A Novel High Order Fuzzy Time Series Forecasting Method with Higher Accuracy Rate

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Abstract
Fuzzy time series have been extensively used to make predictions of weather, road accidents, academic enrollments, population, and stock prices. In this paper, we have introduced an improved fuzzy time series forecasting model. This new model is applied in forecasting the University of Alabama student enrollments. Later a comparison has been done with some of the existing fuzzy time series forecasting methods being carried out on the same data set for university student enrollments. It has been observed that the proposed model has improved forecasting accuracy as well as reduced model complexity compared to other methods.

Key words:
soft computing; fuzzy time series; forecasting; education management

1. Introduction

In current years, many researchers used fuzzy time series to address forecasting problems [1]. Gold prices, weather, stock exchange prices, investment, academic enrollments and so many other data sets are needed to forecast to take smart decisions by the decision makers. These kinds of data sets are based over a period of time which is called time series data. A time series is a sequence of data points, measured typically at a successive time spaced at uniform time intervals [2]. Conventional time series models need to verify some statistics. In order to avoid these problems, a method is introduced by Song and Chissom which is called Fuzzy time series [3, 4].

The next section will discuss previous studies in related to fuzzy times-series forecasting. Section III will introduce our proposed and improved fuzzy time-series method. In section IV a comparison of our improved fuzzy time-series method to previously used models for prediction using the accuracy measurements such as ―AFER‖, and ―RMSE has been presented. The last section concludes this paper.

2. Literature Review

They presented the concept of fuzzy time series based on the historical enrollments of the University of Alabama. Song and Chissom presented the time-invariant fuzzy time series model and the time variant fuzzy time series model based on the fuzzy set theory for forecasting the enrollments of the University of Alabama [5]. Chen presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series [6]. It has the advantage of reducing the calculation, time and simplifying the calculation process using simple fuzzy number arithmetic operations. Chen in another paper presented a forecasting method for time series predicting using the high order fuzzy time series [7]. Huarng used simplified calculations with the addition of heuristic rules to forecast the enrollments [8].

Jillani and Burney [9-11] presented forecasting method to get higher accuracy in forecasting of the enrollment of University of Alabama, TAIFEX, and car accidents casualties in Belgium. Lee et al. introduced a method for temperature forecasting based on two factor high order fuzzy time series [12]. Reference [13] presented a forecasting method for Gold price changes by using ADAPTVE network fuzzy inference system and compare with the traditional statistics ARIMA model. Novák [14] presented the integration of two soft computing techniques namely the ―F-transform and Fuzzy trending‖ modeling used for time series analysis and forecasting. Kumar [15] fuzzy logic based model used when insurers are diabetic patients. Sasu [16] presented a forecasted method to predict the Romania population. Bell shaped membership function used in probability fuzzy set by Huang [17]. Another research used linguistic terms for performance appraisal using multi valued evaluation [18].

Liu [19] proposed a fuzzy time series method which is based over the trapezoidal fuzzy number instead of a single point value with high accuracy rate as compare to

Manuscript received May 5, 2018
Manuscript revised May 20, 2018

other proposed methods by different authors over the Alabama university student's enrollments. In fuzzy time series, suitable number of interval over the universe of discourse was an issue which is now solved by Huarng [7]. Liu in [18] used Huarng procedure in [7] to finding the suitable number of interval for the universe of discourse.

3. Proposed Fuzzy Time Series Method

The new proposed fuzzy time series model is designed on the basis of membership function which is called Trapezoidal [22]. The improved model's provide the forecasted values in terms of Trapezoidal fuzzy numbers and generate more accurate forecasted results. We chose Chen [6] methods as a foundation to develop the proposed method. Several modifications between the proposed method and Chen [6] method are listed below:

- Determine the number of equal-length intervals using advance method which is proposed by Huarng [8].
- Use Trapezoidal fuzzy numbers to define the fuzzy sets in fuzzy time series.
- Establish the Second Order Fuzzy logical relationship.

