Simulation of Collective Motion of Self Propelled Particles in Homogeneous and Heterogeneous Medium

Israr Ahmed†, Inayatullah Soomro††, Syed Baqer Shah†††, Hisamuddin Shaikh††††

SALU Khairpur Pakistan

Abstract
The concept of self-propelled particles is used to study the collective motion of different organisms such as flocking of birds, swimming of schools of fish or migrating of bacteria. The collective motion of self-propelled particles is investigated in the presence of obstacles and without obstacles. A comparison of the effects of interaction radius, speed and noise on the collective motion of self-propelled particles is conducted. It is found that in the presence of obstacles, mean square displacement of the particles shows large fluctuation, whereas without obstacles fluctuation is less. It is also shown that in the presence of the obstacles, an optimal noise, which maximizes the collective motion of the particles, exists

Key words:
Self-propelled particles, Collective motion, Obstacles.

1. Introduction

Movements in dynamic and complex environments are an integral part of the daily life activities; this includes walking in crowded spaces, playing sports, etc. Many of these tasks that humans perform in such environments involve interactions with stationary or moving obstacles. In this context, it is important to note that there is always a need to coordinate the information with other individuals to deal with the obstacles [1]. There are other examples also exist where living things deal with the obstacles such as flocks of birds facing obstacles while moving collectively. Understanding the detection and avoidance from the obstacles when there is noise in the system is of extreme importance. Many studies have addressed the interaction of individuals with obstacles from different perspectives. For example, in computer science field several studies have been carried out where robots interact with obstacles [2-4]. There are limited studies done to investigate the dynamics of natural systems which deal with the obstacles. Dynamics of the particles in the presence of the obstacles is often observed. Complex collective behaviour is exhibited by the bacteria in the presence of the obstacles. For example, collective behaviour of bacteria was found in the heterogeneous medium such as highly complex tissues in gastrointestinal tract or soil.
Large groups of animals moves very long distances and they cross rivers and forest [5]. Inspite of these facts, there has been limited research done on the effect of heterogeneous media on the collective behaviour of self-propelled particles [6]. Avoidance behaviour of the particle from the obstacle was simulated by the croft et al [7]. In this work measurement of effect was carried out when a single particle collided with the static obstacles. It was found that there are higher chances of collision of social interactions with the obstacles. This is due to the huge supposition and the occurrence of large parameter values. Moreover, the creation of motion due to the social interaction produces key effects on the metrics used to inform management and policy decisions.Mecholsky et al [8] worked on a continuous model in which flocks of birds were considered. Study was done on the linearized interaction of the flocks with an obstacle. Behaviour of the flocks was shown by the density disturbance when there was a interaction with the obstacles. The disturbance was similar to the Mach cones where order was demonstrated by the anistropic spread of waves of flocking. Chepizkho et al [9] focused on the dynamics of the self-propelled particles in the heterogeneous medium. In their model obstacles were randomly distributed in two dimensional spaces. In this model, particle showed avoidance from the obstacle, and the particle's avoiding behaviour was expressed by the turning speed γ . The mean square displacement of the SPPs produced two regimes as a function of obstacle density ρ_o and the particle's turning speed γ. It was found that, in first regime small value of γ produces diffusive movement of particle and determined by diffusive coefficient which demonstrated a least at an intermediate obstacle density ρ_o. In second regime, it was found that for large obstacle densities and for higher values of γ, spontaneous trapping of the SPPs occurred. In their model it was also shown that the presence of fixed and moving obstacles can change the dynamics of the collective behaviour of the particles.
Moreover, optimal noise amplitude took place which maximised the collective motion. This kind of optimality does not appear in the homogeneous medium. Due to the smaller obstacle densities, order parameter showed a single

