Modeling Political and Economic Relations between Russia and Turkey: A Game Theory Approach

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Summary

Analysis of political and economic relations between the two countries has always been one of the concerns of elites and analysts of both countries as well as third countries. Game theory is a powerful tool for analyzing international relations and achieving the desired goals. In this article, by using dynamic system of strategic games, which is a new modeling approach in game theory, political and economic relations of the two countries, Russia and Turkey, are examined over time and in seven stages, since Cold War era up to now.

Key words:

Game theory, 2×2 games, Nash equilibrium, Dynamic system of strategic games.

1. Introduction

Relations between countries are affected by different factors including geopolitical position, national benefits, leaders' ideology, regional conditions and international system structure. The international system includes a set of governments or international players that regularly have interactions with each other and as a result, they partially have a mutual dependency on each other [18]. Models that are distinguishable from these relations and processes include power model (single power, double powers, multi-powers), politics model (revolutionary or moderate), level and model of conflict, level and model of classification (polarization level), and level of cooperation or development of international systems. In the recent years, the international conflicts have become highly increased resulting in dangerous political games [4]. Countries prefer to avoid military confrontation and solve problems through peaceful negotiations [9] and [10]. Since the conditions and activities of every player (country) influence other players (other countries), cooperation is useful for a country when other countries also follow such a strategy. For this purpose, the game theory is used to describe strategic interactions [12]. Fundamental concepts of game theory were suggested by Von Neumann in 1944 [20]. Based on the strategies in a special game like chess, Neumann could model the actions between two countries

of US and Soviet Union during Cold War, considering them as two players in a zero sum game. Then, development of this knowledge was followed more rapidly in various fields.

Some conflicts between players have the aspects which cannot be modeled by static models. Thus, it is necessary the use of dynamic models in which there are limitations and pressures over time on players resulting in changes of strategies and games between them, [2] and [6]. Most studies have been done on the components of the game and on the modeling issues through games. The study of games can be classified into two groups. The first class relies on the studies that speak about simple games with a few players, some few possible actions and a way of modeling the structure by only one game. The second class examines the relationships between games, [13] and [14]. There are few studies that examine the relations between games [1], [9], [10] and [15]. Among them, Meta games and topology of 2×2 games [8] have been considered. In the 2×2 game topology, the games are classified in a systematic way according to their topologic closeness. Then, by using reflections, rotations and transitions reach from one game to another one. Finally, the result of the modeling issues is applied in practical problems [16]. In Meta games (or games about the games), a game is selected initially and, using Meta strategies of n type, Meta games are developed while this structure is used to analyze the game, [8]. In fact, Meta games are approaches to a game that transcend, or operate activations, outside the prescribed rules of the game. In this regard, relations between Russia and Turkey since the emergence of the Republic of Turkey in 1923 up to now have seen many ups and downs in their evolution and development. Russia is the eighth greatest economy of the world having most of the raw energy resources of the world, including gas, and the greatest producer of Non-OPEC oil and the most spacious country of the world that throughout the history always has intended to play an effective role in the world and its surrounding region as one of the most powerful countries of the world. Turkey also as an inheritor of the Ottoman Empire and the only Muslim member of NATO. It is a country that has a

special strategic geographical Asian-European position because of its position in one of the most sensitive regions of the world and, having the two strategic straits of Bosporus and Dardanelles, it is one of the most important goods and energy transit pathway in the world. Over time, several factors have influenced Moscow-Ankara relations. Turkey has an important and inseparable part in Russia's foreign policy and Russia also plays an important role in Turkey's foreign policy. Ankara and Moscow's political officials consider the improvement of relationships and developing mutual dependency based on joint benefits as useful for both parties' long-term benefits. Hence, the development of the two countries is more a function of long-term regional strategies than merely a short-term tactic and tool in a foreign policy context. The study of the two countries' relationships illustrates that the two countries' relations during years has fluctuated from tension and conflict to cooperation and mutual dependency. So, the analysis of the two countries' relations has a particular importance and complexity. In this article, the relations of the two countries are examined using a new system named Dynamic system of strategic games.

2. Preliminary

Dynamic system of strategic games is a model to examine interaction between decision makers more exactly. Every decision maker is a player in this model. To describe this system, we use the games graph. In each node of the graph, there is a strategic game with perfect information in which players can make decisions. Each node of this graph can be the generator of the next game through the two following methods and be related to it:

- strategies,
- pairs of rational actions.

