Efficient Hybrid Multi-Objective Evolutionary Algorithm

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Summary
In the artificial intelligence community the multi-objective optimization problem become very common and has been rapidly increasing attention. This significant is due to the fact that there is high number of real-world applications having optimization problems that include more than one objective function. As has been evident in the last ten years, the evolutionary algorithms are one of the best choices to solve multi-objective optimization problems. Although evolutionary algorithms are the most common approach to solve multi-objective optimization problems, there is still many issues and drawbacks that need solving and enhancing. In this paper a set of improved hybrid Memetic evolutionary algorithms are proposed to solve multi-objective optimization problems. The proposed algorithms enhance the performance of NSGA-II algorithm by using different new proposed and simple search schemes. Merging a simple and efficient search technique to NSGA-II significantly enhances the convergence ability and speed of the algorithm. To assess the performance of proposed algorithms, three multi-objective test problems are used from ZDT set. Our empirical results in this paper show that the proposed algorithms significantly enhance the NSGA-II algorithm performance in both diversity and convergence.

Keywords: Evolutionary algorithms; Memetic algorithms; multi-objective optimization; high dimensional problems; hybrid algorithms

1. Introduction
Evolutionary algorithms (EAs) have proven to be a popular and useful choice when we need to get an estimated optimal solution for complex optimization problems [1]. The real-world optimization problems can be single objective and multi-objective and for each type there are different evolutionary algorithms to solve it. The high complexity of multi-objective optimization problems (MOPs) over the single objective optimization problems prevents the use of traditional single objective evolutionary algorithms in solving MOPs. Therefore, solving MOPs becomes a separated research field and has its own techniques, test problems and performance metrics. Recently the Multi-objective Optimization Evolutionary Algorithms (MOEA) become very important and widely used since there is many real world problems that include multi-objective functions such as telecommunication networks design problems [2], gas turbine combustion optimization problems [3], antenna design issues [4], scheduling optimization problems [5], and many other applications [6]. Recently, the MOPs again rose in many real world applications and many algorithms are proposed to solve such these problems as will be discussed in next sections. In other research work, researchers proposed to solve big data problems using multi-objective evolutionary algorithms [25] using different mechanisms such as fuzzy systems [23][24].

Formally, a general MOP can be defined as minimizing or maximizing a set of two or more objective functions where at least two of them must be in conflict with each other.

Equation 1 shows the definition of MOP.

\[
\min/\max F(x) = (f_1(x), f_2(x) \ldots, f_k(x)) \ldots (1)
\]

Subject to
\[
g_i(x) < 0, i=1,2,3\ldots M
\]
\[
k_j(x) = 0, j=1,2,3\ldots N
\]

The MOP consists of k objectives where k has to be greater than two and M is the number of equality constrains of the problem where N is the number of inequality constrains of the problem on the objective functions. The objective functions of the MOP can be linear or nonlinear, continuous or discrete and stationary or dynamic. In linear problems, the POF is linear due to the existence of linear functions in the objective functions of the MOP. In more complex scenario the POF of a problem may be discrete which increase the difficulty to solve such these problems. Recently the dynamic problems also become very important since there is many real world problems that may be dynamic in nature and continuously changing during the run of the evolutionary algorithm. In this study work we will focus only on the static problem where the proposed algorithms can be applicable on dynamic optimization problems after taking into account the dynamic environment conditions.

In order to solve MOP using genetic algorithms, MOEAs evolve a set of candidate solutions in order to find the optimal solutions (more than one solution) instead of finding a single optimal solution as in the single objective evolutionary algorithms. The collected best solution should be the best solutions and cover the entire true POF of the solved problem. After that the resulted set of best solutions is given to the decision maker who is responsible to select one of these solutions according to his
requirements and plans. In literature POF is used to indicate the set of best solutions in the objective space where POS is used for best solutions in decision space [6]. In MOP solving approaches there is an additional difficulty when selecting best solutions in the population since there is more than one objective function (we cannot select only the highest value in maximization problem and the lowest value in minimization problem as in single objective problems).

Different approaches were developed to estimate the best solutions from the population set where the non-dominating sorting strategy is one of the best and popular sorting strategies that has been proposed. The non-dominating sorting algorithm extracts the best solutions from population by selecting the non-dominated solutions. The non-dominated solution is defined as the solution that is not dominated by any other solutions in the population. Using this approach in each dominance level, many solutions may be exist if they dominated by equal number of other candidate solutions. The first level (which is the POF) is considered to be the best solution set since this means that no other solution could dominate the solutions of that set.