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critical point, under this point the model showed long range order akin to the homogeneous medium. In the case of large obstacle densities two critical points appeared and made the motion disordered at both smaller and larger noise amplitudes. Furthermore, there was the existence of quasi long range order in between these critical points [5].
In this research work, computer simulation is done for the self-propelled particles in homogeneous media and in heterogeneous media. Here the term heterogeneous applies when the collective motion of the particles is investigated in the presence of the obstacles whereas the term homogeneous is consideredWhen collective behaviour of the particles is studied without obstacles. We compare the effects of the interaction radius, speed and noise on the collective motion of self-propelled particles with and without obstacles. It is found that, in the existence of the obstacles, mean square displacement of the particles exhibit huge fluctuations in the system, whereas without obstacle density this displacement of the SPPs shows very less fluctuating behaviour. It is also observed that in the existence of the obstacles, optimal noise exists which maximises the collective motion of the particles. Optimal noise that increases the collective motion is helpful in developing and understanding migration and navigation strategies in moveable or non-moveable heterogeneous media, which help to understand evolution and adaptation of stochastic components in natural systems which show collective motion.

2. Methodology

A two dimensional continuum time model is introduced for N_b self-propelled Particles (SPPs). The model demonstrates the effects of various parameters on the collective behaviour of the SPPs. Motion of the particles is restricted to the two dimensional box of size L having periodic boundary conditions. Particles interact with each other according to the same rule as in Vicsek et al [10] where particle follows the average direction of the neighbours within the radius r .Spatial heterogeneity is given by the presence of the N_o fixed obstacles. The new element in the equation of motion of self-propelled particles is introduced by the obstacle avoidance interaction as it is given in Ref. [5]. Obstacles are randomly distributed in the system. Noise parameter is also introduced in the system, which is randomly given and has values between $[-\pi,\pi]$. At the initial time-step each particle has a random position and a random direction. Particles update their positions as follows:

$$\mathbf{x}_i(t+\Delta t)=\mathbf{x}_i(t)+\mathbf{v}_i(t)\Delta t, \qquad (1)$$

and direction of the particle is given by the following equation:

$$\theta_i(t+\Delta t)=\langle\theta(t)\rangle_r+h(\mathbf{x}_i)+\eta\Delta\theta \qquad (2)$$

In equation 1, $\mathbf{x}_i$ represents the position of ith particle, $\mathbf{v}_i(t)$ is the velocity of the particle with absolute velocity v_o . Δt is the time interval that particles take to move from one point to another.
In equation 2, θ_i represents the direction of the particle and $\Delta\theta$ is the random fluctuation in the system, which is created by noiseand has value in the range of $[-\pi,\pi]$. η is the noise amplitude. $\langle\theta(t)\rangle_r$ represents the average direction of the particles which is within the interaction radius r , where r is the radius of interaction between the self-propelled particles. $\langle\theta(t)\rangle_r$ is given in the following equation:

$$\langle\theta(t)\rangle_r=\arctan\{\langle\sin(\theta)\rangle_r/\langle\cos(\theta)\rangle_r\} \qquad (3)$$

Equation 3 has been taken from [10].The function $h(\mathbf{x}_i)$ in equation 2 defines the interaction of particle with obstacles. Through this function particles avoid the obstacles that are located in its neighborhood:

$$h(\mathbf{x}_i)=\begin{cases}\dfrac{\gamma_0}{n_o(\mathbf{x}_i)}\sum_{|\mathbf{x}_i-\mathbf{y}_k|<R_o}\sin(\alpha_{k,i}-\theta_i), & if\ n_o(\mathbf{x}_i)>0\\ 0, & if\ n_o(\mathbf{x}_i)=0.\end{cases} \qquad (4)$$

The equation 4 is taken from [5]. Through this equation obstacle is introduced. Here $\mathbf{x}_i$ is the position of ith particle, and $\mathbf{y}_k$ is the position of kth obstacle. R_o is known as the interaction radius between particle and obstacle. $n_o(\mathbf{x}_i)$ represents the number of the obstacles located at the distance less than R_o from $\mathbf{x}_i$. In the above equation two conditions are given. Firstly, if $n_o(\mathbf{x}_i)$ is greater than zero, $h(\mathbf{x}_i)$ will show interaction with obstacles, secondly, if $n_o(\mathbf{x}_i)$ is equal to zero, $h(\mathbf{x}_i)$ will be zero. In equation 4, the term $|\mathbf{x}_i-\mathbf{y}_k|<R_o$, means that if the distance between the obstacle and particle is less than the interaction radius R_o , then the summation of sine

