensive effect on their singular values. Therefore, these methods have high resistance against the usual image processing operations like: compacting, filtering, and correcting the histogram and

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geometric changes like: turning, movement, and change of dimensions.

The most important problem of the method based on SVD is the calculation complexity of this transformation which have direct relationship with the dimension of image. In this paper two new methods for decreasing the complexity of calculation of watermarking based on transformation is presented. In the first method, first the DCT transformation will be applied on the host image after the output scan, n initial coefficient from one dimensional array will be chosen in form of zigzag in which the n is equal with the number of pixels of the image of watermark. We divide them with the dimensions of half of image of watermark. We apply the SVD transformation on obtained blocks and after dividing watermark to the blocks with the same sizes we attempt to insert the watermark. In second method first we apply the Hadarmard transformation on host image then by the obtained matrix we choose a matrix with dimensions of watermark image they we apply SVD transformation on it and insert the watermark in the matrix of singular values.

The structure paper is as following: in section 2 we have discussed the watermarking methods based on the SVD transformation. The suggested method will be presented in section 3. In section 4 we discuss the scientific simulation and comparing it with basic method in SVD domain. Finally in section 5 we will sum up.

2. Review of past works

2.1 Summary of done works

- The methods which insert the watermark or its singular values in singular values of host image.
- The methods which insert the watermark by changing the singular right or left vectors of host image.
- The methods which are based on change of the singular vectors or singular values. The watermark got inserted for controlling the special parameters in singular vectors.
- The methods which are the combination of SVD transformation and other methods.

2.2 The watermark algorithms based on change of singular values

The simplest insertion method is adding watermark to singular values of host image. The watermark is a binary logo or a random sequence [14-11].

In another method after applying the SVD transformation on host image, singular values of watermark is inserted in singular values of host image [15].

First time Liu et al. presented a method based on SVD transform [16]. In this method if the image is shown by A without losing the generality, in future discussions it is supposed that A is a square image and is shown in form of $A \in F^{N\times N}$. The F is sign of a real number or a complex number. In SVD the image A is defined as following:

$$\mathrm{A} = \mathrm{USV^T} \tag{4}$$

In which the $U \in F^{N\times N}$ and $V \in F^{N\times N}$ are unit matrix and $S \in F^{N\times N}$ is a diagonal matrix. For inserting the watermark first the SVD transform of image A is calculated. The output of this transformation is two U and V orthogonal matrix and a S diagonal matrix. Watermark is added to matrix S using the $\mathrm{S} + \alpha\mathrm{W}$ relationship. The SVD transformation is applied on obtained matrix which its output is three U_W, V_W, S_W matrices. In this relationship α controls the insertion of watermark. After multiplying the three U, S_W, V^T matrices the watermarked A_W is obtained. If A and W be the main image and watermark respectively and A_W be the watermarked image the stages of watermark insertion is as following.

$$\mathrm{A} \Rightarrow \mathrm{USV^T} \tag{5}$$

$$\mathrm{S} + \alpha\mathrm{W} \Rightarrow \mathrm{U_W S_W V_W^T} \tag{6}$$

$$\mathrm{A_W} \Rightarrow \mathrm{US_W V^T} \tag{7}$$

For extracting the watermark by U_W, S, V_W and by watermarked image that possibly is attacked (A_W^*) the watermark image can be extract in following form:

$$\mathrm{A_W^*} \Rightarrow \mathrm{U^* S_W^* V^{*H}} \tag{8}$$

$$\mathrm{D^*} \Leftarrow \mathrm{U_W S_W^* V_W^H} \tag{9}$$

$$\mathrm{W^*} \Leftarrow \frac{1}{\alpha}(\mathrm{D^*} - \mathrm{S}) \tag{10}$$

The biggest problem of this algorithm is calculation complex of SVD transformation. Instead of using SVD transformation in all of the image the host image can be transformed to not spanning blocks and then the SVD transform be applied on the blocks. In this method the singular values of watermark is added to singular values of host image blocks [17] [18] in this method the watermark can blind [19] semi blind [18] and or ublinded [20].