The MOEA has to take into account two important tasks during the evolution process which are the convergence and diversity. The convergence means that the generated new solutions have to be close to the true Pareto-optimal front. Where the diversity means that the generated solutions should be distributed over all the POF and not gathered in clusters. In this paper, the MOEA is enhanced by merging the power of search algorithms which increase the convergence speed of the algorithm.

This paper is organized as follow; in the second section a short summary of the MOEAs is previewed. Our proposed algorithms and their steps are explained in section three. In section four, the experimental study and results discussion are given. Finally the paper will be concluded in section five.

2. Related work

The researchers give MOPs a high significant since they can be found in many applications. Therefore, in literature there are hundreds of papers published to address MOPs. In this section we will summarize the most important evolutionary algorithms used to solve MOPs.

A. The Vector Evaluated Genetic Algorithm (VEGA)

VEGA [7, 8] is one of the first algorithms that proposed to solve MOPs. The VEGA works by generating a separated sub population for each objective function. VEGA tries to guide the solutions toward local optima for each objective function separately. Each sub population of VEGA uses only the fitness function of their own objective for fitness assignment. After that a selection mechanism is used to select the best solutions and to generate the new mating pool. All sub populations are gathered in one population which is the next generation population.

B. The non-dominated sorting genetic algorithm (NSGA)

NSGA is a multi-objective algorithm developed and proposed in [9]. NSGA algorithm uses the concept of non-dominating sorting to sort the solutions and it has a good performance but it suffers from some problems such as the non-existence of elitism process and the high complexity of the algorithm. Because of these problems, Deb proposed his modified version of NSGA and names it as NSGA-II [10]. The Modified algorithm becomes one of the most popular algorithms in multi-objective domain. NSGA-II solved the two issues of NSGA algorithm and improved the implementation of NSGA to become less computational expensive. Because of the importance of this algorithm our proposed algorithms are developed based on it.

The NSGA-II is described briefly in the following steps:-

Step 1: Previous iteration parent and children populations are merged to obtain one new population with size 2N.

Step 2: The obtained population is sorted using non-dominating sorting algorithm to give ranks for each solution where rank one indicates the best solutions.

Step 3: Transfer best solutions rank by rank starting with rank one to next generation population until we reach size N.

Step 4: Sort The last rank in the population (which if we add their solutions the size of the new population will be more than N) using crowding distance.

Step 5: Apply mutation and crossover operators and generate the new mating

Because of the high performance of NSGA-II, it is used to solve many real-world problems and to develop many other versions of this algorithm to solve stationary [11] and dynamic [12] optimization problems.

C. Strength Pareto Evolutionary Algorithm (SPEA)

SPEA is an efficient MOEA which introduced by Eckart Zitzler in [13]. This MOEA can be considered as a hybrid algorithm which integrates different MOEAs. The SPEA algorithm uses an external archive to gather the non-dominated solutions that have been found previously. At each generation, the non-dominated solutions are copied to the archive. For each solution in the archive individuals, a strength value is computed as described in [14]. In SPEA, the fitness value of each individual from population is computed using the value of all archive non-dominated solutions that dominate it. This new fitness
assignment method takes into account the closeness of fitness values to the true POF and solutions distribution over the fitness space. This approach approves its effectiveness but sometimes it depends on the size of the archive of non-dominated solutions. In [15] the authors proposed an enhanced version of SPEA which is SPEA2. The new revised algorithm solves many issues of the first version and consequently the performance is enhanced.

D. Strength Pareto Evolutionary Algorithm (SPEA)

This algorithm is developed by modifying the normal genetic algorithm (GA) to deal with multiple objectives scenarios. This is done by adding the concept of Pareto domination solutions as in the previous algorithms to the selection operator, and after that a niching process is applied to spread the solutions of the population over the true Pareto optimal to make tradeoff between the objective functions [26]. The authors proved and demonstrated the ability of this algorithm to find and maintain a diverse "Pareto optimal solutions" using many real-world hydro systems problems and test problems.