$\left(\sum \sin\left(\alpha_{k,i} - \theta_i\right)\right)$ will take place. Number of sine values that will be summed, depends on the number of obstacles $n_o(\mathbf{x}_i)$ that are located in the interaction radius range of the particle $\mathbf{x}_i$. The term $\alpha_{k,i}$ shows the angle in polar coordinates of the vector $\mathbf{x}_i - \mathbf{y}_k$. Also the parameter γ_o which is for interaction purpose, known as the particle's turning speed when it interacts with obstacle.

2.1 Order parameter

The order parameter, w, is used to characterize the macroscopic collective movement of the particles [5].

$$w = \langle w(t) \rangle_t = \left\langle \left| \frac{1}{N_b} \sum_{i=1}^{N_b} e^{i\theta_i(t)} \right| \right\rangle_t \qquad (5)$$

Here $\langle \ \rangle$ demonstrates temporal average. Equation 5 is a unitary complex number. There appear two possibilities, value of the order parameter can be either near to zero or near to 1. If the value is near to zero, we interpret it as the disordered motion. If the value is near to 1, we say collective motion is in state of order. This is the tool by which we scale the collective motion of SPP. The equation (5) is also termed as the average normalised velocity. The obstacle density ρ_o can be interpreted by using the following equation:

$$\rho_o = N_o / L^2, \qquad (6)$$

here N_o is the number of obstacles, and L is the box length. The description of the parameters that were used in the model is given in the Table 1.

Table 1: Parameters used in the simulation.

Symbol	Description
L	Length of box
N_b	Number of particles
N_o	Number of obstacles
t	Time step
η	Noise amplitude
R_o	Interaction radius between the particle and the obstacles
r	Interaction radius between the particles
v_o	Absolute velocity
γ_o	Particle's turning speed when it interact with obstacle
Δt	Time interval
w	Collective motion parameter

3. Results and Discussion

Simulation was performed in a square box of length L. We first considered the case in which noise was, $\eta = 0.01$.At initial time steps particles moved randomly with constant speed. After that each particle adopted an average direction of the particles which were in its neighborhood. Particles interacted with the obstacles and they turned away from the obstacles when they came closer to obstacle. There was no any repulsion force given to the particles. They showed alignment when they were close to each other. It was observed that at lower noise particles make groups. There was strong coordination in the particles as shown in the Figure 1. Strong coordination means particles had higher interaction with each other. Phase exhibited by the system is known as the clustered phase. At the same noise value, clustered phase was also observed in other work [5]. If we compare the order parameter in our model and in Ref [5] we found that our model exhibits more collective motion.

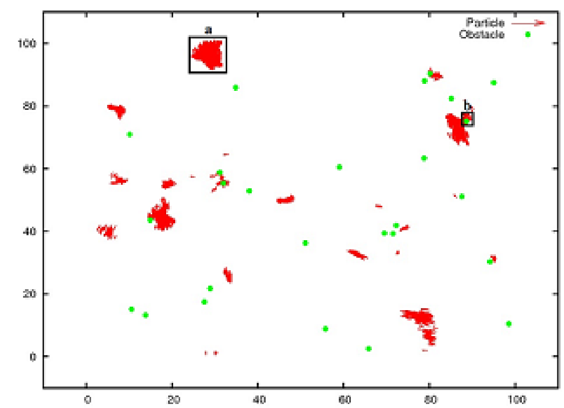

Fig. 1 Collective motion of the particles in groups. Box length $L = 100$, noise amplitude $\eta = 0.01$, time $t = 10000$, particles $N_b = 10000$, obstacles $N_o = 26$, interaction radius $r = 1$, avoidance radius $R_o = 1$, speed $v_o = 1$, particle's turning speed $\gamma_o = 1$, time interval $\Delta t = 0.1$. The crops of areas “a”and “b” are shown in the Figure 2.