The selection of the length of the interval was subjective in Chen [6] method, but now we use Huarng [13] method which designed on the average-based length method. Therefore, this is the first changed in our proposed method. Second, the fuzzy time series models presented in references [6], [9], [10], [11], and [12] used discrete fuzzy sets which are presented in step 5 but we used Fuzzy numbers defined in step 6. The third modification we propose is to use high order fuzzy sets which described in step 7. The following are the proposed method steps:

- Step 1: Collect the time series historical data (Y(t)).
- Step 2: Universe of discourse (U) is defined as follows:

$$U = [D_{min} - D1, D_{max} + D2]$$

Where:

D_{min} = minimum value of Y(t)

D_{max} = maximum value of Y(t)

D1 and D2 = Proper values to make universe of discourse

- Step 3: Determine the length of interval "l" computed using "Average Based length".

(i) Take the average of the first absolute difference of time series data.

(ii) Take one half of the average of the first absolute difference as the length.

(iii) Locate the range of the length using the table defined by Huarng [8].

(iv) Round off the length of interval "l" according to the base value find out in step 3.

Table 1: Average based length method

Range	Base
0.1–1.0	0.1
1.1–10	1
11–100	10
101–1000	100
1001–10000	1,000

In literature review, researchers used discrete fuzzy sets to forecast their fuzzy time series.

- Step 4: Determine the number of intervals "m" using the appropriate length of interval "l" using the following formula:

$$m = (Dmax + D2 - Dmin + D1) / l$$

Intervals "m" are presented by u1, u2, … , um. Assume that the "m" intervals are u1 = [d1, d2], u2 = [d2, d3], ... , um−2 = [dm−2, dm−1], um−1 = [dm−1, dm], and um = [dm, dm+1]. Therefore, the discrete fuzzy sets $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_m$ are defined as follows:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \cdots + \frac{0}{u_{m-1}} + \frac{0}{u_m}$$

$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \cdots + \frac{0}{u_{m-1}} + \frac{0}{u_m}$$

$$\vdots$$

$$A_m = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \cdots + \frac{0.5}{u_{m-1}} + \frac{1}{u_m}$$

Where :

Ui = The midpoints of intervals.

- Step 5: Determine the fuzzy numbers:

This study replaces the discrete fuzzy sets by fuzzy numbers with Trapezoidal. The Trapezoidal fuzzy number

$\tilde{A}$ [23] obtained using the Matlab software (R12a) $\tilde{A} = (a, b, c, d)$ with Trapezoidal membership function:

$$\mu A(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{c-x}{c-b} & c \le x \le d \\ 0, & x > d \end{cases}$$

The Trapezoidal membership function consists of parameters “a”, “b”, “c” and “d”. The fuzzy numbers A_1, A_2 , …, Am can be defined as follows:

$$A_1 = (do, d1, d2, d3)$$

$$A_2 = (d1, d2, d3, d4)$$

.

.

.

$$A_m = (dm - 2, dm - 1, dm, dm + 1)$$

The following figure 4 shows the fuzzy numbers A1 and A2 using Trapezoidal membership function:

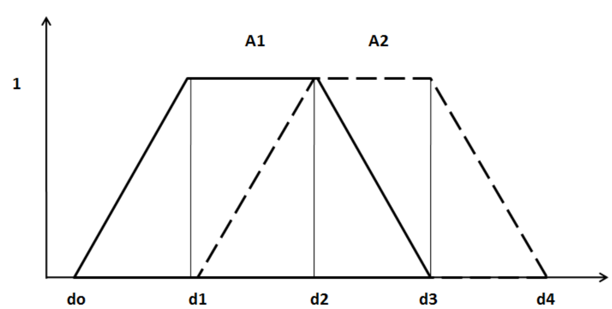

Fig. 1 Fuzzy numbers A1 and A2

- **Step 6:** Fuzzify the time series data Y(t):

Fuzzify the time series historical data using the following rule:

If the value of time series Dvt lies in the range of interval Uj, then it belongs to fuzzy number $\tilde{A}$j. We use this rule and apply over all the values of time series data.

- **Step 7:** Fuzzy logical relationships of Type-1 Fuzzy Time Series:

First Order Fuzzy logical relationship can be generating using the following relationship:

$$\tilde{A}j \rightarrow \tilde{A}k$$

Where $\tilde{A}$j denotes the left hand side and $\tilde{A}$k denotes the right hand side of the logical relationship.

If Y(t-1) is the value of time (t-1) then it is represented by $\tilde{A}$j and if Y(t) is the value of time (t) then it is represented by $\tilde{A}$k. Using the definition 6, we derive the fuzzy logical

Second Order Fuzzy logical relationship can be generating using the following relationship:

$$\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}k$$

where $\tilde{A}i$ and $\tilde{A}$j denote the left hand side and $\tilde{A}$k denotes the right hand side of the logical relationship.