The main problems of this method can be shown as following;

- False result: as it is mentioned in [21-26] these methods have the problem of false results and extract the watermark from the content which they was not in it [27]. Therefore the problems of copyright cannot be solved [28]. The methods which insert the singular values of watermark in the singular values of host image are irreversible [22].

- The quality of watermarked image: In these methods change of the biggest singular value might decrease the quality of watermarked image. Therefore it is needed that a coefficient be used for insertion process. This coefficient decreases the energy of watermark therefore the resistance of this method against the different methods decreas.
- The high complexity of calculation

2.3 The watermarking algorithms based on singular vectors

In this algorithms the water marks in U and V matrices and after SVD transformation of host image is inserted [29].

2.4 The watermarking algorithm based on vectors and singular values

In the year 2008 Kandara [30] invented a twofold method based on blocking. The host image is divided to 4 block. Initially, using the quantization and the method written by Chang [31] the watermark (a binary image) is inserted in biggest singular coefficient on the top left block and then in bottom right block. For improving the security of this method the watermark gets disassembled before the insertion process. This algorithm against the usual geometric attacks is resistant but the change of the top left blocks and bottom right blocks of host image in some cases lead to creating some changes in border of blocks in water marked image.

2.5 combined algorithms

The SVD can be used with other transformation [26 & 32-35]. The written method in [26] decompose the colorful host image to three U, V, and Y and then divide the Y to not spanning blocks. For inserting the bits of watermark the biggest singular value of each blocks gets quantized. This method is resistant against most of attacks and the main image is not need in process of detecting. In another method [26] the singular values of watermark get added to image singular coefficient, meaning that the image is consist of DC coefficient of each block. For preventing the false result problem, the matrix of right vectors of watermark get inserted in second and third AC coefficient of each block. In [37] two watermarking methods based on homomorphic transformation and decomposition of singular values (SVD) are presented.

In the year 2006, the [32] paper showed that the detecting process written in [35] has false result problem. There are similar conditions in [32] too. In The [26] method by attention to limitation of DC coefficient the capacity decrease extremely. In the first method [37] there is the calculation complexity problem but in second method of it despite of decreasing the complexity of calculations the quality of watermarked image and extracted watermark considerably decrease.

3. Suggested algorithm

As it is shown in equation three the calculation complexity of SVD transformation is directly related to dimensions of matrix. Therefore, for decreasing the calculation complexity of it, in watermark a part of host image can be used for inserting the watermark. For example, if the image of cameraman with 256x256 dimensions be chosen for inserting the Lena watermark image with 64x64 dimensions, in this method a segment of image with 64x64 gets chosen, then we divide it to four 32x32 blocks in which the blocks of watermark is inserted therefore, by attention to equation 3 the computational complexity of process of inserting the watermark decreases from $256 \times 256^2 = 16777216$ multiplication to $4 \times (32 \times 32^2) = 131072$ multiplication. but distortion of that part of image can disturb the watermark (figure 1-A). image blocking leads to decrease of quality of watermark image and creation of disturbance in border of blocks (figure 1-B) and decrease of its resistant against the attacks. Therefore, before SVD the image is taken to transformation domain.

A

B

Figure.1 The watermarked image with dividing the host image with 256x256 to not spanning blocks with 64x64 blocks and c=0.5 coefficient in (A) watermarking in a part of host image (B) the watermarked image by host image blocking method

This transformation must be in a way that itself not lead to increase of computational complexity, on the other hand with inserting the image of watermark in more important coefficient the resistance against the attacks like adding noise and JPEG compacting get increased. The energy distribution of image over the coefficients in different transformations are different. The figure 2 illustrates the energy distribution of an image with 256x256 dimensions over the transformation coefficients DFT, DCT, Waslh-Hadamard, and HAAR.

a b c d

Figure.2 Energy distribution over the coefficient in different transformations a) the selected piece b) Haar transformation c) DFT transformation d) DCT transformation

The curve of energy distribution of coefficients of fast transformation is shown in figure 3.