Players proceed to move from one node to another node by selecting the strategy or pair of rational actions. Graph **G** is binary of (G,M) in which the first coordinate, $G = (g_1, g_2, .., g_n)$, is a finite set of nodes each being a strategic game. The second coordinate M is a finite set named edges which are produced strategies or pair of rational actions.

The set of all strategies produced by k-th game is represented by $_kS =_k S_1 \cup_k S_2 \cup \phi$. The Set of all pairs of players' actions in k-th game is shown with $_kA =_k A_1 \cup_k A_2$. Set of all pairs of rational actions for player i is shown with $_kA_i'$, that is a subset of $_kA$, for all $k \in \{1,2,...,n\}$.

Let $A =_1 A \cup_2 A... \cup_n A \cup \phi$ and $S =_1 S \cup_2 S... \cup_n S$ be two sets. The set valued functions, rational actions pair $\varphi_i' : G \to A$ and the strategy maker $\varphi_i : G \to S$ for players are defined as follows:

$$\varphi_i'(g_k) =_k A_i' = \begin{cases} \{(_k a_i,_k a_{-i})_i | (_k a_i,_k a_{-i})_i \in_k A\} \\ \phi \end{cases}$$

and

$$\varphi_i(g_k) =_k S_i = \begin{cases} \{_k S_i^j |_k S_i^j \in_k S\} \\ \phi \end{cases}$$

for all $i \in N$ and $j,k \in I = \{1,2,...,n\}$, where g_k shows the k-th game, $(_k a_i,_k a_{-i})_i$ shows the rational actions pair of the i-th player from k-th game and $_k S_i^j$ shows the j-th strategy of the i-th player from k-th game [4]. Every move of system as a member of set M is as follows:

$$M = \begin{cases} m_k^j \Big| m_k^j =_k S_i^j \ or \ m_k^j = (_k a_i,_k a_{-i})_i \ or \ m_k^j = (_k a_i,_k a_{-i})_i \\ \quad or \ m_k^j = (_k a_i,_k a_{-i})_{i,j}, \\ , \forall_k S_i^j \in_k S_i, (_k a_i,_k a_{-i})_i \in_k A_i, (_k a_i,_k a_{-i})_j \in_k A_j, \end{cases}$$

where m_k^j shows the j-th move of the k-th game and $(_k a_i,_k a_{-i})_{i,j}$ shows the pair of rational actions selected by players i and j of the k-th game. Players' move function $\alpha_i : M \to G^2$ and $\alpha_{i,j} : M \to G^2 \cup \phi$, with $\alpha_{i,j}(_k S_i^j) = \phi$, is defined as follows:

$$\alpha_i(m_k^j) = \begin{cases} (g_k, g_p) = g_k g_p \ if \ m_k^j =_k S_i^j \\ (g_k, g_q) = g_k g_q \ if \ m_k^j = (_k a_i,_k a_{-i})_i, \end{cases}$$

and $\alpha_{i,j}(m_k^j) = \begin{cases} (g_k, g_p) = \phi \quad if \ m_k^j =_k S_i^j \\ (g_k, g_s) = g_k g_s \ if \ m_k^j = (_k a_i,_k a_{-i})_{i,j}. \end{cases}$

The above functions show what move connects two play nodes to each other by either one or both players. Consequently, it can be said that in move $m_k^j =_k S_i^j$, the nodes g_k and g_p have been connected through the strategy selected by player i to each other. In

move $m_k^j = (_k a_{i,k} a_{-i})_i$ the nodes g_k and g_q have been connected by pair of rational action selected by player i to each other. In the move $m_k^j = (_k a_{i,k} a_{-i})_{i,j}$ the nodes g_k and g_s have been connected through a pair of rational actions selected by players i and j to each other [4]. Consider that H is a set including all series (finite and infinite) that hold true in the following conditions:

1. $\phi \in H$,

2. Sequence $\{ m_i^j, \{ g_k, m_k^j \} \}_{i,j,k \in I}$ for all $i,j,k \in I = \{1,2,...,n\}$, is a member of H. Each member of H is called a history and is represented by h,

3. History $h = \{ m_i^j, \{ g_k, m_k^j \} \}_{i,j,k \in I}$ is called final history if it is infinite or there isn't g_{k+1} that is a member of h.

The set H is called System History. Preferences of each node of a game system that are exactly the same preferences on the pairs of strategic game actions are called node preferences or tactical preferences. Preferences on strategies set or set of rational actions pair of a game are called systemic preferences or strategic preferences [4].