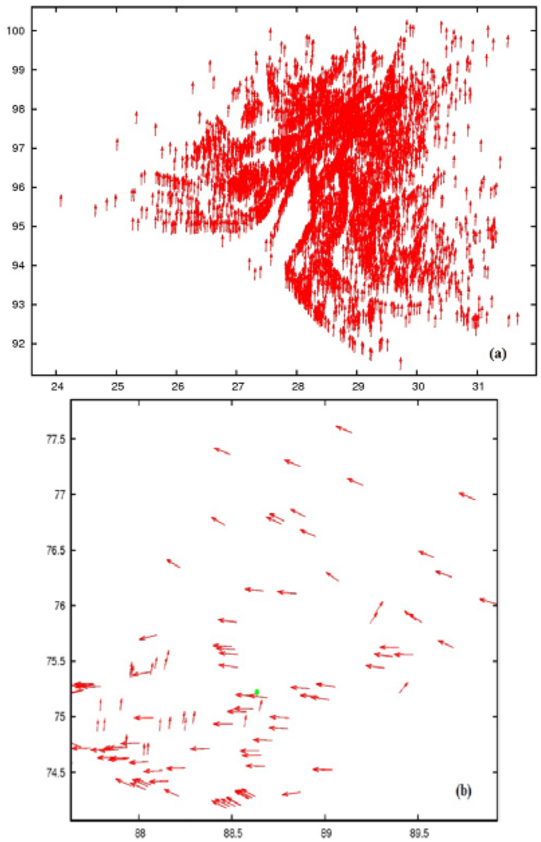

Fig. 2 Close-up from areas “a”and “b” in the Figure 1.

In the Figure 1 green circles show obstacles, whereas red arrows represent particles. Collective motion exhibited by the particles was calculated to be 0.65. This figure shows the result of the movement of the particles at 10000th time step. In the system dense clusters were formed. We can see that when particles moved closer to the obstacles they tried to avoid from the obstacle. At initial time steps particles had random motion, after some time-steps they started developing coordination with each other. When particles collide or near to the obstacles they scatter and their collective motion is disturbed. After the scattering they again tried to move together. Interaction between the particles followed the rule of interaction as shown in the Vicsek model [10] where velocities of the particles were summed when they were in the interaction radius.

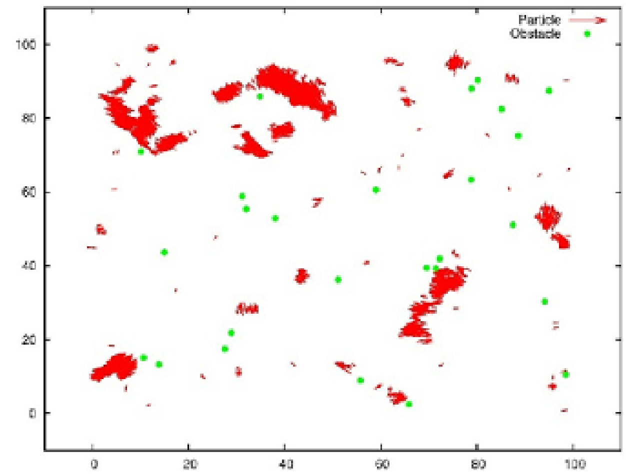

Fig. 3 Increasing noise from 0.01 to 0.3.L=100, t =10000, Nb =10000, N0 = 26, r =1, R0, = v0=1, $\gamma_o = 1, \Delta t = 0.1$.

The Figure 3 demonstrates the effect of noise when it was increased from 0.01 to 0.3. It was observed that there appears a slight randomness in the orientation. It can be clearly seen that particles formed some smaller clusters which were the result of the increase in the noise. Here each cluster has different direction and the collective motion of the particles is decreased, which shows that there is an effect of the noise on the system. This can be easily seen by comparing the Figures 1 and 3. It was obtained that the value of order parameter wasw=0.65 for $\eta = 0.01$ while it wasw=0.22 for $\eta = 0.3$.

