

# Robust and Fast Fault Tolerant Control for Systems with Multi Actuator and Parameters Faults

Manel Allous<sup>†</sup> and Nadia Zanzouri<sup>††</sup>

<sup>†</sup>University of Tunis El Manar, National Engineering School of Tunis  
Box 37, Le Belvedere 1002 Tunis, Tunisia

<sup>††</sup>University of Tunis, Preparatory Engineering Institute of Tunis 2,  
Street Jawaher Lel Nahrou-Monfleury, 1089 Tunis, Tunisia

## Summary

This paper deals with a major problem related to the modern control systems, which is the fault-tolerant control. Firstly, a novel fault-tolerant control method for systems with multi actuator and parameter faults is proposed. The fault-tolerant controller is synthesized by using the Lyapunov function and a linear matrix inequality approach. The novelty of the present study resides that the fault estimation is not necessary and especially for systems with complex multi-fault. The only fault diagnosis module based on Luenberger observer is used for residual generation which is injected into the control loop. Next, based on the fault diagnosis results, a new fast fault-tolerant controller is designed for aims to construct a reconfiguration block and to achieve some prescribed specifications. Finally, the proposed fault-tolerant control technique is applied to the vertical take-off and landing (VTOL) aircraft system which demonstrates the effectiveness of the developed techniques.

## Key words:

*Diagnosis, Fault tolerant control, Actuator and parameter faults, Lyapunov stability tools*

## 1. Introduction

Modern control systems can malfunction due to possible faults and failures in actuators, sensors or other components. Therefore, the fault-tolerant controller design has been an active research area for several years, which aims to design a controller guaranteeing a satisfactory performance for a given system under both normal and fault conditions [1]. The fault tolerant control (FTC) and its complementary part, fault detection and diagnosis (FDD) are particularly important for safety-critical systems, such as aircraft, spacecraft, chemical plants and power networks [2-6]. However, reconfiguration methods shown above must require fast and accurate fault information. For this reason, fault detection and diagnosis (FDD) technique is generally introduced. In the literature, many approaches have been proposed for FDD. Recently, the observer based fault detection and isolation (FDI) approach has gained a lot of research attention [7-9]. Many authors have approached the FDI problem using Luenberger observers [10]. On the other hand, the

extended Kalman filters (EKFs) is used to detect the predefined actuator fault and estimate the unknown fault parameters [11].

In general, fault-tolerant control systems can be classified as passive and active approaches. In the passive fault-tolerant control (PFTC) strategy, the controller is designed to be sufficiently robust to pre-specified faults so that no modification in the control process is needed [12], [13]. However, active FTC can reconfigure the controller structure according to the information provided by a fault diagnosis module [14], [15]. In AFTC, different fault tolerant design as the Pseudo Inverse Model (PIM), the Linear Quadratic (LQ) approach and the Linear Matrix Inequality (LMI) are introduced [16].

Many techniques of compensation of multi-faults are developed and the technique of estimation and fault isolability are considered in several problems of FTC. Among existing approaches, the authors in [17], [18] consider the case when some actuators are always functional, while every combination of the remaining actuators are allowed to fail. They design reliable controllers to guarantee satisfactory linear-quadratic and  $H_\infty$  performances under the failure of any subset of susceptible actuators. Another typical technique for fault compensation is based on the adaptive method [19], [20]. An adaptive FTC scheme of fault actuator with application to flight control is established [21]. Also, system identification schemes appropriated to adaptive and reconfigurable control are proposed [22]. On the other hand, the fault estimation is not necessary and especially for systems with complex multi-fault. Our new technique is not considered in the other existing works and the novelty approach can avoid the delay and improve the rapidity of fault compensation.

In this paper, a novel fault tolerant control design approach is proposed for systems in the presence of multi-actuator and parameter faults. A reference signal  $r$  is used to generate a nominal control input ( $u_n$ ) and a new control design is developed to generate the additive control signal ( $u_{add}$ ). In this proposed approach, the fault estimation and fault isolability are not considered in the system

recoverability, just the residual signal provided by the fault diagnosis unit is then used to generate the additive control signal. The advantage of our proposed approach is to enhance the rapidity of fault compensation and also can guarantee both satisfactory dynamical and steady state performances.

The rest of this paper is organized as follows. in Section 2, we briefly introduce the problem formulation. Section 3 contains the main results, including the new fault tolerant control design. A vertical take-off and landing (VTOL) aircraft system showed the effectiveness of the proposed methods is given in Section 4 and some concluding remarks are provided in Section 5.