2.1 Example

Suppose two countries' relations have entered Self-Serving game g_1. For more accurate expression, the row player is called player 1 and column player is called player 2. Each player has two actions. Players can choose cooperation action $_1C$ or defect $_1D$. Players obtain a payoff by choosing each action as shown in Fig. 1. Game g_1 is producer of Stag Hunt game g_2 and Prisoners Dilemma g_3. Using dynamic system of games, we model relations between two countries. The game Nash equilibrium is $(_1C,_1D)$. In this game, dominant strategy $_1S_2^2$ for player 2 is defect and dominated strategy $_1S_2^1$ is cooperation. Considering dominant strategy $_1S_2^2$ player 2, produces responses of player 1 to this strategy pair of rational actions. So, pairs of rational actions for player 1 is $(_1C,_1D)_1$ and $(_1D,_1D)_1$. Game g_1 isn't strategy maker for player 1. Thus, game g_1 is strategy maker of order (2,1). Based on players being rational, player 1 chooses pair of rational actions $(_1C,_1D)_1$ and player 2 chooses dominant strategy $_1S_2^2$ to continue the game. In summary, strategies, Players' move function and pair of rational actions of game g_1 are as follows:

$\varphi_1(g_1) =_1 S_1 = \phi,$

$\varphi_2(g_1) =_1 S_2 = \{ _1S_2^2 \},$

$\varphi_1'(g_1) =_1 A_1' = \{ (_1C,_1D)_1, (_1D,_1D)_1 \}$
$\qquad = \{ (C,D),(D,D) \} = \{ (3,4),(2,2) \},$

$\varphi_2'(g_1) =_1 A_2' = \{ (_1C,_1D)_2 \} = \{ (3,4) \}.$

Pair of rational actions $(_1C,_1D)_1$ in game g_1 for player 1 ends to Stag Hunt game g_2. The game Nash equilibria are $(_2C,_2D)$ and $(_2D,_2D)$. Game g_2 isn't strategy maker for players. Therefore, g_2 is strategy maker of order (2,0). The players' pairs of rational actions are $(_2C,_2D)_{1,2}$ and $(_2D,_2D)_{1,2}$. So each player can select one of the pairs of rational actions to continue the game. Thus

$\varphi_1(g_2) =_2 S_1 = \phi$

$\varphi_2(g_2) =_2 S_2 = \phi$

$\varphi_1'(g_2) =_2 A_1' = \{ (_2C,_2C)_1, (_2D,_2D)_1 \} = \{ (4,4),(2,2) \}$

$\varphi_2'(g_2) =_2 A_2' = \{ (_2C,_2C)_2, (_2D,_2D)_2 \} = \{ (4,4),(2,2) \}.$

Strategy $_1S_2^2$ for player 2 in game g_1 ends to Prisoner's Dilemma g_3. In g_3 the dominant strategy $_3S_i^2$, is defect and dominated strategy $_3S_i^1$, is cooperation for two players. In other words, the Prisoner's Dilemma is a strategy maker game of order (2,2). Nash equilibrium of the game g_3 is $(_3D,_3D)$. Pair of actions $(_3C,_3C)$ is dominant Pareto compared to pair of actions $(_3D,_3D)$. So pairs of rational actions are $(_3C,_3C)_{1,2}$ and $(_3D,_3D)_{1,2}$. We have

$\varphi_1(g_3) =_3 S_1 = \{ _3S_1^1, _3S_1^2)$

$\varphi_2(g_3) =_3 S_2 = \{ _3S_2^1, _3S_2^2 \}$

$\varphi_1'(g_3) =_3 A_1' = \{ (_3C,_3C)_1, (_3D,_3D)_1 \} = \{ (3,3),(2,2) \}$

$\varphi_2'(g_3) =_3 A_2' = \{ (_3C,_3C)_2, (_3D,_3D)_2 \} = \{ (3,3),(2,2) \}.$

Functions of the game move are as follows:

$\alpha_{1,2}(m_1^2) = \alpha_{1,2}((_1C,_1D)_{1,2}) = g_1 g_2$

$\alpha_2(m_1^1) = \alpha_2(_1S_2^2) = g_1 g_3.$

The system history is as follows:

$$H = \left\{ \begin{array}{l} \phi, \{ g_1, (_1C,_1D)_{1,2}, _1S_2^2 \} \{ (_1C,_1D)_{1,2}, \{ g_2 \} \} \\ , \{ _1S_2^2, \{ g_3 \} \} \end{array} \right\}$$
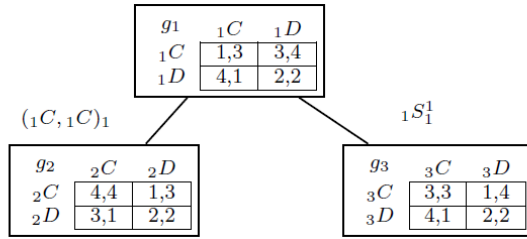
Fig. 1 Dynamic system of games for Self- Serving game g_1.

3. Modeling political relations

3.1 Cold war era

Throughout the history, Turkey and Russia relationships always have had ups and downs so that mainly, a cool space has governed on the two countries' relations until the collapse of Soviet. During the cold war, Russia was treated as a geopolitical rival and always was considered as a security concerns of Turkey's ruling elites. Often during this time, the two countries following their own benefit, particularly in regional level, were in conflict and disagreement. As a result, the two countries entered total conflict game and didn't have much tendency to cooperation and preferred non-cooperation. These variations are demonstrated in game g_1 (Fig. 2). Turkey is assumed as row player (player 1) and Russia is assumed column player (player 2). The players' set of actions include cooperation $_1C$ and defect $_1D$. Players' preferences in this node are the same order preferences of strategic game g_1. This game is producer of dominant strategy of defect $_1S_1^2$ and dominated strategy of cooperation $_1S_1^1$ for player 1 and also dominant strategy $_1S_2^2$ for player 2 is defect and dominated strategy $_1S_2^1$ is cooperation. Therefore g_1 is strategy maker of order (2, 1). The game Nash equilibrium is $(_1D,_1D)$. The only pair of rational actions for both players is $(_1D,_1D)_{1,2}$.

3.2 After Soviet collapse (1990s and early 2000)

After the collapse of the Soviet Union, a new season in Russia and Turkey relationships was started. These developments led to a reduction in the role and influence of Russia in the World (especially in central Asia and Caucasus). Thus, Russia wasn't considered like the past as a threat to security and territorial integrity of Turkey. As a result of these changes, Ankara-Moscow relations saw an ascendant trend and the two countries while having conflicting interests desired to expand and deepen mutual relations and extend strategic cooperation. The political relations between the two countries began with the visit of Turkish Foreign Minister "Hikmet Cetin" to Moscow on 20 January 1992. A month later, Russian Foreign Minister "Andrei Kozyrev" traveled to Turkey for a political meeting. Then, Turkish Prime Minister "Slüeyman Demirel", has visited Russia after a few months. During this trip, the general lines document of Russia and Turkey relations was signed. According to this document, the two countries signed more than 20 documents in economic, technological, scientific, cultural, health and tourism areas from 1992 to 1996 .

Turkey following its cooperation strategy intended to utilize and revive its lost role in improving relationships with Russia. Successive trips of Turkey officials to Turkish language republics of Central Asia and Caucasus, holding the first meeting of Turkish countries in Ankara (in 1992) and the third summit of Azerbaijan, Turkmenistan, Kyrgyzstan and Uzbekistan republics along with Turkey in Kyrgyzstan (in 1995) resulted in sensitivities for Moscow. Russia considered these actions of Ankara as an attempt to promote pan-Turkism thoughts in central Asia and Caucasus region. Boris Yeltsin Russia' President in response to Turkish language meeting in Bishkek constituted the Parliament in exile of Turkey's Kurds in Moscow (in November 1, 1995). These actions cause tension in relations between two countries. Generally, throughout the 1990s, Russia and Turkey have experienced tense relationships, sometimes cooperation and sometimes defect.

Turkey's non-cooperation dominant strategy $_1S_1^2$ in game g_1 results in Battle of the Sexes game g_2. Turkey in this game has two actions, either trusts in Russia and develops cooperation and increases its role playing in the region $_2C$ or goes ahead to unilateralism and non-cooperation $_2D$ in order to increase its regional benefits. In return, Russia also has two actions in this game. It either goes ahead to extend relationships and benefits coordination with Turkey $_2C$ or experiences defect $_2D$. The game Nash equilibria are $(_2C,_2D)$ and $(_2D,_2C)$.