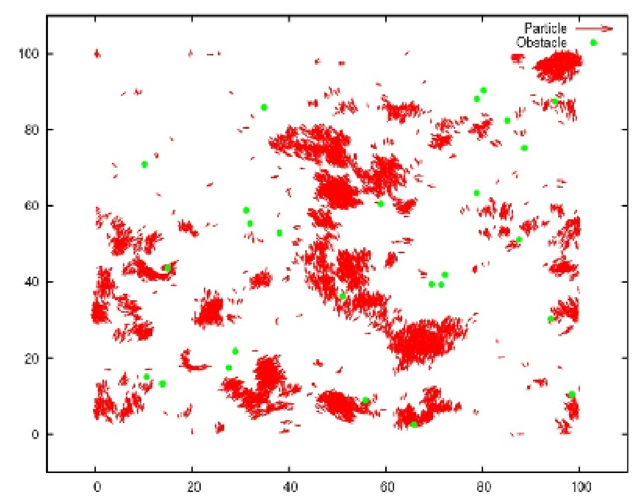

Fig. 4 Segregation of the particles. L=100, t =10000, Nb =10000, N0 = 26, r =1, R0, = v0=1, $\gamma_o = 1, \Delta t = 0.1, \eta = 0.6$

By increasing the noise to 0.6, system showed that particles scatter at larger scale (see Figure 4). There was more cluster formation. By applying higher noise the system exhibited an interesting behaviour as the collective motion reachedto w = 0.77, which was higher than the result of the previous two noise values. This higher value is attributed to the random distribution of the obstacles. This is contradictory to the fact when particles move in the homogeneous medium where increasing noise results inthe decline of the collective motion of the particles [6]. It is in good agreement with the results obtained in Ref. [5].

3.1 Effect of parameters

The model that has been introduced in this work has some parameters which are described in the Table 1. In this study we have investigated the effect of three parameters on the collective motion of the self-propelled particles. These three parameters are the interaction radius r, speed of the particles v_o and noise η. Table 2 shows the values of the parameters that were used in the simulation.

Table 2: Parameters values used in the calculation

Parameter	Value
L	40
N_b	1000
N_o	20
t	2000
η	0
R_o	1
r	1
v_o	1
γ_o	10
Δt	0.1

3.1.1 Effect of the Interaction radius

The interaction radius is the distance at which particles contact with each other. Each particle in the system had the same interaction radius. The larger value of the interaction radius encouraged the collective motion in the system. In Figure 5 the collective motion as a function of interaction radius is plotted for the system where obstacle density was $\rho_o = 0$ (circles)and $\rho_o = 0.0125$ (triangles).

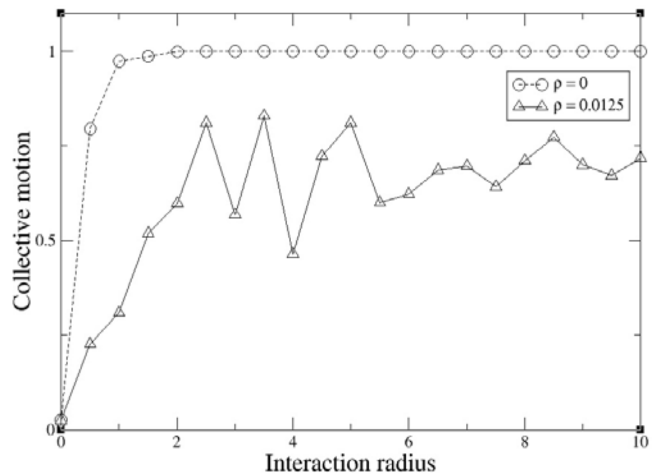

Fig. 5 Collective motion as a function of the interaction radius r.