In literature, there are still many MOEAs proposed such as the PAES [16], Niched-Pareto Genetic Algorithm (NPGA) [17] and Multiobjective Messy Genetic Algorithm (MOMGA) [18]. Although there is a big number of proposed algorithms to solve MOP, many comparative experiments in MOEA research domain show that SPEA2 and NSGA-II are the most reliable and efficient algorithms and so they are widely used in comparing newly designed MOEAs.

3. Proposed Algorithms

As explained in previous section, NSGA-II algorithm is one of the best algorithms used to solve MOPs and because of this we selected this algorithm as a base of our new proposed algorithms. NSGA-II algorithm as many other MOEAs try to converges to true Pareto optimal front with keeping the solutions distributed as can as possible. Keeping the solutions distributed may slow the convergence process for NSGA-II algorithm. Our new proposed algorithms are developed mainly to enhance the NSGA-II algorithm convergence and keeping the solutions distributed during the evolution iterations. In order to increase the convergence speed of the NSGA-II algorithm an additional efficient search strategy is used to achieve long jumps toward the true Pareto optimal set.

E. The Proposed Search Algorithm

The proposed search strategy is described in Algorithm 1. Firstly, the value that will be added to each variable is computed. This value must be large in the early generations and then starts to decrease gradually while evolving. Therefore, it is multiplied by adapting factor which is 0.9 in our algorithm. Secondly this value is added or decreased from each variable in the considered solution. Finally the current solution, the solution results from adding the specified value and the solution results from decreasing the specified value are compared to determine which solution is better. Determining the best solution process is based on simple mechanism which is the sum of fitness function values. If the problem is minimization the solution with the minimum fitness value will be the best one.

F. Combing Search Algorithm with NSGA-II

To get the best performance from the search algorithm we need to merge the search algorithm in a convenience way to not negatively affecting the NSGA-II performance. Algorithm 2 shows the steps of the proposed algorithm after adding the efficient search mechanism. As shown in the algorithm our proposed search mechanism is applied in
the first step of each generation on the parent’s individuals, after that best solutions are selected using non-domination strategy and crowding distance. In the last step, the crossover operator followed by mutation operator is applied to generate the new child population.

In this paper we designed four strategies to apply the efficient search mechanism in order to test different scheme and select the best one. The fore proposed scheme are different based on when and where to apply the search mechanism.

1) **NSGA-II-M1**: In the first version of proposed algorithm, the search algorithm is applied in each generation on all variables and all individuals in the population.

2) **NSGA-II-M2**: The second proposed memetic algorithm uses the search algorithm only on half of generations. So the search algorithm is applied on one generation and the next generation is performed without search mechanisms.

3) **NSGA-II-M3**: In this version of proposed algorithm, the search algorithm is applied in each generation on all variables but using only the half individuals of the population. This strategy can gives the NSGA-II algorithm a chance to run 50% of population using its own mechanism which will avoid the full dependence on search algorithm.

4) **NSGA-II-AD**: The last proposed memetic algorithm works adaptively and dynamic. In order to avoid degrading the best solutions on the population this algorithm applies the search mechanism only on the dominated solutions. Therefore the non-dominated solutions will remains without changing and solutions that have rank greater than 1 will be enhanced by the proposed search strategy. Using this mechanism the number of changed solutions is determined in each generation adaptively depending on the number of dominated solutions.

4. Experiments and discussions

In this section we will test the performance of proposed algorithms by performing a comprehensive comparison. The proposed algorithms will be compared with NSGA-II, using three test benchmarks.

G. Test Problems and Performance Metric

To evaluate the performance of the proposed algorithms, three multiobjective test problems were used. The three test problems are selected from ZDT test suite, (ZDT1 to ZDT3) [19]. The ZDT test suite is one of the most popular MOP test problems. These problems designed based on the principles of Deb in [20], the MOP in ZDT suite are constructed by using three functions, including a distribution function $f_1$, a distance function $g$ and a shape function $h$, function $f_1$ is designated to test the ability of diversity maintain, function $g$ is for testing the ability of convergence and finally function $h$ for defining the shape of the PF [21]. The following equations define the three ZDT test problems:-

- **ZDT1**:
  
  \[
  f_1(x_1) = x_1 \\
  g(x_2,\ldots,x_m) = 1 + 9 \cdot \frac{\sum_{i=2}^{m} x_i}{m-1} \\
  h(f_1,g) = 1 - \sqrt{\frac{f_1}{g}}
  \]

  Where $x \in [0,1]$ and POF can be generated when $g(x) = 1$.