## 2. Problem formulations

Consider the nominal control system given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the nominal input vector,  $y(t) \in \mathbb{R}^p$  is the output vector.  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times m}$  and  $C \in \mathbb{R}^{p \times n}$  are constant matrices with appropriate dimensions.

In presence of the actuator fault as an event that changes the nominal input vector  $u(t)$  to the faulty input vector  $u_f(t)$ , the faulty system with actuator faults can be represented as:

$$\begin{cases} \dot{x}_f(t) = Ax_f(t) + Bu_f(t) + Dw(t) \\ y_f(t) = Cx_f(t) \end{cases} \quad (2)$$

and

$$u_f(t) = u(t) + f(t) \quad (3)$$

where  $x_f(t) \in \mathbb{R}^n$  is the faulty state vector,  $u_f(t) \in \mathbb{R}^m$  is the control input,  $y_f(t) \in \mathbb{R}^p$  is the measured output vector,  $f(t) \in \mathbb{R}^m$  is the actuator fault and  $w(t) \in \mathbb{R}^d$  is the disturbance vector.

The controller is chosen as follows:

$$u_{FTC}(t) = u(t) + u_{add}(t) \quad (4)$$

where

$$u(t) = K_x x(t) + K_r r(t) + K_f (r(t) - x_f(t)) \quad (5)$$

and

$$u_{add}(t) = -K \Delta y_f(t) \quad (6)$$

where  $r(t) \in \mathbb{R}^p$  is the reference input vector,  $Kx \in \mathbb{R}^{m \times n}$  is the state feedback control matrix to keep system stable,  $K_r \in \mathbb{R}^{m \times p}$  is feedforward control matrix to keep system tracking the reference offset-free and  $u_{add}$  denotes the additive control part to be designed in order to remove the faults effect. The gain  $K$  of additive control  $u_{add}$  and the term  $\Delta y_f(t)$  will be designed in section 3.

The purpose of this paper is to deal with the following two interrelated problems:

- Design a new FTC method to achieve the compensation of multi actuators faults and parameters faults.

- The proposed FTC should be fast and robust against disturbances and fulfills some prescribed specification.

In this paper, it is assumed that all the state variables are measurable. Moreover, multi-actuators and parameter faults are considered.

**Notation:** In symmetric block matrices, we use “\*” to represent a term that is induced by symmetry matrix and if their dimensions are not explicitly stated, they are assumed to be compatible for algebraic operations.  $S\{z\}$  is the mathematical expectation of a stochastic variable  $z$ .

## 3. Fault tolerant control design

### 3.1 Diagnosis part

In this section, we are interested in the analysis and design problem of an active fault-tolerant controller, which includes a fault diagnosis unit followed by a controller reconfiguration strategy. The principal of our proposed approach is presented in Fig. 1. Once a fault is detected and by the diagnosis unit, the controller is reconfigured so as to guarantee some prescribed specifications.

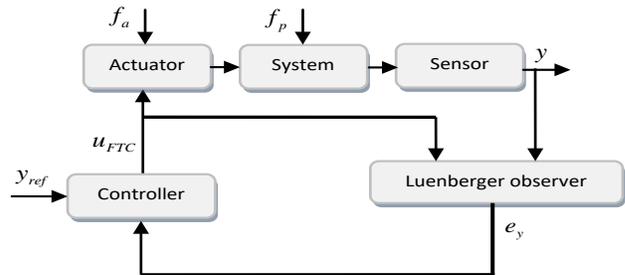


Fig. 1 Fault tolerant control design of system with actuator and parameter fault ( $f_a(t)$  and  $f_p(t)$ )

Fig. 1 Fault tolerant control design of system with actuator and parameter fault (fa (t) and fp(t))

For the system (2), a Luenberger observer is designed as the following form:

$$\begin{cases} \dot{\hat{x}}(t) = Ax(t) + Bu(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = Cx(t) \end{cases} \quad (7)$$

where  $\hat{x}(t)$  is the estimate of the system state,  $\hat{y}(t)$  is the estimated output and L is the gain matrix defined by:

$$L = PC^T R^{-1} \quad (8)$$

where the matrix P is obtained by solving the following Riccati equation [23]:

$$A^T P + PA + PCR^{-1}C^T P + Q < 0 \quad (9)$$

where

$$P > 0, Q > 0 \text{ and } R > 0$$

Using the Schur Complement, we can represent this inequality by LMIs as follows:

$$\begin{bmatrix} A^T P + PA + Q & PC^T \\ CP & -R \end{bmatrix} < 0 \quad (10)$$

### 3.2 Control part

In nominal case: Considered the closed-loop dynamics system as follows:

$$\begin{cases} \dot{x} = (A + BK_x)x \\ y = Cx \end{cases} \quad (11)$$

The feedback matrix  $K_x$  is determined by the simple Lyapunov function method to guarantee the closed-loop system  $(A + BK_x)$  stable.