If Y(t-2) and Y(t-1) are the values of time (t-2) and (t-1) then it is represented by $\tilde{A}i$ and $\tilde{A}$j and if Y(t) is the value of time (t) then it is represented by $\tilde{A}$k. Using the definition 7, we derive the fuzzy logical relationship for all fuzzified data.

- **Step 8:** Fuzzy Logical Relationship Groups of Type-1 Fuzzy Time Series:

The fuzzy logical relationship groups can be formed when left hand side fuzzy numbers are same on the fuzzy logical relationships.

Table 2: fuzzy logical relationship groups

First Order	**Second Order**
$\tilde{A}j \rightarrow \tilde{A}k1$	$\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}k1$
$\tilde{A}j \rightarrow \tilde{A}k2$	$\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}k2$
.	.
$\tilde{A}j \rightarrow \tilde{A}kp$	$\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}kp$

- **Step 9: Defuzzificatoin and Forecasted Output:**

The forecasted output can be found out using the following table 3:

In case 1, the right hand side fuzzy numbers are the forecasted output. When left hand side fuzzy numbers are same then we take the average of the right hand side of the fuzzy numbers like case 2. In case 3, when there is an empty fuzzy number on the right hand side than we use the following rule for forecasting output as shown below in table 4.

Table 3 :forecaseted output

First Order	Second Order
$\tilde{A}j$	$\tilde{A}j$

We used trapezoidal fuzzy numbers in forecasting the enrollments time series. In order to defuzzify these numbers we used the following centroid formula:

$$x^* = \frac{\sum_{i=1}^{n} \mu A(xi) xi}{\sum_{i=1}^{n} \mu A(xi)}$$

Where x* is the defuzzify value. We used the MATLAB software to defuzzify the forecasted fuzzy numbers. To measure the accuracy of forecasting output of the proposed fuzzy time series model, we used average forecasting error rate "AFER" and root mean square error "RMSE" as the evaluation of the performance.

$$\text{AFER} = \frac{\sum_{i=1}^{n} \left| \frac{(\text{Actual Value} - \text{Forecasted Value})}{\text{Actual Value}} \right|}{n}$$

$$\text{RMSE} = \sum_{i=1}^{n} \frac{(\text{Actual Value} - \text{Forecasted Value})^2}{n}$$

Table 4 :

	First Order	Second Order
Case 1	$\tilde{A}j \rightarrow \tilde{A}k$	$\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}k$
Case 2	$\tilde{A}j \rightarrow \tilde{A}k1$ $\tilde{A}j \rightarrow \tilde{A}k2$ $\tilde{A}j \rightarrow \tilde{A}kp$ $\tilde{A}j = \frac{\tilde{A}k1 + \tilde{A}k2 + \cdots + \tilde{A}kp}{p}$	$\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}k1$ $\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}k2$ $\tilde{A}i,\ \tilde{A}j \rightarrow \tilde{A}kp$ $\tilde{A}i, \tilde{A}j = \frac{\tilde{A}k1 + \tilde{A}k2 + \cdots + \tilde{A}kp}{p}$
Case 3	$\tilde{A}j \rightarrow \emptyset$	$\tilde{A}i,\ \tilde{A}j \rightarrow \emptyset$

The proposed method algorithm of Fuzzy Time Series is presented in the following figure:

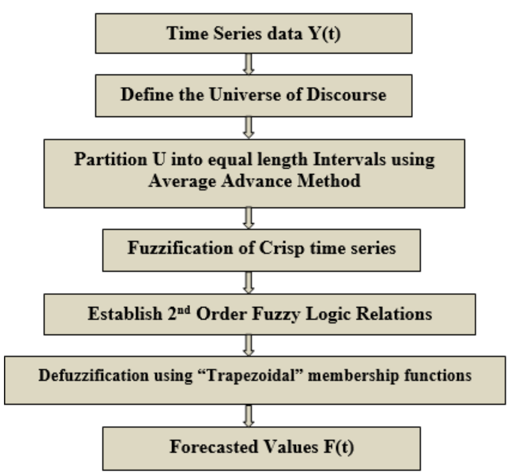

Fig 2. Proposed fuzy time series method

4. Application of Proposed Fuzzy Time Series Method

In this section we applied the proposed method over the University of Alabama enrollments time series from 1970 to 1992. The following steps demonstrate the implementation of fuzzy time series model over the enrollments:

Step 1: The Actual enrollment time series of University of Alamaba present in table 5:

Table 5: Time series of actual student enrollments

Year	Enrollments	Year	Enrollments
1971	13055	1982	15433
1972	13563	1983	15497
1973	13867	1984	15145
1974	14696	1985	15163
1975	15460	1986	15984
1976	15311	1987	16859
1977	15603	1988	18150
1978	15861	1989	18970
1979	16807	1990	19328
1980	16919	1991	19337
1981	16388	1992	18876

Step 2: Universe of discourse (U) is defined as follows:

The universe of discourse (U) is computed using the formula 1:

$$U = [D_{min} - D1, D_{max} + D2]$$

Where:

Dmin = 13055

Dmax = 19355

D1 and D2 = 55 and 45.

U = [13000, 19400]

Step 3: Determine the length of interval "l" computed using "Average Based length" as defined in [13].

(i) The average of the first absolute difference between Dv(t-i) and Dv(t) is 510.33.
(ii) The one half of the average of the first absolute difference is 255.167.
(iii) The base value is 100 because 255.1 lies in the range 101- 1000.
(iv) According to base value, the length of interval “l” is 300.

Step 4: Determine the number of intervals “m” using the appropriate length of interval “l”:

The number of intervals (m) computed:

m = (Dmax + D2 − Dmin + D1) / l = (19400-13000) / 300 = 21.3 ≈ 22.

Intervals “m=22” are presented by u1, u2, … , u22. Intervals defined as u1 = [13000 13300], u2 = [13300 13600], … , u22 = [19100 19400]. These number of intervals present fuzzy sets from A1, A2 to A22 using following equation:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \cdots + \frac{0}{u_{22-1}} + \frac{0}{u_{22}}$$

$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \cdots + \frac{0}{u_{22-1}} + \frac{0}{u_{22}}$$

.
.
.

$$A_{22} = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \cdots + \frac{0.5}{u_{22-1}} + \frac{1}{u_{22}}$$

Now we use Liu [15] strategy of using fuzzy numbers instead of fuzzy sets using the Trapezoidal membership function.

Step 5: The fuzzy numbers using the Trapezoidal membership function can be represented as:

$$\tilde{A}_1 = (12700, 13000, 13300, 13600)$$

$$\tilde{A}_2 = (13000, 13300, 13600, 13900)$$

$$\tilde{A}_3 = (13300, 13600, 13900, 14200)$$

.
.
.

$$\tilde{A}_{22} = (18800, 19100, 19400, 19700)$$

Step 6: In this step, we fuzzify the time series data by mapping data values corresponding to the fuzzy numbers. For example, when we consider year 1971 the number of enrollment is 13055 which is located in the interval range U1 = [13000 13300]. Therefore, the corresponding fuzzy number for given year 1971 is A1. Similarly we have to locate the range of each data value and present’s fuzzy numbers which are mentioned in the following table:

Table 6: fuzzy numbers

Year	Enrollments	Fuzzy Numbers
1971	13055	A1
1972	13563	A2
1973	13867	A3
1974	14696	A6
1975	15460	A9
1976	15311	A8
1977	15603	A9
1978	15861	A10
1979	16807	A13
1980	16919	A14
1981	16388	A12
1982	15433	A9
1983	15497	A9
1984	15145	A8
1985	15163	A8
1986	15984	A10
1987	16859	A13
1988	18150	A18
1989	18970	A20
1990	19328	A22
1991	19337	A22
1992	18876	A20

The following figure 6 shows the fuzzy numbers A1 and A2 with Trapezoidal membership function:

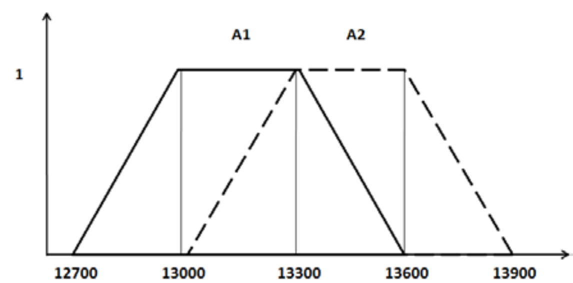

Fig. 3 Fuzzy numbers A1 & A2 using Trapezoidal membership function

Step 7: In this step, we constructed fuzzy logical relationships using the table 6. The First and Second order fuzzy logical relationships are mentioned in the following table 7, and 8.