- **ZDT2**:
  
  \[
  f_1(x_1) = x_1 \\
  g(x_2,\ldots,x_m) = 1 + 9 \cdot \sum_{i=2}^{m} x_i/(m-1) \\
  h(f_1,g) = 1 - (\frac{f_1}{g})^2
  \]

  Where $x \in [0,1]$ and POF can be generated when $g(x) = 1$.

- **ZDT3**:

Algorithm 2. The steps of proposed memetic algorithm.

```plaintext
Begin
  Initialize Population (P)
  for $i = 1$ to genNum do
    //Apply the proposed search mechanism
    fastSearchAlgorithm(Parent)
    // Merge Parent and Child individuals in One 2*N Population
    Pbig = MergeParentChild()
    // Assign Rank (level) for each solution based on Pareto dominance
    NonDominatingSort(Pbig)
    // Add best solutions to next generation starting from the first Rank
    Pnew = SelectBest(Pbig)
    // Use crowding distance to select the remaining solutions from last rank
    Pnew = CrowdingDist(Pbig)
    Pchild = TournamentSelection() //Binary Tournament Selection
    Recombination(Pchild)
    Mutation(Pchild)
  end // Generations end
end // Program end
```
\[ f_i(x) = x_i \]
\[ g(x_2, ..., x_m) = 1 + 9 \cdot \sum_{i=2}^{m} \frac{x_i}{m-1} \]
\[ h_i(x_i) = 1 - \sqrt{x_i/g} - (f_i/g) \sin(10\pi f_i) \]

Where \( x \in [0,1] \) and POF can be generated when \( g(x) = 1\).

In our experiments, the Inverted Generational Distance (IGD) is mainly used [22] to test the performance of the proposed algorithms. The IGD metric can measure the diversity and convergence of the MOEAs. The IGD metric is computed for the final solutions of the population. The following equations define the IGD metric and show how it computed:-

\[ IGD_t = \frac{\sum_{i=1}^{|PF_t|} d_i}{|PF_t|} \]
\[ d_i = \min_{k=1}^{|PF_t|} \left| \sum_{j=1}^{M} \left( f_i^{(k)} - f_i^{(j)} \right)^2 \right| \]

Where the term \( PF_t \) indicates the optimal Pareto Front and \( M \) is the number of objective functions used in the problem. \( d_i \) is computed by using the Euclidean distance.

### H. Results and Discussion

To test the performance of the proposed algorithms we implemented the proposed algorithms based on NSGA-II. The population size is selected to be 10, 20, 30 and 40. The crossover operator is fixed to be 0.7. The number of variables is selected to be 10 and two objectives are used in each test problem.

Table I, Table II and Table III show the performance comparison between the four proposed algorithms and the results of the three test problems show that the performance of NSGA-II is enhanced significantly when our efficient search algorithm applied on the half of the population and when it’s applied on the bad solutions (the adaptive version). In addition, it is clear from results on tables that the NSGA-II-M1 and NSGA-II-M2 algorithms are still better than the NSGA-II in many cases. This result proves that adding a simple search algorithm on the evolutionary algorithms is very important and can be very helpful to guide the search mechanism of the evolutionary algorithms. In addition the usage of search algorithm in every generation may have negative effects on the diversity of solutions so it is preferable to restrict the use of the local search only in the needed situations as in the NSGA-II-M3 and NSGA-II-AD proposed algorithms.

To more investigate the performance of proposed algorithms Figure 2, Figure 3 and Figure 4 show the distribution of the solutions in last population (using 100 NSGA-II for ZDT1, ZDT2 and ZDT3 problems respectively. For each experiment, the best result is marked by bold font. The values in tables represent the averages of 10 independent runs for each instance of the experiments. As shown in the results NSGA-II-M3 and NSGA-II-AD are the best two algorithms since they outperform the other algorithms in most of cases. The results also show that the NSGA-II algorithm without any searching algorithm is the worst algorithm. The NSGA-II-M3 algorithm which apply the search algorithm on half of population individuals outperforms the other algorithms in 9 cases out of 12, where NSGA-II-AD is the best algorithm in 8 cases.