For given a positive scalar  $\alpha$ , if there exist matrices Y and positive definite symmetric matrices  $X_n \in \mathbb{R}^{n \times n}$ , subject to

$$AX_n + X_n A^T + BY + Y^T B^T + \alpha^2 X_n < 0 \quad (12)$$

and the gain matrix  $K_x = YX_n^{-1}$

Taking a candidate Lyapunov function as:

$$V = x^T P_n x \quad (13)$$

From (13), the derivative of V is given as follows:

$$\dot{V} = x^T (P_n (A + BK_x) + (A + BK_x)^T P_n) x \quad (14)$$

If there exists a positive definite matrix  $P_n$  to make the following inequality satisfied

$$P_n (A + BK_x) + (A + BK_x)^T P_n + \alpha^2 P_n < 0 \quad (16)$$

Then pre- and post-multiplying by  $P_n^{-1}$  and setting  $X_n = P_n^{-1}$ ,  $Y = K_x X_n$ , so (12) has been proved.

Then the feedforward matrix  $K_r$  is designed by the following equation:

$$K_r = -[C(A + BK_x)^{-1} B]^+ \quad (17)$$

In faulty case: In this part, a novel FTC algorithm is proposed to improve the compensation of fault. So  $u_{add}$  will be chosen to remove the influence of the fault. We start by the vector  $\Delta y_f(t)$  which defined as follows:

$$\begin{aligned} \Delta y_f &= e_y + \Delta e_y \\ &= C(e_x + \Delta e_x) \end{aligned} \quad (18)$$

where  $\Delta e_x$  is given as follow:

$$\Delta \dot{x} = A\Delta e_x + Le_y \quad (19)$$

where

$$\begin{cases} e_y = y - \hat{y} \\ \Delta e_y = C\Delta e_x \end{cases} \quad (20)$$

So Eq. (6) can be rewritten as follows

$$u_{add} = -K(e_y + \Delta e_y) \quad (21)$$

Our objective is to find the additive control gain K. Considered the closed-loop dynamics system as follows:

$$\begin{cases} \Delta \dot{x}_f = (A + BKC)\Delta x_f + Dw(t) \\ \Delta y_f = C\Delta x_f \end{cases} \quad (22)$$

where

$$u_{add} = -KC\Delta x_f \quad (23)$$

Consider the following Lyapunov function

$$V = \Delta x_f^T P \Delta x_f \quad (24)$$

Then, the derivative of  $V$  is given as follow:

$$\begin{aligned} \dot{V} &= \Delta x_f^T (P(A+BKC) + (A+BK C)^T P) \Delta x_f \\ &\quad + \Delta x_f^T PDw + w^T D^T P \Delta x_f \\ &< \gamma^2 w^T w \end{aligned} \quad (25)$$

So Eq. (25) can be rewritten as follows:

$$\dot{V} - \gamma^2 w^T w < 0 \quad (26)$$

Substituting (25) into (26), yields

$$\begin{bmatrix} P(A+BKC) + (A+BK C)^T P & PD \\ * & -\gamma^2 I \end{bmatrix} < 0 \quad (27)$$

Then pre- and post-multiplying by  $\text{diag}(P^{-1}, I)$  and setting  $X = P^{-1}$ ,  $\bar{K} = KCX$ , then (27) can be re-written as

$$\begin{bmatrix} \Psi & D \\ * & -\gamma^2 I \end{bmatrix} < 0 \quad (28)$$

$$\Psi = AX + XA^T + B\bar{K} + \bar{K}^T B^T \quad (29)$$

The amplitude of the additive control matrix  $K$  is likely to be too higher and this is not suitable for practical application. In order to prevent the problem, the following constraint is added:

$$\delta^2 \|\Psi\| < I \quad (30)$$

where  $\|\Psi\| = \sqrt{\Psi^T \Psi}$  denotes its L2 norm, and  $0 < \delta < 1$  is a positive scalar.