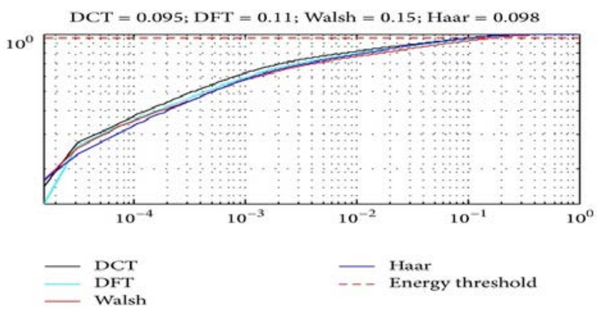
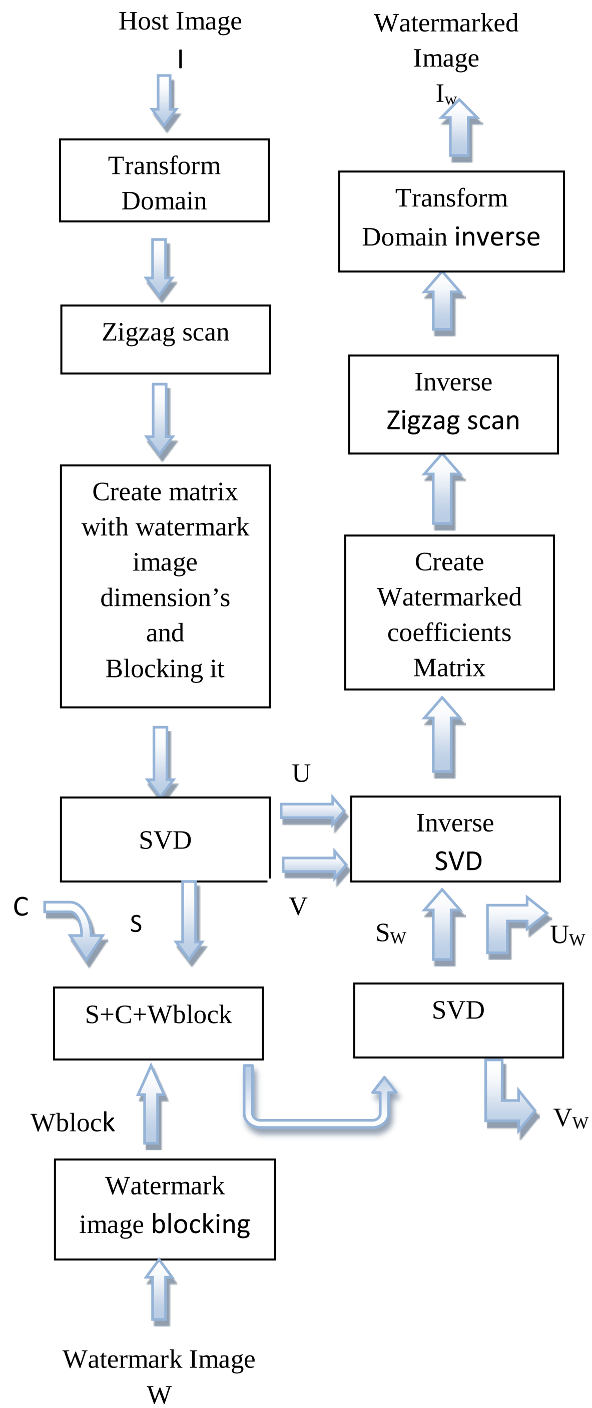


Figure.3 the curve of energy distribution of coefficients of fast transformation for images with 256x256 dimensions

As it is illustrated the DCT transformation have the best energy distribution between the fast methods. The DCT transformation is one of the transformations that while having low complexity, have appropriate energy distribution for watermarking. Considering the mentioned problems we suggest two methods. The block of suggested algorithm diagram including insert algorithm and extract algorithm is illustrated in figure 4 (a and b).