Game g_2 isn't strategy maker for players. In other words, g_2 is strategy maker of order (2, 0). The players' pairs of rational actions are $(_2C,_2D)_{1,2}$ and $(_2D,_2C)_{1,2}$.

Russia's $_1S_2^2$ strategy in game g_1 results in Self- Serving game g_3. In this game, Russia has two actions, either by trusting in Turkey go ahead to increase joint benefits $_3C$ or exploiting the appeared opportunity merely to revive its role in the region and increase national and regional benefits $_3D$. Turkey also has two actions in this game. It

either leaves tension-making actions make confidence $_3C$ or continues to its tension-making policies $_3D$. The game Nash equilibrium is $(_3C,_3D)$. In g_3, dominant strategy $_3S_2^2$ is defect and dominated strategy $_3S_2^1$ is cooperation for Russia. But this game isn't strategy maker for Turkey. Thus, g_3 is strategy maker of order (2, 1) The only pair of rational actions is $(_3C,_3D)_1$.

3.3 Toward cooperation

In late 1990s and early 2000s, the two countries took fundamental steps to improve relationships by leaving tension-making policies. After justice and development party's coming to power and following up the foreign policy of looking to east of this party, the parties' relationships developed dramatically [7]. During this period, political and economic changes in internal arenas resulted in change of activities and review of the past foreign policy. The strategic plan of Ahmet Davutoğlu, Turkey's foreign minister, regarding zero tension with neighbors became the driving force to develop Turkey's relations with the most of the neighbor countries. On the other hand, strategic depth theory also suggested by Ahmet Davutoğlu was based on increasing Turkey's influence in the region, particularly central Asian countries that had cultural, linguistic, religious and ethnical points of similarity with Turkey. Over this time and specially in direct presence of West in the region and Iraq and reduction of Turkey's strategic role, Ankara made extensive attempts based on the "look to the east" policy, to redefine its historical and geographical role, play a bold regional presence and become a bridge and focus of transferring energy from Russia and central Asian regions and Caucasus to Europe. In return, Russia also has taken many steps to cooperate with Ankara. In the process of improving relationships, Putin was the first president of Russia who visited Turkey in 2004 after 32 years.

During these years, the two countries' relations became so powerful that 15 agreements and 7 protocols were signed in 2009 following the Putin's trip to Turkey and Erdoğan called the nature of the two countries' relationships and cooperation as strategic. Dramatic growth of economic relationships over these years resulted in political cooperation development so that in, two countries constituted the High Cooperation Council in May 2010 during the visit of Medvedev by the then Russian President to Turkey and they consequently could confront with common problems [5]. Considering what was said, change of the two countries' behavior in the course of relationships improvement led to more tendency of Turkey to reinforce cooperation. The Mixed Harmony game, g_4 was produced by the same attitude. In this game,

Turkey have two actions either by changing attitude and taking step in trust making, the strategy of zero tension and reinforcing cooperation $_4C$ or continuing its policies and considering its own benefits $_4D$. Russia has two actions in the game g_4 in response to change of strategy and desire to Turkey's cooperation: either implementing cooperative policies and relationships development $_4C$ or doing non-cooperation $_4D$. Game g_4 is strategy maker of order (2,2). The game Nash equilibrium and pair of rational actions is $(_4C,_4C)_{1,2}$.

Dominant strategy $_3S_2^2$ in game g_3 ends to Stag Hunt game, g_5. In this game, Russia has two actions. It either looks to Turkey as a strategic partner and attempts to develop cooperation $_5C$ or still considers Turkey as a rival and makes non-collaboration $_5D$. In this game, Turkey also has two actions: either changing its policies in order to attract Russia's satisfaction and to develop collaborations $_5C$ or following its Pan Turkism policies and choosing non-cooperation $_5D$. g_5 is strategy maker of order (2,2). The game Nash equilibria and pairs of rational actions for players are $(_5C,_5C)_{1,2}$ and $(_5D,_5D)_{1,2}$.