Based on the definition of the L2 norm, Eq. (30) is equivalent to

$$\delta^2 \Psi^T \Psi - I < 0 \quad (31)$$

Using Schur complement lemma, Eq. (31) is equivalent to

$$\begin{bmatrix} -\delta^2 I & \Psi \\ * & -I \end{bmatrix} < 0 \quad (32)$$

Then, there exist a symmetric and positive definite matrix  $\bar{X} = \text{diag}(X, X)$  such that

$$\begin{bmatrix} -\delta^2 I & \Psi \\ * & -I \end{bmatrix} - \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix} < 0 \quad (33)$$

then Eq. (33) can be rewritten as

$$\begin{bmatrix} -(\delta^2 I + X) & \Psi \\ * & -(I + X) \end{bmatrix} < 0 \quad (34)$$

Based in Eq. (28) and (34), we get

$$\begin{bmatrix} -(\delta^2 I + X) & \Psi & D \\ * & -(I + X) & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (35)$$

Moreover, if (35) holds, The additive control matrix  $K$  can be obtained from the following equation:

$$K = \bar{K} X^{-1} C^+ \quad (36)$$

where  $C^+$  is the pseudo-inverse of matrix  $C$  calculated as following :

$$C^+ = C^T (CC^T)^{-1} \quad (37)$$

### 3.3 Robust and fast tolerant control design

Now, we need to achieve a fast and robust fault tolerant control. Based in Eq. (19) and (21), the new additive controller is determined as follows:

$$\begin{cases} u_{add} = -[K(e_y + \Delta e_y) + K(e_y + \Delta e_y)] \\ \Delta \mathcal{E}_x = A\Delta e_x + Le_y - BK(e_y + \Delta e_y) \end{cases}$$

$$\begin{cases} u_{add} = -2K(e_y + \Delta e_y) \\ \Delta \mathcal{E}_x = A\Delta e_x + Le_y - BK(e_y + \Delta e_y) \end{cases}$$

$$\begin{cases} u_{add} = -3K(e_y + \Delta e_y) \\ \Delta \mathcal{E}_x = A\Delta e_x + Le_y - 2BK(e_y + \Delta e_y) \end{cases}$$

⋮

$$\begin{cases} u_{add} = -nK(e_y + \Delta e_y) \\ \Delta \mathcal{E}_x = A\Delta e_x + Le_y - (n-1)BK(e_y + \Delta e_y) \end{cases} \quad (38)$$

So, there exist a scalar  $n$  such as the rapidity of fault compensation is guaranteed. Then (38) can be rewritten as follows:

$$\begin{cases} u_{add} = -\Gamma K(e_y + \Delta e_y) \\ \Delta \mathcal{E}_x = A\Delta e_x + Le_y - (\Gamma - I)BK(e_y + \Delta e_y) \end{cases} \quad (39)$$

where the symmetric positive definite matrix  $\Gamma \in \mathbb{R}^{m \times m}$  is the learning rate and  $\Gamma = \text{diag}(n_1, \dots, n_m)$ , where  $m$  is the number of input control.

#### 4. Simulation results

To demonstrate the efficiency of the proposed approach, we consider the dynamic model of a VTOL aircraft in the vertical plane. The state space model of the VTOL [24] is given below:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + Bu(t) + Ef(t) + Dw(t) \\ y(t) = Cx(t) \end{cases} \quad (40)$$

where  $x(t) = [Vh, Vv, q, \theta]$ ,  $u(t) = [\delta c, \delta l]$  and  $y(t)$  are respectively the state, the input vector and the output vector.  $f(t)$  and  $\Delta A$  are respectively the actuator fault and the parameters.  $Vh$  is the horizontal velocity,  $Vv$  is the vertical velocity,  $q$  is the pitch rate, and  $\theta$  is the pitch angle;  $\delta c$  is the collective pitch control and the longitudinal cyclic pitch control is  $\delta l$ . In addition,  $w(t)$  is the process disturbance shown in Fig. 2.