A

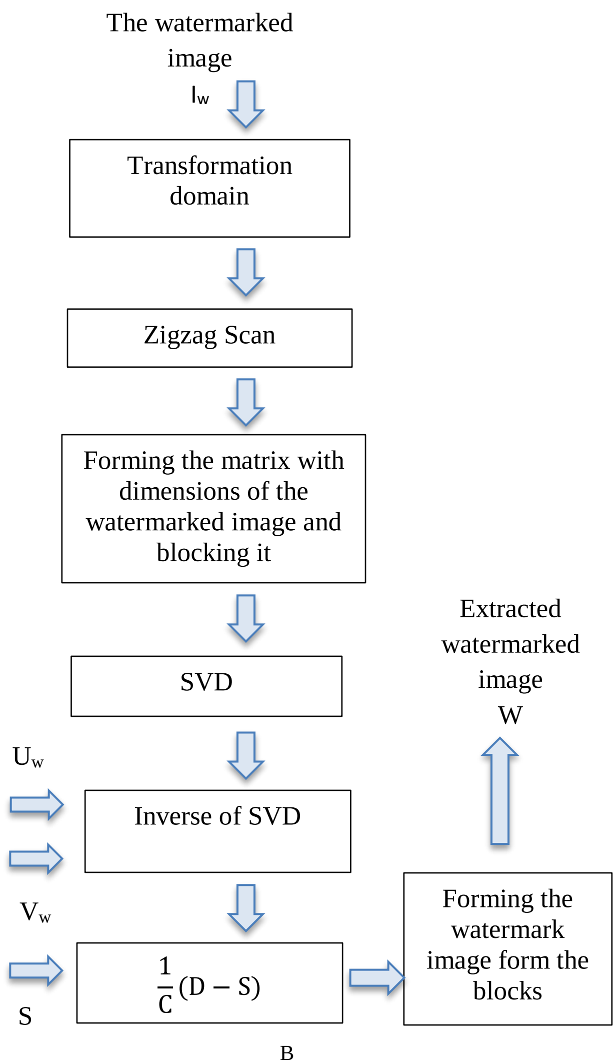


Figure.4 The block of suggested algorithm diagram (a) Insertion algorithm (b) Extraction algorithm

3.1 The first suggested algorithm

3.1.1 Watermark image insertion

DCT transformation of A host image

$$B = DCT(A) \quad (11)$$

Scan of matrix of B obtained coefficients in form of zigzag and putting it in a one dimensional array

$$B2 = ZigZagScan(B) \quad (12)$$

Choosing n initial coefficient from one dimensional B2 array in which n is equal with the number of watermark pixel and creating the D matrix with dimension of watermark image from this coefficient. We divided this D matrix to not spanning blocks.

The SVD transformation of obtained blocks

$$[U\ S\ V] = SVD(block) \quad (13)$$

Dividing the watermark too not spanning blocks with the size of obtained blocks in previous stage and inserting them by equation 14

$$SU = S + C * Wbloc \quad (14)$$

C is a positive coefficient and Wblock is watermark blocks.

Decomposing the matrix obtained from the previous stage with SVD

$$[U_W, S_W, V_W] = SVD(SU) \quad (15)$$

The inverse of SVD and forming the BAW watermark coefficient blocks

$$BAW = U * S_W * V^T \quad (16)$$

Forming the BW matrix obtained from previous stage

Transforming the BW matrix to one dimensional array and then inversing its zigzag broom and creating the B coefficient matrix.