Table 7: first order fuzzy logic relationship

$\tilde{A}_1 \to \tilde{A}_2$	$\tilde{A}_{10} \to \tilde{A}_{13}$	$\tilde{A}_8 \to \tilde{A}_{10}$
$\tilde{A}_2 \to \tilde{A}_3$	$\tilde{A}_{13} \to \tilde{A}_{14}$	$\tilde{A}_{10} \to \tilde{A}_{13}$
$\tilde{A}_3 \to \tilde{A}_6$	$\tilde{A}_{14} \to \tilde{A}_{12}$	$\tilde{A}_{13} \to \tilde{A}_{18}$
$\tilde{A}_6 \to \tilde{A}_9$	$\tilde{A}_{12} \to \tilde{A}_9$	$\tilde{A}_{18} \to \tilde{A}_{20}$
$\tilde{A}_9 \to \tilde{A}_8$	$\tilde{A}_9 \to \tilde{A}_9$	$\tilde{A}_{20} \to \tilde{A}_{22}$
$\tilde{A}_8 \to \tilde{A}_9$	$\tilde{A}_9 \to \tilde{A}_8$	$\tilde{A}_{22} \to \tilde{A}_{22}$
$\tilde{A}_9 \to \tilde{A}_{10}$	$\tilde{A}_8 \to \tilde{A}_8$	$\tilde{A}_{22} \to \tilde{A}_{20}$

Table 8: second order fuzzy logic relationship

	$\tilde{A}_{9},\tilde{A}_{10} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{8},\tilde{A}_{8} \rightarrow \tilde{A}_{10}$
$\tilde{A}_{1},\tilde{A}_{2} \rightarrow \tilde{A}_{3}$	$\tilde{A}_{10},\tilde{A}_{13} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{8},\tilde{A}_{10} \rightarrow \tilde{A}_{13}$
$\tilde{A}_{2},\tilde{A}_{3} \rightarrow \tilde{A}_{6}$	$\tilde{A}_{13},\tilde{A}_{14} \rightarrow \tilde{A}_{12}$	$\tilde{A}_{10},\tilde{A}_{13} \rightarrow \tilde{A}_{18}$
$\tilde{A}_{3},\tilde{A}_{6} \rightarrow \tilde{A}_{9}$	$\tilde{A}_{14},\tilde{A}_{12} \rightarrow \tilde{A}_{9}$	$\tilde{A}_{13},\tilde{A}_{18} \rightarrow \tilde{A}_{20}$
$\tilde{A}_{6},\tilde{A}_{9} \rightarrow \tilde{A}_{8}$	$\tilde{A}_{12},\tilde{A}_{9} \rightarrow \tilde{A}_{9}$	$\tilde{A}_{18},\tilde{A}_{20} \rightarrow \tilde{A}_{22}$
$\tilde{A}_{9},\tilde{A}_{8} \rightarrow \tilde{A}_{9}$	$\tilde{A}_{9},\tilde{A}_{9} \rightarrow \tilde{A}_{8}$	$\tilde{A}_{20},\tilde{A}_{22} \rightarrow \tilde{A}_{22}$
$\tilde{A}_{8},\tilde{A}_{9} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{9},\tilde{A}_{8} \rightarrow \tilde{A}_{8}$	$\tilde{A}_{22},\tilde{A}_{22} \rightarrow \tilde{A}_{20}$

Step 8: Fuzzy Logical Relationship Groups of Type-1 Fuzzy Time Series:

The fuzzy logical relations groups constructed using the similar fuzzy number on the left hand side of the fuzzy logical relationships in step 7. We have used second order therefore which is mentioned in the following table 9:

Talbe 9: fuzzy logical relationship groups of second order

Groups	Second Order
1	$\tilde{A}_{1},\tilde{A}_{2} \rightarrow \tilde{A}_{3}$
2	$\tilde{A}_{2},\tilde{A}_{3} \rightarrow \tilde{A}_{6}$
3	$\tilde{A}_{3},\tilde{A}_{6} \rightarrow \tilde{A}_{9}$
4	$\tilde{A}_{6},\tilde{A}_{9} \rightarrow \tilde{A}_{8}$
5	$\tilde{A}_{9},\tilde{A}_{8} \rightarrow \tilde{A}_{9}$
6	$\tilde{A}_{8},\tilde{A}_{9} \rightarrow \tilde{A}_{10}$
7	$\tilde{A}_{9},\tilde{A}_{10} \rightarrow \tilde{A}_{13}$
8	$\tilde{A}_{10},\tilde{A}_{13} \rightarrow \tilde{A}_{14}$
9	$\tilde{A}_{13},\tilde{A}_{14} \rightarrow \tilde{A}_{12}$
10	$\tilde{A}_{14},\tilde{A}_{12} \rightarrow \tilde{A}_{9}$
11	$\tilde{A}_{12},\tilde{A}_{9} \rightarrow \tilde{A}_{9}$
12	$\tilde{A}_{9},\tilde{A}_{9} \rightarrow \tilde{A}_{8}$
13	$\tilde{A}_{9},\tilde{A}_{8} \rightarrow \tilde{A}_{8}$
14	$\tilde{A}_{8},\tilde{A}_{8} \rightarrow \tilde{A}_{10}$
15	$\tilde{A}_{8},\tilde{A}_{10} \rightarrow \tilde{A}_{13}$
16	$\tilde{A}_{10},\tilde{A}_{13} \rightarrow \tilde{A}_{18}$
17	$\tilde{A}_{13},\tilde{A}_{18} \rightarrow \tilde{A}_{20}$
18	$\tilde{A}_{18},\tilde{A}_{20} \rightarrow \tilde{A}_{22}$
19	$\tilde{A}_{20},\tilde{A}_{22} \rightarrow \tilde{A}_{22}$
20	$\tilde{A}_{22},\tilde{A}_{22} \rightarrow \tilde{A}_{20}$

Step 9: Forecasted Output:

We forecast the enrollments values using our proposed second order fuzzy time series method. Table X shows that forecast enrollment values.

MATLAB was being used to defuzzify the fuzzified forecasted enrollment by using the centroid formula given in the step 9. To assess the performance of the proposed forecasting method, there are nine literature review methods used for comparing of their forecasted results using the average forecasted error rate "AFER" and root mean square error "RMSE" are selected to assess the forecasting accuracy. Table XI shows the comparison of proposed method and the literature review methods where as figure 7 show its graphical representation.

5. Conclusion

The model proposed in this paper outperformed the previously used time-series methods for forecasting student enrollments as shown in table XI. In future, a research can be carried out on some other forecasting problems using the presented method to check if this method can become a general method for predicting with accuracy.

Table 10: forecaseted output

Year	Enrollment	Forecasted Enrollment
1971	13055	
1972	13563	
1973	13867	A3 = (13600, 13700, 13800, 13900)
1974	14696	A6 = (14500, 14600, 14700, 14800)
1975	15460	A9 = (15400, 15500, 15600, 15700)
1976	15311	A8 = (15100, 15200, 15300, 15400)
1977	15603	A9 = (15400, 15500, 15600, 15700)
1978	15861	A10 = (15700, 15800, 15900, 16000)
1979	16807	A13 = (16600, 16700, 16800, 16900)
1980	16919	A14 = (16900, 17000, 17100, 17200)
1981	16388	A12 = (16300, 16400, 16500, 16600)
1982	15433	A9 = (15400, 15500, 15600, 15700)
1983	15497	A9 = (15400, 15500, 15600, 15700)
1984	15145	A8 = (15100, 15200, 15300, 15400)
1985	15163	A8 = (15100, 15200, 15300, 15400)
1986	15984	A10 = (15700, 15800, 15900, 16000)
1987	16859	A13 = (16600, 16700, 16800, 16900)
1988	18150	A18 = (18100, 18200, 18300, 18400)
1989	18970	A20 = (18700, 18800, 18900, 19000)
1990	19328	A22 = (19300, 19400, 19500, 19600)
1991	19337	A22 = (19300, 19400, 19500, 19600)
1992	18876	A20 = (18700, 18800, 18900, 19000)