The system matrices  $A$ ,  $B$ ,  $C$ ,  $E$  and  $D$  are given as follows:

$$A = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ 5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.4422 & 0.176 \\ 3.545 & -7.592 \\ 5.520 & 4.490 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.15 \\ 0.30 \\ 0.20 \\ 0 \end{bmatrix}$$

The system is subject to the reference input  $u = [1 \ 1]^T$  and initial value  $x(0) = [0 \ 0 \ 0 \ 0]^T$ . This paper only considers the additive actuator fault, without loss of generality, it is assumed that  $E=B$ .

Figures given below present the obtained results of the proposed FTC approach for different actuator fault scenarios  $f = [f_1(t) \ f_2(t)]^T$  and parameters fault  $\Delta A = A \times f_3$ . The actuator and parameter faults respectively are as follows:

$$f_1(t) = \begin{cases} 0 & t < 45s \\ 1 + 0.05 \sin(2t) & t > 45s \end{cases} \quad (41)$$

$$f_2(t) = \begin{cases} 0 & t < 8s, t > 20s \\ 0.5 + 0.05 \sin(2t) & 8s < t < 20s \end{cases} \quad (42)$$

$$f_3(t) = \begin{cases} 0 & t < 30s, t > 40s \\ 0.3 & 30s < t < 40s \end{cases} \quad (43)$$

Solving the linear matrices inequalities in (10), we obtain the gain  $L$  of Luenberger observer:

$$L = \begin{bmatrix} 0.6559 & 0.0294 & -0.1451 \\ 0.0416 & 0.0127 & -0.0503 \\ 0.1432 & -0.0135 & 0.0706 \\ -0.1385 & -0.0539 & 0.5207 \end{bmatrix}$$

Next, by choosing  $\alpha = 1$  and solving the inequality in (12) the feedback gain matrix  $K_x$  is obtained as:

$$K_x = \begin{bmatrix} -7.5581 & -11.7455 & 6.2787 & 9.6474 \\ -9.3669 & -5.3049 & -7.0222 & -1.6667 \end{bmatrix}$$

and

$$K_r = \begin{bmatrix} -2.8132 & 10.9627 & -3.6674 \\ 2.7850 & 4.6363 & 5.8600 \end{bmatrix}$$

Selecting  $\delta = 0.1$ ,  $\gamma = 10$  and solving the matrix inequality (35), we can obtain the proportional gain:

$$K = \begin{bmatrix} 4.8690 & 0.3938 & -2.1936 \\ -1.2409 & -0.2564 & 1.3279 \end{bmatrix}$$

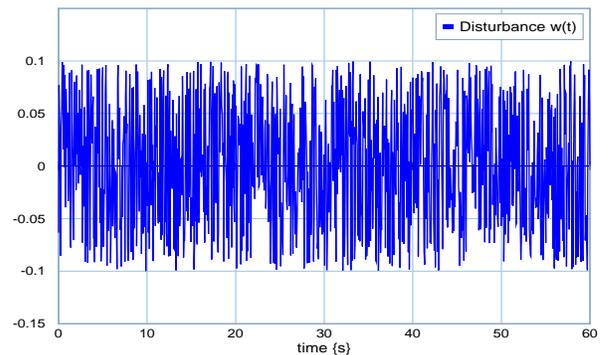


Fig. 2 Disturbance signal

In this paper, the scalar  $n$  is chosen as  $\Gamma = \text{diag}(1, 1)$  and  $\Gamma = \text{diag}(10, 10)$  separately.

The results presented in Fig. 3, show the additive control  $u_{add}(t)$  evolution for  $n=1$  and  $n=10$ .

