Analysis of Downlink Coverage for Multi-Tier Cellular Network under Different Association Criteria Using Stochastic Geometry

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Summary
The regularly rising of the information rates due to the huge usage of cellular applications implies that the legacy mobile network models comprising of macro-cell architecture do not fulfill the demand of the users. The only feasible solution to achieve the increasing demand is by adding small-scale BSs (pico cells, femto cells or both) in the legacy cellular network architecture of macro type BSs. The addition of small cells transforms the network into heterogeneous network. In this research, a heterogeneous network comprised of two-tiers of macro and pico BSs is evaluated for the downlink coverage probability of a user attached or associated to its macro or pico base station using stochastic geometry approach under two camping rules i.e., strongest instantaneous power and physically closet BS. Poisson point process (PPP) is utilized to model/represent the placement of BSs and users in the network. The analytical results of the SIR are presented, and the simulation model is developed accordingly to get the simulation results which are nicely matching with the analytical results. The results provide insights about the behavior of coverage (CCDF of SIR) under different assessed association criteria. It is observed on comparing all the coverage results for macro and pico tiers under both the studied associations that the coverage is highest for macro tier than the pico tier because of its high transmit power in the downlink as compared to pico BSs under both associations. Similarly, when compared the results of coverage for the two associations, the coverage under maximum strongest power association is slightly higher than the nearest association.

Key words:
Stochastic Geometry, Coverage, Association Criteria, Signal-to-Interference Plus Noise Ratio.

1. Introduction

The regularly rising of the information rates due to the huge usage of cellular applications implies that the legacy mobile network models comprising of macro-cell architecture do not fulfill the demand of the users. Due to this demand, the present 4G systems are being changed from 4G to 5G [1]. There are ways to improve the capacity of these legacy mobile networks as per guidelines of the 3GPP (Third Generation Partnership Project) under different LTE releases which is one of the standard body.

The ways suggested by the standard bodies are to utilize additional spectrum, spectral efficiency improvement, or by balancing the load where large-scale BS (Base Station) users are detached from their attached BSs and camped on to small-scale BSs. Although, the easiest way to enhance capacity and coverage which consequently fulfills this rising demand is simply by introducing more bandwidth/spectrum but unluckily, this arrangement is not suitable in terms of cost. Thus, the only solution to achieve the increasing demand is by adding small-scale BSs (pico cells, femto cells or both) in the legacy cellular network architecture of macro BSs type. It should be noted that the new network is not designed only with small cells but it is mixture of existing large-scale BSs (macro BSs) with small-scale BSs (Pico or Femto BSs) and this new network architecture with mixed BSs is called HCN (Heterogeneous Cellular Network) [2] [3] [4]. The reason for this mixed type of network (HCN) which comprises with in excess of one class of base station is that the network of only small-cells is not suitable for low traffic zones. Hence, fundamental network coverage and versatility support specially for high speed users and users in low traffic zones will be kept on being dealt with by macro cells.

The very basic and imperative parameter of the mobile networks is the SINR (Signal-to-Interference Plus Noise Ratio) through which BER (Bit Error Rate) on any channel between the user and its camped BS is determined which consequently helps in calculating the coverage (the set threshold or limit lower than the obtained SINR) and highest obtained data rate (capacity) of the user. Subsequently, a system administrator needs to upgrade the task of an HCN has to know the spatial distribution of the SINR and its reliance on the arrangement parameters of the HCN e.g., downlink SINR from the associated BS to its user has its reliance on transmit powers and densities of the different classes or levels (tiers) of network BSs. The spatial distribution of the SINR under arranged parameters is easily understood by the simulation studies but extensive simulations are required for each scenario with different arrangement parameters to completely understand or get insights about the network which is quite difficult.