The inverse of DCT transformation and obtaining the watermarked image.

$$I_w = IDCT\ (B) \quad (17)$$

3.1.2 Extracting watermark image

DCT transformation of I_W watermarked image

$$BW = DCT(I_W) \quad (18)$$

The scan of obtained *BW* coefficient matrix in form of zigzag and placing it in a one dimensional array

$$CW = ZigZagScan(BW) \quad (19)$$

Choosing n initial coefficient from *CW* one dimensional array in which n is equal to the number of watermark image pixel and creating *DW* matrix with the dimensions of watermark image from these coefficient. This matrix gets divided to individual *BDW* blocks with a size equal to blocks in insertion stage.

SVD transformation of obtained blocks

$$[U2\ S_{W2}\ V2] = SVD(BDW) \quad (20)$$

Then

$$D = U_W * S_{W2} * V_W^T \quad (21)$$

In which the U_W *and* V_W are matrices which were obtained in insert stage.

The constructing blocks of watermark image get obtained by this following equation

$$BW = \frac{1}{c}(D - S) \quad (22)$$

Which S is the matrix obtained from SVD transformation in insert stage and finally the watermark image gets made by this blocks.

3.2 Second suggested algorithm

3.2.1 Inserting watermark image

Walsh-hadamard transformation of A host image

$$B = \text{Whadamard}(A) \quad (23)$$

The scan of B obtained matrix in form of zigzag and placing them in a one dimensional array

$$B2 = \text{ZigZagScan}(B) \quad (24)$$

Choosing n initial coefficient of B2 one dimensional array in which the n is equal to the number of pixels of watermark image and Creating D matrix with dimension of watermark image of these coefficient. We divide This D matrix to not spanning blocks.

SVD transformation of obtained blocks

$$[U\ S\ V] = \text{SVD}(\text{block})\ ; \quad (25)$$

Watermark division to not spanning blocks with the size of obtained blocks in previous stage and inserting them by equation 25

$$SU = S + C * \text{Wblock} \quad (26)$$

In which the C is a constant positive coefficient and $Wblock$ is the blocks of watermark.

Decomposition of matrix obtained from the previous stage by SVD

$$[U_W, S_W, V_W] = \text{SVD}(SU) \quad (27)$$

The inverse of SVD and forming the blocks of BAW watermark coefficient

$$BAW = U * S_W * V^T \quad (28)$$

Forming the BW matrix from the obtained blocks in last stage

Transforming the BW matrix to one dimensional array then its zigzag broom and creating the B coefficient matrix

The inverse of Walsh-hadamard transformation and obtaining the watermarked image

$$I_w = \text{IWhadamard}(B) \quad (29)$$

3.2.2 Extracting watermark image

Walsh- hadamard transformation of I_W watermarked image

$$BW = \text{Whadamard}(I_W) \quad (30)$$

The scan of BW obtained coefficient matrix in form of zigzag and placing it in a one dimensional array

$$CW = \text{ZigZagScan}(BW) \quad (31)$$

Choosing n initial coefficient from CW the one dimensional array in which n is equal to pixels of watermark image and creating the DW matrix with dimension of watermark image from these coefficient. This matrix is divided to individual BDW blocks with equal size of blocks in insert stage.

The SVD transformation of obtained blocks

$$[U2\ S_{W2}\ V2] = \text{SVD}(BDW) \quad (32)$$

Then

$$D = U_W * S_{W2} * V_W^T \quad (33)$$

In which $U_W\ adn\ V_W$ are matrices that are obtained in insert stage.

The constructing blocks of watermark image get obtained using the following equation

$$BW = \frac{1}{c}(D - S) \quad (34)$$

In which the S is the matrix obtained from SVD transformation in insert stage and finally the image of watermark is made by these blocks.