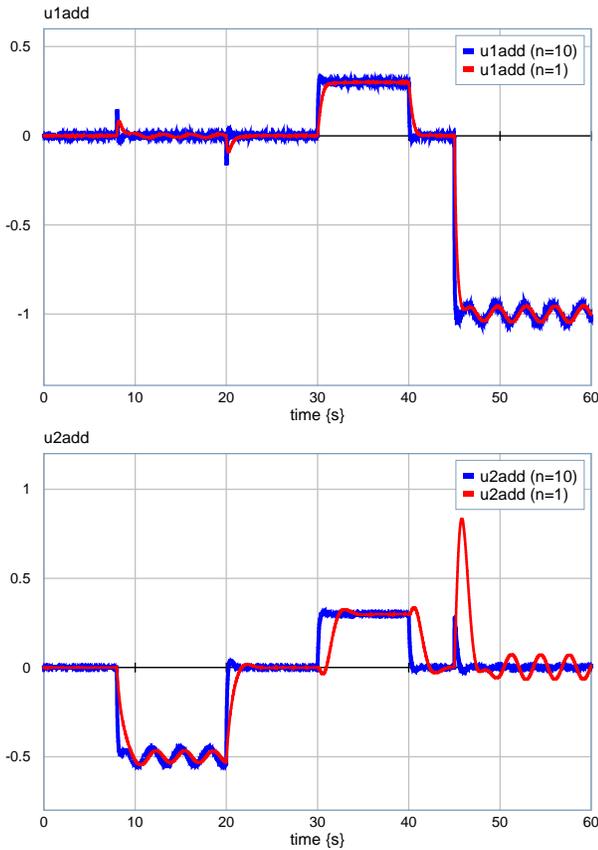


Fig. 3 Additive control signal  $u_{add}(t)$

From Fig. 3, it is clearly to see that the additive control can improve the rapidity of the faults compensation when  $n$  increases from  $n = 1$  to  $n = 10$ .

Figure 4 presents the output responses  $y(t)$  with and without fault compensation in the presence of different faults values.

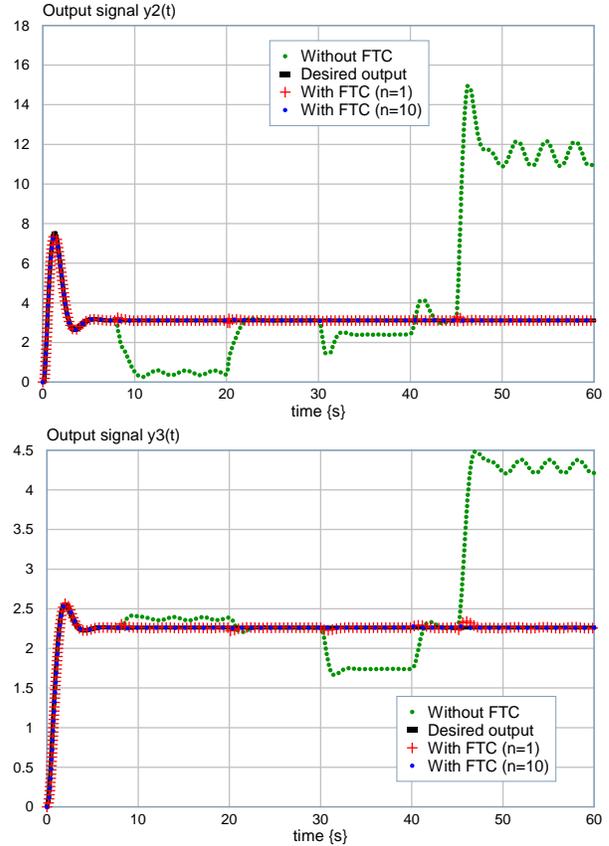
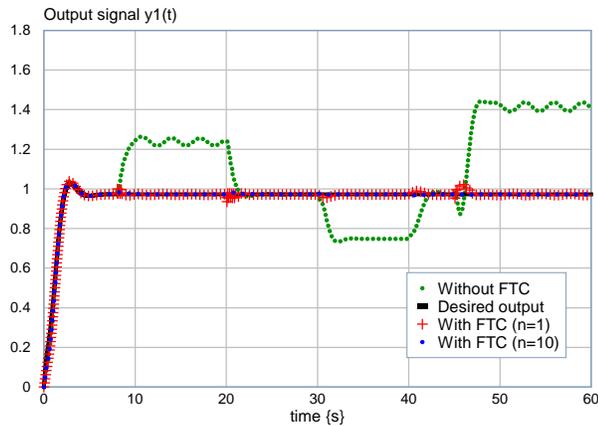
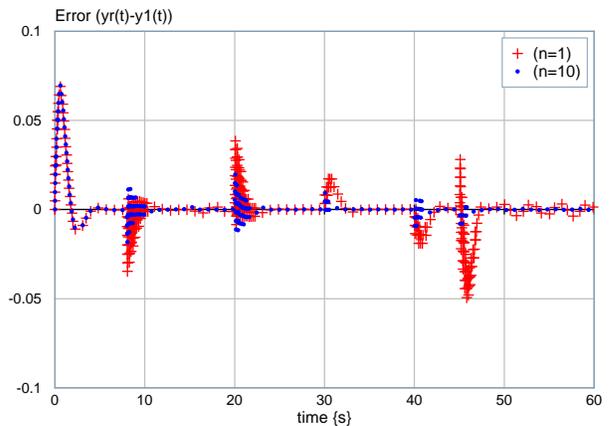


Fig. 4 Output signal with and without compensation respectively of  $y_1(t)$ ,  $y_2(t)$  and  $y_3(t)$ .