Many fields of studies where randomness of objects is involved have been analyzed through a mathematical tool, known as Stochastic Geometry [5]. The network where

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multi-tiers are involved becomes complex enough to analyze them through traditional mathematical tools is usually not possible. Thus, stochastic geometry has now got attention and provides the solution in examining the heterogeneous type of cellular network with multi-tiers where the users' locations are dispersed randomly across the different tiers, following the certain distribution [6] [7]. These random locations of users are modeled by Poisson point process (PPP) which figures the exact solution of SINR distribution with small numerical difficulty and depends just on specific blends of the network parameters.
Coverage and consequently other performance metrics are always affected when users are camped according to the different association schemes [8] [9] [10] [11], also called as association/camping rules, criteria, or schemes. The affiliation or joining of the users in downlink with their BSs happen according to different criteria. In this paper, under the nearest association or instantaneous received power camping rules, the heterogeneous cellular network comprising of two tiers of macro and pico BSs has been investigated analytically and with simulations. The subtle elements of these schemes are further discussed in section 3.
In a large portion of the related work, the outcomes for SINR in downlink (DL) are acquired using average maximum power in DL camping scheme [12] [13] [14] [15], however, very less work related to SINR using instantaneous power has been achieved [8]. In this paper, the logical analytical results/outcomes derived in [8] for various criteria of camping i.e., closest and most received instantaneous power are not quite the same as the vast majority of the accessible literature. The MATLAB® software has been used to develop the two-tier HCN using mentioned association schemes to get results to confirm the expository analytical outcomes presented in [8] through simulations so as to give insights regarding the two-tier HCN and predict the performance of the proposed HCN under the two rules of affiliation specified earlier.
Moreover, this paper is divided into following sections: section-2 explains the proposed network model of HCN comprising of two-tiers of macro and pico BSs. The channel model of the considered network is also discussed to find the SINR for the coverage results. The two schemes of user association which include maximum instantaneous power received by intended user amongst of all tier BSs and physically nearest available BS are described in section-3. Simulation and analytical outcomes/results for coverage are presented using the two schemes of user association or camping mentioned above along with the insights extracted from the obtained results. Finally, the paper is concluded along with the discussion about the future work to extend the proposed research in section-5.

2. Network Model

The proposed network model is shown in Fig. 1 which is formed by large-scale (macro) BSs and small-scale (pico) BSs. This network model is considered in a specific moment of time (one TTI in LTE) which relates to one RE (Resource Element) in LTE (Long-Term Evolution) and one frequency interval for transmission in other standards like HSPA (High Speed Packet Access). An HCN of two-tiers has been considered with t = 1, 2 where t_1 represents macro tier and t_2 represents pico tier. Placement (location) of user (UE) is anywhere in the network which is shown as intended user (UE_0), receiving desired signal from its serving/camped BS b_0 as shown in Fig. 1. The rest of the BSs belonging to macro or pico tier are seen to be as interferers for the intended user (UE_0). This whole scenario is shown in Fig. 1 with solid line representing intended signal and dashed lines are interfering signals. Let Φ_1 with density λ_1 and Φ_2 with density λ_2 form the two separate PPPs which denote the placement of BSs in tier-1 and tier-2 respectively. The placement of UEs forms another PPP, denoted by Φu. The superposition of Φ_1 and Φ_2 also forms a PPP as per superposition theorem of Poison point processes, represented by Φt with total density of BSs of both tiers, $\lambda_t .= \lambda_1 + \lambda_2$. The power of all the BSs in the tier t is P_t^{Tx} which is same and fixed in that tier. The power of tier-1 is greater than tier-2 i.e., $P_1^{Tx} > P_2^{Tx}$. First, UE is selected randomly from all the users available in the network. After selecting UE, the camping BS is selected individually from both the tiers and later one BS is selected out of two BSs for the randomly selected UE as per association criteria.
Two components are required to find the loss of a specific link in a communication system. Those two components are the path loss and fading. Fading is naturally random whereas path loss depends upon the distance d between the UE and the associated BS. The path loss (PL) is usually given by

$$PL = 10\alpha \log_{10}(d) - 10\log_{10} K, \qquad (1)$$

where d, K, and α are separation between UE and its serving BS (distance in meters), intercept, and the path loss slope respectively. Further, assuming the above-mentioned parameters, the received instantaneous power (Rb) when UE is at distance d from the BS b pertaining to Φt and transmitting power of P_t^{Tx} can be written as

$$R_b = \frac{K_t P_t^{Tx} H_b}{d_b^{\alpha_t}}, \qquad (2)$$

where H_b is the fade attenuation on the link between UE and its camped BS. The fade attenuations for all BSs are independent and following the Rayleigh distribution. The thermal noise is denoted by N_0. All the BSs in both the tiers except the intended BS b_0 treat as interferers and thus, total interference (I) is given by

$$I = \sum_{b\in\Phi_t\setminus\{b_0\}} R_b = P_t^{Tx} \sum_{b\in\Phi_t\setminus\{b_0\}} \frac{K_t H_b}{d_b^{\alpha_t}}, \quad (3)$$