4. Applying and scientific results

In this section in order to evaluate the performance of suggested method, we compare this method with others by experimenting in [16]. The host image is the cameraman standard image with 256x256 dimensions and the watermark is the Lena with dimension of 64x64. The both images have gray background with 8 bits depth. The supposed criterion for evaluating the quality of watermark image and host image is PSNR. The value of PSNR is calculated by equation [35].

$$PSNR = 10 \log \frac{(I_{max})^2}{MSE} \quad (35)$$

The value of I_{max} is equal with the biggest pixel in image. MSE is the square of residuals between main image and host image and is calculating using (36) equation.

$$\mathrm{MSE} = \frac{1}{\mathrm{MN}} \sum_{\mathrm{MN}} \left(\mathrm{I}_{\mathrm{M,N}} - \acute{\mathrm{I}}_{\mathrm{M.N}} \right)^2 \quad (36)$$

The $I_{M,N}$ is the value of main image pixel in place of MN and $\acute{I}_{M,N}$ is the value of watermarked image pixels (by attention to main image).

4.1 The results of algorithm simulation

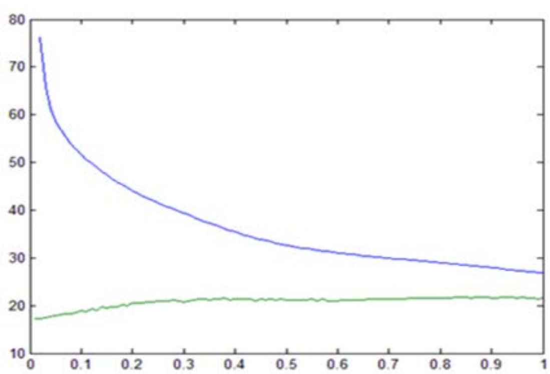
The results of algorithm simulation [16] is shown in figure 5 and the results of first suggested method and second method are shown in figures 6 and 7 respectively. the watermarked image will be attacked by 10 percent noise, 10 degree turning, median filter, and JPEG compacting and then the extracted watermark from this image will be evaluated by PSNR standards.

The host image with 256x256 dimensions

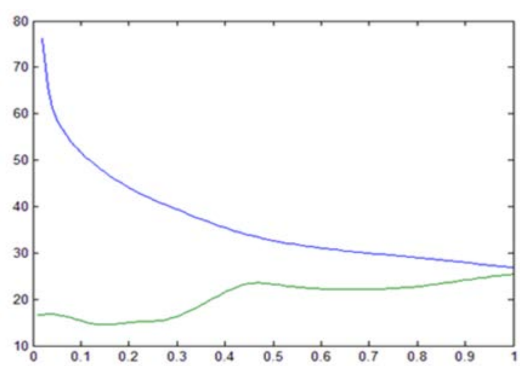
The watermark image with 64x64 dimensions

The watermarked image with coefficient C=0.3
PSNR=42dB

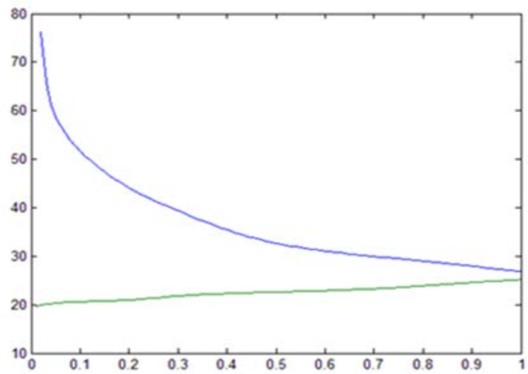
The extracted image after getting attacked by 10% Gaussian noise
PSNR=21dB