The simulation curves in Fig. 4 show that when there are simultaneous faults, we obtain better output performances by using our proposed fault tolerant control.

Figure 5 shows the error  $e(t)$  evolution between the desired and the output signal of system with different multi-fault values.



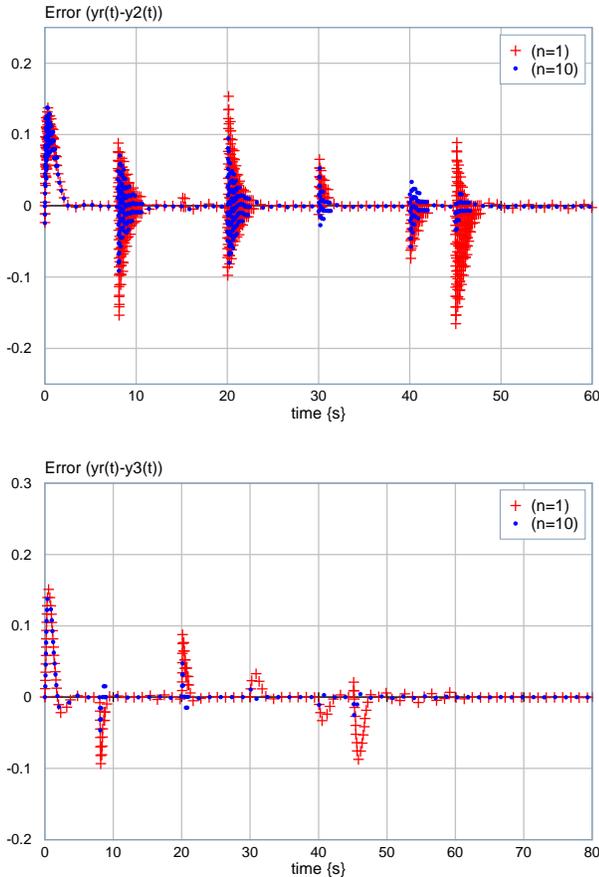


Fig. 5 Error  $e(t)$  between the desired and the output signal respectively of  $y_1$ ,  $y_2$  and  $y_3$ .

In Fig. 5, the error converges quickly to zero with the scalar  $n=10$ .

From the results in Figs. 4 and 5, it is obvious that with the increase scalar  $n=10$ , the convergence rate of the fault compensation is faster. Consequently, the proposed FTC approach can improve the rapidity of the recovery fault. Also, the proposed fault-tolerant control strategy has excellent robustness against disturbances, compared with the known techniques that did not consider the robustness issue or assumed the input disturbances.

## 5. Conclusion

In this paper, a novel fault-tolerant control method for systems with multi actuator and parameter faults is proposed. Firstly, a Luenberger observer is used to generate the residual signal. Then, the residual information has been used to design the new control strategy which guarantees the fault compensation. The robust and fast fault-tolerant control method is well developed. Finally, the proposed approach is applied to a dynamic VTOL

aircraft model and the simulation results show that the proposed FTC fulfills the prescribed specification.

The carried out study confirm that performed analytical and numerical calculations can be used to analyse and simulate systems with complex multi-faults. Additionally, the fast fault tolerant control is necessary to system with delay time.