Thus, SINR in downlink is given by

$$SINR^{DL} = \frac{R_{b_0}}{I + N_0}, \quad (4)$$

Where R_{b0} is the instantaneous received power at intended BS, I is the total inference of all BSs from both tiers, and N_0 is the thermal noise.

3. Camping Rules/Association Criteria

UEs always are affiliated/camped or associated/connected with their BSs which serve them as indicated by various rules, known as camping or association rules/criteria. These rules affect the SINR of the network and thus, change the other performance metrics like coverage, data rate, and spectral efficiency.

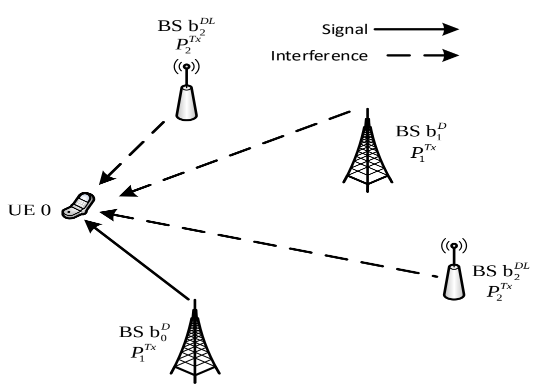

Fig.1 Two-Tier Heterogeneous Cellular Network model with intended and interference signals in the downlink.

In this work, two rules are followed and used to calculate the SINR in DL. In the closest scheme, the UE is physically closet to the BS and connected BS is chosen based on only path loss model given by eq. (1) as fading becomes zero by averaging it from all BSs at the received user while considering the closest scheme. It is given by

$$b_u^{DL} = \arg\min_{b\in\Phi_t}\{d_u^b\} \quad (5)$$

Other affiliation scheme is a high instantaneous power where UE is related to the BS from which it gets most strong instantaneous power. This is defined as

$$b_u^{DL} = \arg\max_{b\in\Phi_t} P_t^{Tx} \quad (6)$$

Further, in the next section-4, probability of a randomly selected user greater than the specific threshold which associates to macro (tier-1) or pico (tier-2) BS in the downlink is calculated under nearest or strongest instantaneous power camping scheme.

4. Numerical and Simulation Results

In this paper, a two-tier macro-pico BSs deployed network has been considered. The modeling of the BSs has been done using PPP (Poisson Point Process). The simulation parameters used to develop the proposed model are shown in Table-1. The coverage under different associations is discussed as under.

Table 1: Simulation parameters with their notations and considered values

Parameters with Notation	Values Considered with Units
Macro-tier BS density (λ_1)	4.6 (BSs/Km)
Pico-tier BS density (λ_2)	3*Macro BS density = 4.6*3 = 13.8 (BSs/Km)
Tier BSs Power (P_1^{Tx} , P_2^{Tx})	[46, 30] dBm
Tier BSs Antenna Gains (G_1, G_2)	[0, 0] dB (Unity Gains)
Path loss Intercepts (K_1, K_2)	[0, 0] dB
Path loss Slopes (α_1, $\alpha_2 = \alpha$)	4
Carrier Frequency	2 GHz
Minimum Distance of UE from BSs (d_{min})	0.01 Km