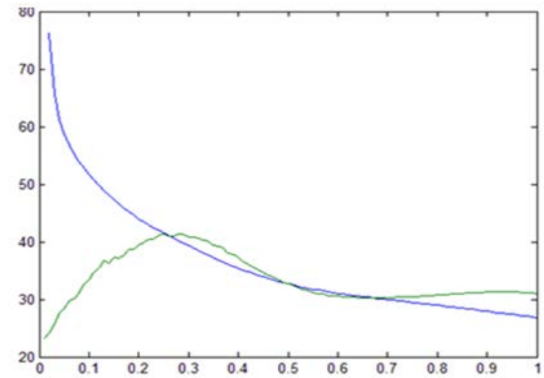
The PSNR changes of host image (blue) and the extracted watermarked image (green) after applying 10 % Gaussian noise to watermarked image with changing the C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after 10 degree turning of watermarked image with change of C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after applying the Median filter to watermarked image with change of C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after compacting the watermarked image with change of C coefficient

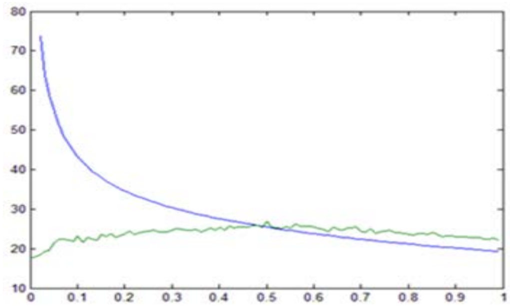
Figure.5 The results of algorithm simulation based on SVD [16]

The host image with 256x256 dimensions

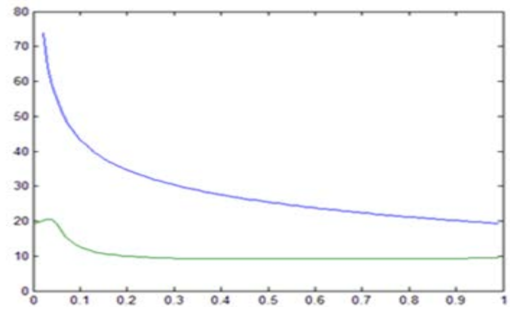
The watermark image with 64x64 dimensions

The watermarked image with c=0.1
PSNR=47dB

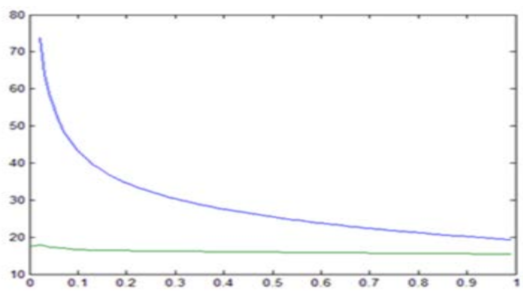
The extracted image after getting attacked by 10% Gaussian noise
PSNR=23dB



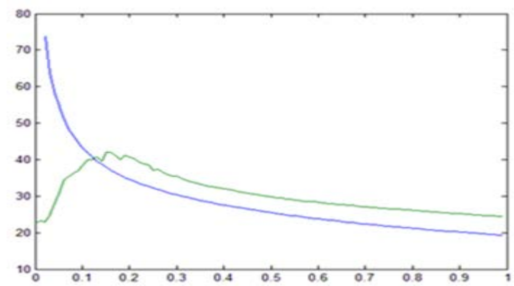
The PSNR changes of host image (blue) and the extracted watermarked image (green) after applying 10 % Gaussian noise to watermarked image with changing the C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after 10 degree turning of watermarked image with change of C coefficient



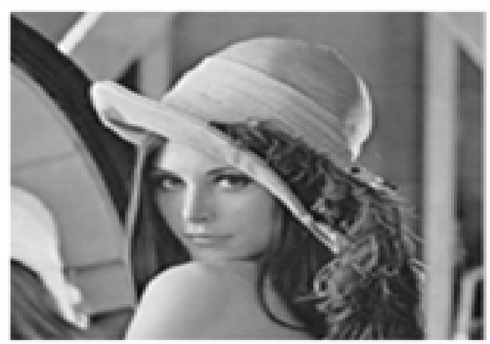
The PSNR changes of host image (blue) and the extracted watermarked image (green) after applying the Median filter to watermarked image with change of C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after compacting the watermarked image with change of C coefficient