## References

- [1] M. Blanke, M. Kinnaert, J. Lunze and M. Staroswiecki, "Diagnosis and fault-tolerant control." Berlin, Germany: Springer, 2006.
- [2] S. Qikun, J. Bin, S. Peng and L. Cheng-Chew, "Novel Neural Networks-Based Fault Tolerant Control Scheme With Fault Alarm," IEEE Trans on Cybernetics, vol.44, no.11, pp.2190-2201, 2014.
- [3] L. Ming, C. Xibin and S. Peng, "Fuzzy-Model Based Fault-Tolerant Design for Nonlinear Stochastic Systems Against Simultaneous Sensor and Actuator Faults," IEEE Trans on Fuzzy Systems, vol.21, no.5, pp.789-799, 2013.
- [4] T. Jain, J. Yame and D. Sauter, "A Novel Approach to Real-Time Fault Accommodation in NREL's 5-MW Wind Turbine Systems," IEEE Trans on Sustainable Energy, vol.4, no.4, pp.1082-1090, 2013.
- [5] Y. Kobayashi, M. Ikeda and Y. Fujisaki, "Stability of large space structures preserved under failures of local controllers," IEEE Transactions on Automatic Control, vol.52, no.2, pp.318-322, 2007.
- [6] M. Sami and R. Patton, "Active Fault Tolerant Control for Nonlinear Systems with Simultaneous Actuator and Sensor Faults," Int J of Control Automation and Systems, vol.11, no.6, pp.1149-1161, 2013.
- [7] R. J. Patton, L. Chen and S. Klinkhieo, "An LPV pole-placement approach to friction compensation as an FTC problem," Int J Appl Math Comput Sci, vol.22, no.1, pp.149-160, 2012.
- [8] H. Alwi, C. Edwards and A. Marcos, "Fault reconstruction using a LPV sliding mode observer for a class of LPV systems," J. of the Franklin Institute, vol.349, no.2, pp.510-530, 2012.
- [9] X. Wei and M. Verhaegen, "Sensor and actuator fault diagnosis for wind turbine systems by using robust observer and filter," Wind Energy, vol.14, no.4, pp.491-516, 2011.
- [10] R. Dixon, "Observer-based FDI: Application to an electromechanical positioning system," Control Engineering Practice, vol.12, pp.1113-1125, 2004.
- [11] S.M. Kargar, K. Salahshoor and M.J. Yazdanpanah, "Integration of multiple model based fault detection and nonlinear model predictive fault-tolerant control," IEEE Transactions on Electrical and Electronic Engineering, vol.10, no.5, pp.547-553, 2015.
- [12] H. Niemann and J. Stoustrup, "Passive fault tolerant control of a double inverted pendulum case study," Control engineering practice, vol.13, no.8, pp.1047-1059, 2005.
- [13] J. Jiang and X. Yu, "Fault-tolerant control systems: A comparative study between active and passive approaches," Annual Reviews in control, vol.36, no.1, pp. 60-72, 2012.
- [14] Y. Zhang Y and J. Jiang, "Fault tolerant control system design with explicit consideration of performance

- degradation,” *IEEE Transactions on Aerospace and Electronic Systems*, vol.39, no.3, pp.838–848, 2003.
- [15] D. Du, B. Jiang and P. Shi, “Fault estimation and accommodation for switched systems with time-varying delay,” *International Journal of Control Automation and Systems*, vol.9, no.3, pp.442–451, 2011.
- [16] K. Zhang, B. Jiang and V. Cocquempot, “Adaptive observer-based fast fault estimation,” *International Journal of Control Automation and Systems*, vol.6, no.3, pp.320–326, 2008.
- [17] R.J. Veillette, J. Medanid and W. Perkins, “Design of reliable control systems,” *IEEE Transactions on Automatic Control*, vol.37, no.3, pp.290-304, 1992.
- [18] R. Veillette, “Reliable linear-quadratic state-feedback control,” *Automatica*, vol.31, no.1, pp.137-143, 1995.
- [19] M. Bodson and j. Groszkiewicz, “Multivariable Adaptive Algorithms for Reconfigurable Flight Control. *IEEE Transactions on Control Systems Technology*, vol.5, no.2, pp.217-229, 1997.
- [20] W. Cai, X. Liao and D. Song, “Indirect Robust Adaptive Fault Tolerant Control for Attitude Tracking of Spacecraft,” *Journal of Guidance Control and Dynamics*, vol.31, no.5, pp.1456-1463, 2008.
- [21] D. Ye and G. Yang, “Adaptive Fault-Tolerant Tracking Control against Actuator Faults with Application to Flight Control,” *IEEE Transactions on Control Systems Technology*, vol.14, no.6, pp.1088-1096, 2006.
- [22] P. Chandler, M. Pachter and M. Mears, “System Identification for Adaptive and Reconfigurable Control,” *Journal of Guidance Control and Dynamics*, vol.18, no.3, pp.516-524, 1995.
- [23] J. Willems, “Least squares stationary optimal control and the algebraic Riccati equation,” *IEEE Tran Aut Control*, vol.16, no.6, pp.621-634, 1971.
- [24] T. Park and K. Lee, “Process fault isolation for linear systems with unknown inputs,” *IEE Proc Control Theory Appl*, vol.151, no.6, pp.720-726, 2004.