4.1 Physically closet UE to its associated BS

The thresholds of macro and pico tiers are same and denoted by γ. The coverage (CCDF) of SIR when N_0 is zero for the user connected to its closet macro BS that is its (UE) SIR is greater than the threshold is given in [8] while considering minimum distance (d_{min}) as

$$P[SIR_M^{DL} > \gamma] = \frac{\exp(-m_1\{1+\gamma^{2/\alpha}[G_\alpha(\gamma^{-2/\alpha}) + \beta / \sin c(2/\alpha)]\})}{1+\gamma^{2/\alpha}[G_\alpha(\gamma^{-2/\alpha}) + \beta / \sin c(2/\alpha)]}, \quad (7)$$

where M denotes the macro BS, m_1 is the mean BSs number within d_{min} radius, and β is the ratio of densities and powers of pico and macro BSs and are given by

$$m_1 = \lambda_1 \pi d_{\min}^2 \tag{8}$$

$$\beta = \frac{\lambda_2}{\lambda_1}\left(\frac{K_2 P_2^{Tx}}{K_1 P_1^{Tx}}\right)^{2/\alpha} = \frac{\lambda_2}{\lambda_1}\left(\frac{P_2^{Tx}}{P_1^{Tx}}\right)^{2/\alpha}, \tag{9}$$

K terms have been cancelled in eq. (9) as $K_1 = K_2 = K$. The simulation results for eq. (7) which indicates the SIR of a user connected to its closet macro BS are shown in Fig. 2.
Generally, the results for coverage show that as the threshold is increased, the coverage reduces because threshold cannot be surpassed by the received signal at the user. The both numerical and simulation results are overlapping each other very well as shown in Fig. 2. It should also be noted that the results for the user to connect

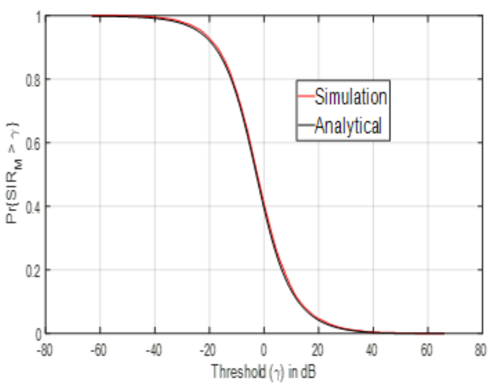

Fig. 2 CCDF of SIR (Coverage) greater than γ for its different values and α = 4 under closet association when UE associates to macro tier.

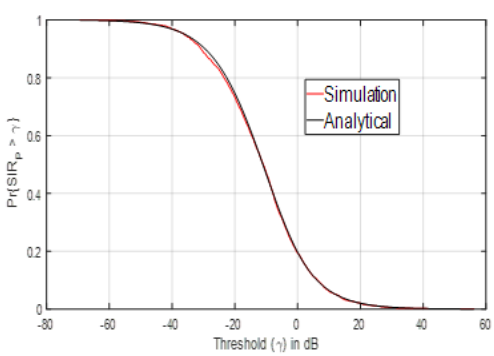

Fig. 3 CCDF of SIR (Coverage) greater than γ for its different values and α = 4 under closet association when UE associates to pico tier.

to its closet serving BS is totally described by m_1 and β mentioned in eqs. (8) and (9) respectively. Moreover, the user connected to its physically closet pico BS as in [8] is given by

$$P[SIR_p^{DL} > \gamma] = \int_0^{\infty} \exp\left(-u\left\{1+\gamma^{2/\alpha}\left[G_\alpha\left(\frac{1}{\gamma^{2/\alpha}}\right)+\frac{1/p-1}{\operatorname{sinc}(2/\alpha)}+\frac{1}{p\beta}G_\alpha\left(\frac{m_1 p\beta}{\gamma^{2/\alpha}u}\right)\right]\right\}\right)du, \tag{10}$$

Where p stands for fraction of pico BSs which is considered 1 because pico BSs are open loop BSs. P would have different values if small BSs considered to be femto BSs which belongs to CSG (Closed Subscribers Group). The simulation results for eq. (10) are shown in Fig. 3.
If the results of coverage (CCDF of SIR) for macro and pico tiers under nearest association are compared as shown in Fig. 2 and Fig. 3 respectively, it is quite evident that the coverage of macro tier is highest than pico tier. For example, coverage of macro tier at γ = 0 dB is 40% whereas coverage of pico tier is 20% for the same value of γ = 0 dB when compared the results of Fig. 2 and Fig. 3 respectively. The reason is that the macro tier BSs transmits high power in the downlink than the pico BSs in the pico tier and therefore, intended UE receives high power which improves its SIR and in result improves its coverage than the UE if it is attached to pico tier.