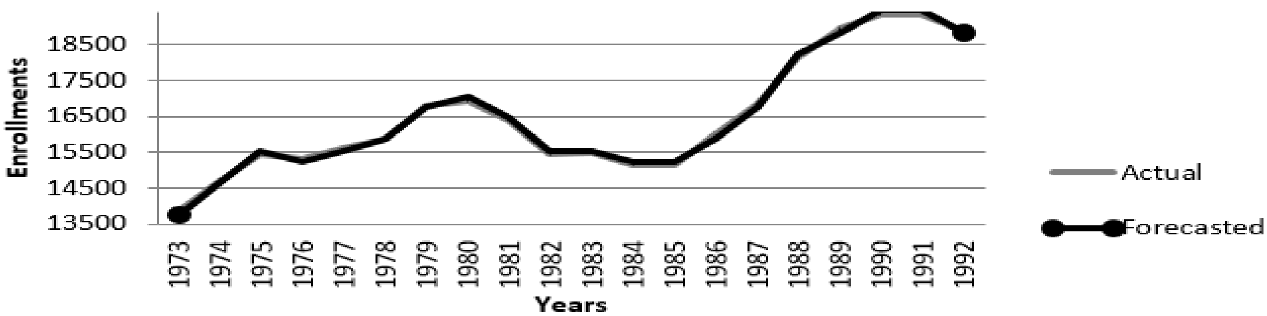

Fig. 4 Actual and Forecaseted Output Chart

Table 11: Comparison of time series

Year	Enrollments	Song Chissom [2]	Song Chissom [3]	Chen[5]	Hwan Chen Lee [22]	Chen [6]	Burney [8]	Burney [9]	Burney [10]	Proposed Method
1971	13055	-	-	-	-	-	-	14464	13579	-
1972	13563	14000	-	14000	-	-	-	14464	13798	-
1973	13867	14000	-	14000	-	-	-	14464	13798	14650
1974	14696	14000	-	14000	-	14500	14730	14710	14452	15550
1975	15460	15500	14700	15500	-	15500	15615	15606	15373	15250
1976	15311	16000	14800	16000	16260	15500	15614	15606	15373	15550
1977	15603	16000	15400	16000	15511	15500	15611	15606	15623	15850
1978	15861	16000	15500	16000	16003	15500	15611	15606	15883	16750
1979	16807	16000	15500	16000	16261	16500	16484	16470	17079	17050
1980	16919	16813	16800	16833	17407	16500	16476	16470	17079	16450
1981	16388	16813	16200	16833	17119	16500	16469	16470	16497	15550
1982	15433	16789	16400	16833	16188	15500	15609	15606	15737	15550
1983	15497	16000	16800	16000	14833	15500	15614	15606	15737	15250
1984	15145	16000	16400	16000	15497	15500	15612	15606	15024	15250
1985	15163	16000	15500	16000	14745	15500	15609	15606	15024	15850
1986	15984	16000	15500	16000	15163	15500	15606	15606	15883	16750
1987	16859	16000	15500	16000	16384	16500	16477	16470	17079	18250
1988	18150	16813	16800	16833	17659	18500	18482	18473	17991	18850
1989	18970	19000	19300	19000	19150	18500	18481	18473	18802	19450
1990	19328	19000	17800	19000	19770	19500	19158	19155	18994	19450
1991	19337	19000	19300	19000	19928	19500	19155	19155	18994	18850
1992	18876	-	19600	19000	15837	18500	18475	18473	18916	14650
	AFER	4.38%	3.11%	3.11%	2.44%	1.52%	1.40%	2.38%	1.02%	**0.521%**
	RMSE	880.73	638.36	566.93	476.03	294.43	286.82	476.64	203.53	**92.862**

References

[1] Burney, SM Aqil, Syed Mubashir Ali, and Shamaila Burney. "A Survey of Soft Computing Applications for Decision Making in Supply Chain Management."2017 3rd IEEE International Conference on Engineering Technologies and Social Sciences (ICETSS), Bangkok, Thailand. (2017)

[2] Raza, S. Akhter, and S. M. Burney. Time Series Analysis of High Speed Wireless Networks: Forecasting network load using time series models. LAP Lambert Academic Publishing, 2011.

[3] Song, Qiang, and Brad S. Chissom. "Forecasting enrollments with fuzzy time series—part I." Fuzzy sets and systems 54, no. 1 (1993): 1-9.