Figure.6 The results of first suggested algorithm simulation

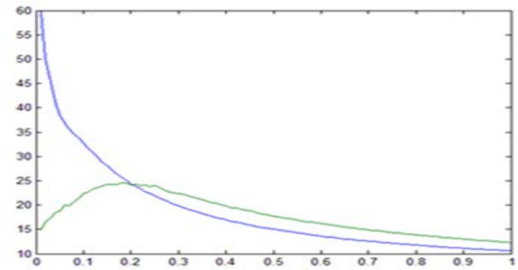
The host image with 256x256 dimensions



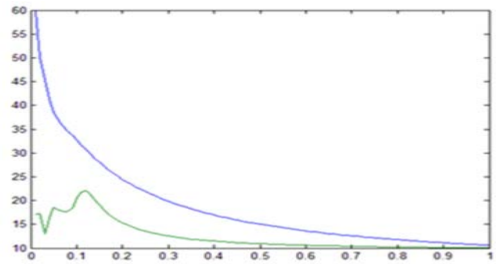
The watermark image with 64x64 dimensions

The watermarked image with 256x256 dimensions with c =0.02
PSNR=41dB

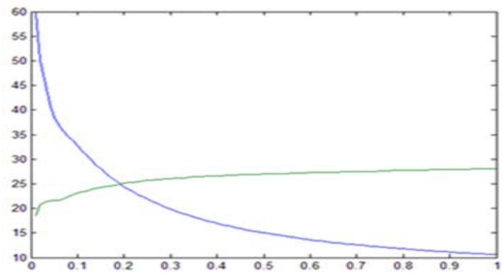
The extracted image after getting attacked by 10% Gaussian noise
PSNR=19dB



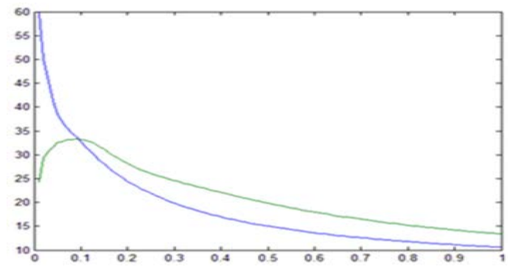
The PSNR changes of host image (blue) and the extracted watermarked image (green) after applying 10 % Gaussian noise to watermarked image with changing the C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after 10 degree turning of watermarked image with change of C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after applying the Median filter to watermarked image with change of C coefficient



The PSNR changes of host image (blue) and the extracted watermarked image (green) after compacting the watermarked image with change of C coefficient

Figure.7 The results of second algorithm simulation

Computational complexity of process of [16] algorithm and suggested algorithms is shown in table 1.

Table.1 Computational complexity of insertion process

Algorithm	Host image dimensions	Watermark image dimension	Computational complexity
[16**]** basic	256x256	64x64	$256 \times 256^2 = 16777216$ multiplication
First suggested	256x256	64x64	$4 \times \left((32 \times 32^2) + 32 \times \log 32\right) = 131120$ multiplication
Second suggested	256x256	64x64	$4 \times (32 \times 32^2) = 131072$ multiplication $32 \times \log 32 = 48$ sum

As it is clear from the results of table 1 our suggested algorithms has led to improvement of computational complexity.

5. Conclusion

In this paper a new watermark for watermarking in SVD transformation domain is presented, which has less computational complexity in comparison with algorithms of this domain. Moreover, the suggested method led to better quality and resistance in comparison with basic method based on SVD. The evaluation criterion for evaluating is the quality of watermarked image and image of watermark extracted from PSNR. The watermarked image was attacked by 10% noise, 10 degree turning, median filter, and JPEG compacting. The extracted watermark from this image is evaluated by PSNR criterion. The scientific results confirmed the efficiency of suggested method.

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