4.2 UE associated to its strongest BS

When user is attached to a BS from which UE receives strongest power in the DL with thermal noise equals to zero ($N_0 = 0$) with path loss exponents for both tiers considered to be same, and γ ≥ 1 then

$$P[SIR_t > \gamma] = \beta_t \frac{\operatorname{sinc}(2/\alpha)}{\gamma^{2/\gamma}}, \quad \gamma \geq 1, \quad t = 1,2 \tag{11}$$

The simulation results for macro and pico tiers for their affiliated UEs under strongest camping scheme when γ ≥ 1 and $\alpha_1 = \alpha_2 = \alpha = 4$ are shown in Fig. 4 and Fig. 5 respectively. It should be noted that in these results, the marginal coverage of both macro and pico tiers is fitting with the analytical results from the value of 1 (0 dB) and onwards (for positive values of threshold) as per eq. (11) because to obtain the analytical results (derivation) of joint CCDF require absolutely positive values of γ which is not in the scope of this paper. Further, when the coverage results of macro and pico tiers are compared under maximum instantaneous power association scheme in Fig.

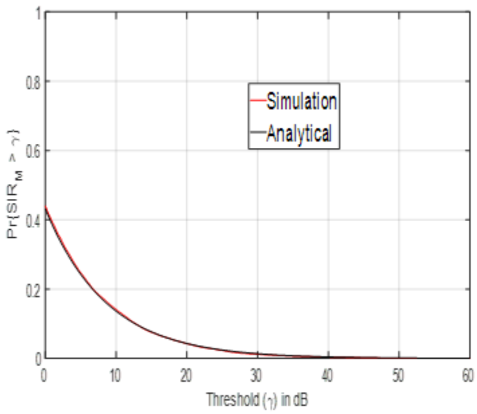

Fig. 4 CCDF of SIR (Coverage) for $\gamma \geq 1$ and $\alpha = 4$ under strongest association when UE associates to macro tier.

4 and Fig. 5, the coverage for macro tier is higher than pico tier for the same reason as discussed under nearest association results.

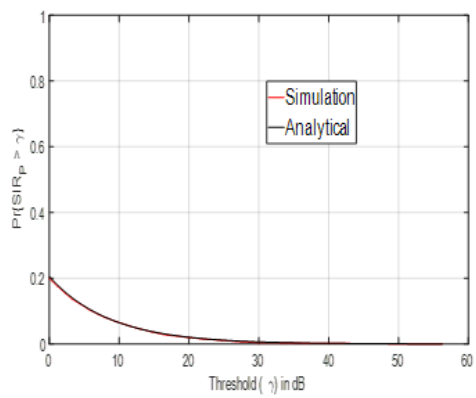

Fig. 5 CCDF of SIR (Coverage) for $\gamma \geq 1$ and $\alpha = 4$ under strongest association when UE associates to pico tier.

Furthermore, when the results of nearest and strongest instantaneous power are compared for macro tier in Fig. 2 and Fig. 4 respectively (because the results for macro tier are highest under both camping schemes), the coverage under strongest instantaneous power is slightly higher than the nearest association scheme which seems to be quite negligible and not worthy compared to its counterpart.

5. Conclusion

In this work, the downlink coverage probability of a user attached or associated to its macro or pico base station was studied using stochastic geometry approach under two camping rules i.e., strongest instantaneous power and physically closet BS. The proposed network was a heterogeneous network comprised of two-tiers of macro and pico BSs. Poisson point process (PPP) was utilized to represent the placement of BSs and users in the network. The analytical results of the SIR were presented, and the simulation model was developed accordingly to get the simulation results which were nicely matching with the analytical results. Those results provide insights about the behavior of coverage (CCDF of SIR) under different assessed association criteria. It was observed on comparing all the coverage results for macro and pico tiers under both the studied associations that the coverage is highest for macro tier than the pico tier because of its high transmit power in the downlink as compared to pico BSs under both associations. Similarly, when compared the results of coverage for the two associations, the coverage under maximum strongest power association was slightly higher than the nearest association. The research presented was only regarding the marginal coverage (marginal CCDF of SIR) for macro-pico tier heterogeneous network. In future, the joint CCDF of SIR (joint coverage) will be investigated for the proposed network under different camping rules.

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