[4] Song, Qiang, and Brad S. Chissom. "Fuzzy time series and its models." Fuzzy sets and systems 54, no. 3 (1993): 269-277.

[5] Song, Qiang, and Brad S. Chissom. "Forecasting enrollments with fuzzy time series—part II." Fuzzy sets and systems 62, no. 1 (1994): 1-8.

[6] Chen, Shyi-Ming. "Forecasting enrollments based on fuzzy time series." Fuzzy sets and systems 81, no. 3 (1996): 311-319.

[7] Chen, Shyi-Ming. "Forecasting enrollments based on high-order fuzzy time series." Cybernetics and Systems 33, no. 1 (2002): 1-16.

[8] Huarng, Kunhuang. "Effective lengths of intervals to improve forecasting in fuzzy time series." Fuzzy sets and systems 123, no. 3 (2001): 387-394.

[9] Jilani, Tahseen Ahmed, and Syed Muhammad Aqil Burney. "M-factor high order fuzzy time series forecasting for road accident data." In Analysis and design of intelligent systems using soft computing techniques, pp. 246-254. Springer Berlin Heidelberg, 2007.

[10] Jilani, Tahseen Ahmed, Syed Muhammad Aqil Burney, and Cemal Ardil. "Fuzzy metric approach for fuzzy time series forecasting based on frequency density based partitioning." International Journal of Computational Intelligence 4, no. 2 (2008): 112-117.

[11] Jilani, Tahseen A., Aqil S. Burney, and Cemal Ardil. "A new quantile based fuzzy time series forecasting model."

International Journal of Intelligent Systems and Technologies 3, no. 4 (2008): 201-207.

[12] Lee, Li-Wei, Li-Hui Wang, and Shyi-Ming Chen. "Temperature prediction and TAIFEX forecasting based on high-order fuzzy logical relationships and genetic simulated annealing techniques." Expert Systems with Applications 34, no. 1 (2008): 328-336.

[13] Yazdani-Chamzini, Abdolreza, Siamak Haji Yakhchali, Diana Volungevičienė, and Edmundas Kazimieras Zavadskas. "Forecasting gold price changes by using adaptive network fuzzy inference system." Journal of Business Economics and Management 13, no. 5 (2012): 994-1010.

[14] Novák, Vilém, Martin Štěpnička, Antonín Dvořák, Irina Perfilieva, Viktor Pavliska, and Lenka Vavříčková. "Analysis of seasonal time series using fuzzy approach." International Journal of General Systems 39, no. 3 (2010): 305-328.

[15] Kumar, Sanjeev, and Hemlata Jain. "A fuzzy logic based model for life insurance underwriting when insurer is diabetic." European Journal of Applied Sciences 4, no. 5 (2012): 196-202.

[16] Sasu, Adela. "An Application of Fuzzy Time Series to The Romanian Population." Bulletin of the Transilvania University of Brasov• Vol 3 (2010): 52.

[17] Huang, Wenjing, and Yihua Li. "Bell-Shaped probabilistic fuzzy set for uncertainties modeling." Journal of Theoretical & Applied Information Technology 46, no. 2 (2012).

[18] Burney, Shamaila. "An Integrated Conceptual Model of Performance Appraisals and Succession Planning using Multi valued Evaluation." Pakistan Journal of Engineering, Technology & Science 2, no. 2 (2016).

[19] Liu, Hao-Tien. "An improved fuzzy time series forecasting method using trapezoidal fuzzy numbers." Fuzzy Optimization and Decision Making 6, no. 1 (2007): 63-80.

[20] Zadeh, Lotfi Asker. "The concept of a linguistic variable and its application to approximate reasoning—I." Information sciences 8, no. 3 (1975): 199-249.

[21] Pedrycz, Witold. "Fuzzy Sets of Higher Type and Higher Order in Fuzzy Modeling." In Frontiers of Higher Order Fuzzy Sets, pp. 31-49. Springer New York, 2015.

[22] Monil, Mohammad Alaul Haque, and Rashedur M. Rahman. "Vm consolidation approach based on heuristics fuzzy logic, and migration control." Journal of Cloud Computing 5, no. 1 (2016): 1-18.

[23] Hwang, Jeng-Ren, Shyi-Ming Chen, and Chia-Hoang Lee. "Handling forecasting problems using fuzzy time series." Fuzzy sets and systems 100, no. 1-3 (1998): 217-228.