The interaction radiuswas varied from 0 to 10 with an interval of 0.5. It was observed that particles show higher coordination with each other when radius increases. This coordination among particles made the system stable.For $\rho_o = 0$, it can be clearly seen that at the value of r equal to zero, the system is completely in disordered state, there was no emergence of the collective motion of the particles in the system. Increasing the radius of the particles makes the system more stable, because particles move collectively with proper coordination without any hindrance.From $r = 2$, order parameter had gained a very consistent value which was equal to 0.99. This value was the evidence of the stable system. When there was a presence of obstacles in the system, at $\rho_o = 0.0125$, the collective motion was smaller than the previous case of $\rho_o = 0$. Despite of the obstacle's existence, particles showed collective motion andit never went to zero. Fluctuation of the collective motion as a function of interaction radius was due to the number of the particles used in the calculation was not so large. We believe that if we increase the number of particles in the system, fluctuation will occur at smaller scale.

3.1.2 Effect of the speed

In the model each particle carried a constant speed. Speed parameter has a significant effect on the collective behaviour of the particles. At zero speed, particles come into the static positon and there is no any movement in the system. The Figure 6 demonstrates the collective motion as a function of speed for obstacle density $\rho_o = 0$ (circles) and $\rho_o = 0.0125$, (triangles).

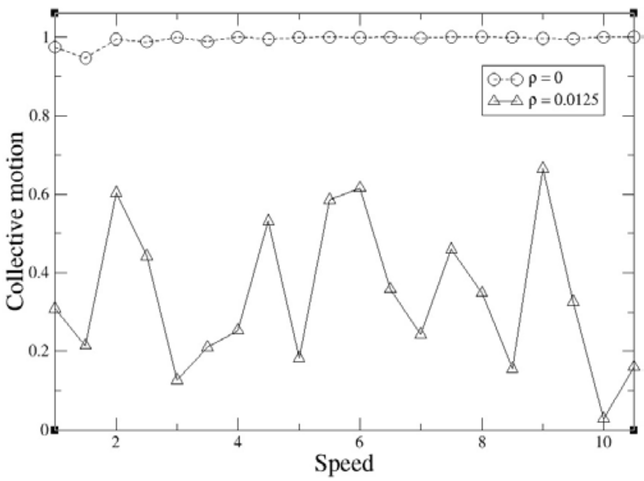

Figure 6:Collective motion as a function of the speed for obstacle density, for $\rho_o = 0$ and $\rho_o = 0.0125$ (20 obstacles).

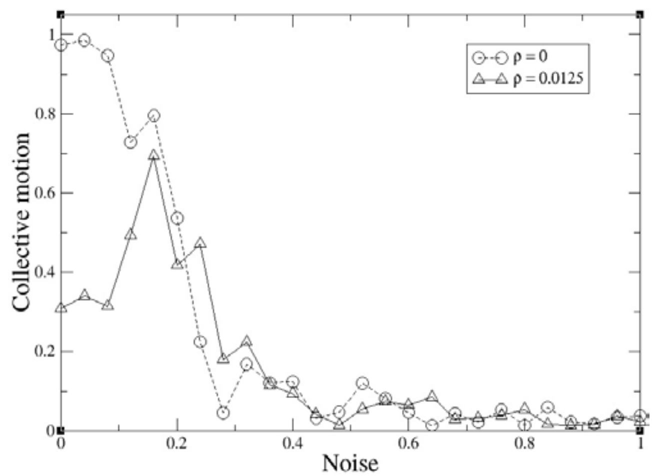

Figure 7:Collective motion as a function of the noise for two values of obstacle density, $\rho_o = 0$ and $\rho_o = 0.0125$.

In the Figure 6, for $\rho_o = 0$ (circles) line demonstrates the results for zero obstacle density. Particles moved faster when higher value of speed parameter was provided. At initial values of v_o, system showed some fluctuations, from $v_o = 3$ collective motion had consistent value which remained near to 1. It can be clearly seen that by increasing the speed parameter the system showed long range order, particles gained more coordination quickly, as a result the system became stable. There was no any hindrance in the movement of the particles because there was no any obstacle present in the system. In the absence of noise and obstacles, particles moved freely and they show ordered phase. In the case of $\rho_o = 0.0125$ order parameter,w, showed a non-monotonic behaviour because there appeared large fluctuations in the system. This happened because obstacles were randomly distributed in the system. Collective motion of the self-propelled particles was hugely distributed and the system was completely in a disordered state.