| Table I: The performance comparison between the proposed algorithms and NSGA-II algorithm for ZDT1 test problem |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| GEN    | NSGA-II | NSGA-II-M1 | NSGA-II-M2 | NSGA-II-M3 | NSGA-II-AD |
| 10     | 0.1709   | 0.0138      | 0.0422      | 0.0223       | 0.0141       |
| 20     | 0.0329   | 0.0190      | 0.0151      | 0.0102       | 0.0104       |
| 30     | 0.0170   | 0.0147      | 0.0149      | 0.0145       | 0.0147       |
| 40     | 0.0193   | 0.0193      | 0.0199      | 0.0191       | 0.0192       |

| Table II: The performance comparison between the proposed algorithms and NSGA-II algorithm for ZDT2 test problem |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| GEN    | NSGA-II | NSGA-II-M1 | NSGA-II-M2 | NSGA-II-M3 | NSGA-II-AD |
| 10     | 0.5996   | 0.3251      | 0.5437      | 0.3880       | 0.4442       |
| 20     | 0.4082   | 0.2547      | 0.1984      | 0.1034       | 0.1775       |
| 30     | 0.0284   | 0.1538      | 0.0215      | 0.0196       | 0.0189       |
| 40     | 0.1577   | 0.1394      | 0.0262      | 0.0269       | 0.0256       |

| Table III: The performance comparison between the proposed algorithms and NSGA-II algorithm for ZDT3 test problem |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| GEN    | NSGA-II | NSGA-II-M1 | NSGA-II-M2 | NSGA-II-M3 | NSGA-II-AD |
| 10     | 0.1846   | 0.0144      | 0.0235      | 0.0184       | 0.0147       |
| 20     | 0.0272   | 0.0095      | 0.0088      | 0.0086       | 0.0068       |
| 30     | 0.0137   | 0.0117      | 0.0113      | 0.0114       | 0.0112       |
| 40     | 0.0141   | 0.0143      | 0.0144      | 0.0143       | 0.0144       |

solutions in the population for each algorithm) after running 10, 20 and 30 generations and for three test problems ZDT1, ZDT2 and ZDT3 respectively. The figures compare the results of The NSGA-II algorithm and NSGA-II-AD which gains the best performance among the other proposed algorithms. Firstly, Figure 1 shows the true POF of the three used test problems, as shown in the figure the first two problems are continuous and one problem is concave where the other one is convex. On the other hand, ZDT3 is a non-continuous POF problem as shown in Figure 1. From Figure 2, Figure 3 and Figure 4 we can see clearly that NSGA-II-AD is very fast in convergence process since the solutions started to be very close to the true Pareto optimal front only after 10 generations. On the other hand the NSGA-II algorithm performance after 10 generations is still not good and the solutions are far from the true POF. Figure 2, Figure 3 and Figure 4 also show that our
proposed algorithm NSGA-II-AD can fasten the process of convergence significantly without degrading the diversity of solutions.

Fig. 1 The True POF distribution of ZDT1, ZDT2 and ZDT3 problems.

Fig. 2 The POF distribution of population after 10, 20 and 30 generations respectively of ZDT1 problem. The first column is for NSGA-II-AD and the second column is for NSGA-II algorithm

5. Conclusion

In this paper a set of new efficient and improved hybrid Memetic evolutionary algorithms are proposed to solve multi-objective optimization problems. The multi-objective optimization problems become very common and have been rapidly using in different domains.

The proposed algorithms in this paper enhance the performance of NSGA-II algorithm by using different search schemes. In this paper we designed four strategies to apply the efficient search mechanism in order to test different scheme and select the best one. The four proposed scheme are different based on when and where to apply the search mechanism. In the first version of proposed algorithm, the search algorithm is applied in each generation on all variables and all individuals in the population. In the second proposed algorithm we used the search algorithm only on half of generations. So the search algorithm is applied on one generation and the next generation is performed without search mechanism. Version three of the proposed algorithm applies the search algorithm in each generation on all variables but using only the half individuals of the population. The last proposed algorithm works adaptively and dynamic. In order to avoid degrading the best solutions on the population this algorithm applies the search mechanism only on the dominated solutions. Therefore the non-dominated solutions will remains without changing

The results show that merging simple and efficient search techniques to NSGA-II significantly enhance the convergence ability to the set of optimal solutions. To
assess the performance of proposed algorithms, three multi-objective test problems are used from ZDT set. Our empirical results show that the proposed algorithms significantly enhance the performance of NSGA-II algorithm and able to efficiently solve MOPs in less number of generations. The results also show that the proposed algorithm performs better than NSGA-II algorithm in both diversity and convergence.

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