3.1.3 Effect of noise

Noise effect wasinvestigated for both homogeneous and heterogeneous systems. Order parameter, w is plotted against noise values in the homogeneous medium where obstacle density is $\rho_o = 0$ and in heterogeneous medium where obstacle density is $\rho_o = 0.0125$. Noise value was chosen from the range $[-\pi, \pi]$, by using uniform probability distribution.

The Figure 7 demonstrates the effect of the noise on the collective motion of the self-propelled particels.Noise amplitude had been varied from the 0 to 1 with an interval length of 0.04. In the first case where $\rho_o = 0$ (circles line) we see there appears huge randomness in the system. Order parameter reached to zero when higher value of noise was appiled. From the above result we see that at lower nosie values, system is in a state of order because collective motion has value near to 1. When the noise was increased, the system started to showthe disordered phase.This can be clearly seen that when the noise is from 0.48 to 1, system showed collective motion approaches to zero.

In the case of $\rho_o = 0.0125$ (triangle line), it can be clearly seen that at the noise value 0.16, order parameter wreached to the maximum. At the starting value of the noise such as $\eta = 0$ collective motion had smaller value than at $\eta = 0.16$. Due to the random distribution of the obstacles, there existed an optimal noise which maximised the collective motion of the self-propelled particles. Such type of behaviour does not exist in the homogeneous medium. It was also observed that with the increase of the noise, there was decrease in order parameter. System was completely in the state of disorder when noise is larger than 0.4.

4. Conclusions

Collective behaviour of self-propelled particles was investigated for both heterogeneous and homogeneous medium. We investigated the effects of the interaction radius, speed and noise on the collective motion of the self-propelled particles. It was shown that in the homogeneous medium the order parameter gains larger values when the interaction radius and speed are increased, whereas in the case of noise we found that noise has caused fluctuations in the order parameter. In the case of heterogeneous medium large fluctuations take place when the interaction radius of the particles is small.By increasingthe interaction radius the fluctuation is getting smaller. Furthermore, in heterogeneous medium, the variation of noisecausesthe collective motion to behave in a non-monotonic manner. This is because of the randomly distribution of the obstacles in the system. It is observed that the collective motion is always less in the case of the presence of obstacles. It is also observed that there exists an optimal noise which maximises the collective motion of the self-propelled particles. At noise =0.16, the order parameter has reached its maximum value.

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ISRAR AHMED
PhD Applied Mathematics
University of Central Lancashire, Preston, United Kingdom
Research interest
Numerical solutions of PDE's
Mathematical Modelling of soft and active matters, Number Theory and Geometry
Computational Mathematics
Current position.
Assistant Professor Department of Mathematics Shah Abdul Latif University Khairpur

Inayatullah Soomro
PhD Applied Mathematics
University of Central Lancashire, Preston, United Kingdom
Research interest
Numerical solutions of PDE's
Mathematical Modelling
Computational Mathematics
Current position.
Assistant Professor Department of Mathematics Shah Abdul Latif University Khairpur

SYED BAQER SHAH
PhD Applied Mathematics
Shah Abdul Latif University Khairpur
Research interest
Numerical solutions of PDE's
Mathematical Modelling
Computational Fluid
Current Position
Associate Professor,
Department of Mathematics
Shah Abdul Latif University Khairpur

HISAMUDDIN SHAIKH
PhD Applied Mathematics
Shah Abdul Latif University Khairpur
Research interest
Numerical solutions of PDE's
Mathematical Modelling
Computational Fluid
Current Position
Associate Professor,
Chairman, Department of Mathematics
Shah Abdul Latif University Khairpur