3.4 Toward political tension

Turkey and Russia's cooperative procedure continued with a mild process until start of changes called Arab Spring. Russia showed a particular sensitivity to protect Assad Ba'athist government against a various spectrum of opponents due to historical long relations with Damascus and having a sea station in Tartus port. On the other hand, Turkey concentrated on eliminating Assad from authority and attempted several diplomatic and military strategies to operate out its approach based on its Brotherhood approach and for the purpose of preserving its regional superiority seeking. Disagreements between Russia and Turkey again started with suggestion of Syria Constitution draft by Russia in the Astana meeting and increasingly increased more than ever. The grounds for Russia and Syria disagreement don't end to just solution of Syria crisis. Fundamental disagreements in different political areas appeared between two countries among which we can refer to Turkey adjacency to West and Patriot missile deployment in Turkey, launching a part of America's missile defense shield site in Turkey's Corsican station, Turkey's influence over the countries of Caspian Sea, Russia's support of internal opposite groups of Turkey (such as Kurdistan Workers' party), Turkey's fear of reinforcement of its opponent countries including weapons sale to Cyprus, killing of about 9 Turkish soldiers in the

Russian action against ISIS. Turkey's behavior toward the arisen issues ended to Alibi game g_6. Turkey has two options in this game: either taking silence regarding the arisen changes or attempting to solve problems $_6C$ or deciding to confront with Russia and an opportunity to show itself $_6D$. On the contrary, Russia also has two options: either resolving the arisen disagreements $_6C$ or regardless of Turkey's concerns, continuing its policies regarding the region issues $_6D$. The game Nash equilibrium is $(_6D,_6D)$. g_6 is producer of dominant strategy of defect $_6S_2^2$ and dominated strategy of cooperation $_6S_2^1$ for player 2. This game isn't strategy maker for player 1. Thus, g_6 is strategy maker of order (2,1). Pair of actions $(_6C,_6C)$ is dominant Pareto compared to pair of actions $(_6D,_6D)$. So, pairs of rational actions for players are $(_6C,_6C)_{1,2}$ and $(_6D,_6D)_{1,2}$.

The pair of rational actions $(_5D,_5D)$ in game g_5 results in Bully game g_7. Russia has two options in this game: either considering Turkey's benefits and coordinating its regional policies with Turkey and abandoning supporting the groups opponent with Turkey and preventing from tension increase $_7C$ or only seeking its own benefits $_7D$ and continuing its policies. Turkey also has two options: either refusing to approach to West and insisting on disagreements that result in more tensions $_7C$ or choosing defect and tendency to West $_7D$. g_7 is strategy maker of order (2,1). Nash equilibrium of game and the only pair of rational action for both players is $(_7C,_7D)_{1,2}$.

3.5 Creation of a tragedy

Russia's dominant strategy $_7S_2^2$ in game g_7 and the players' pair of rational actions $(_6D,_6D)_{1,2}$ in game g_6 resulted in creation of a tragedy and Tragedy game, g_8. On November 24, 2015, some hours after Putin's one-day trip to Iran, Turkey overthrew one of the Russian fighters. This accident created a new situation in relationships of the two countries, Russia and Turkey. The two countries' relationships that mainly were based on economic transactions entered a new phase of political tension, such that Russia authorities threatened Ankara politically. In game g_8, Turkey has two options: either not retaliating Russia's behavior $_8C$ or attacking to Russia's fighter in response to Russia's behavior $_8D$. On the contrary, Russia

also has two actions: either ignoring Turkey's behavior $_8C$ or confronting seriously with it $_8D$.

In g_8, the dominant strategy $_8S_i^2$ is defect and the dominated strategy $_8S_i^1$ is cooperation for two players. Thus, g_8 is strategy maker of order (2,2). The pair of rational actions is $(_8D,_8D)_{1,2}$.

3.6 Cutting relationships

Russia president signed the Turkey boycott law called the law of "securing Russia national security and applying particular economic measures about Turkey" three days after this accident. Russia used economic relations with Turkey somehow as a hostage and stroke a heavy hit to Turkey's economy by applying the sanction. Besides the sanction, Russia made the condition that Ankara must apologize for the Russia's bomber overthrown in order to resume collaborations with Moscow. After some months, Erdoğan's first letter was sent to Putin on the occasion of Russia national day. Kremlin Palace announced that considering the letter being ceremonial and not being subject of any of the clauses set by Moscow, this letter didn't need any answer. Ten days after submission of the first letter, Turkey sent the second letter to Russian authorities. Russia accepted this letter as a formal apology letter from Turkey. Turkey's behavior to rebuild relationships resulted in Assurance game, g_9. In this game, Turkey has two options: either apologizing Russia $_9C$ and reconstructing relationships or not apologizing and continuing non-collaboration $_9D$. Russia also has two options in this game: accepting Turkey's apology $_9C$ or not accepting it $_9D$. Game g_9 isn't a strategy maker for any of the players. Pairs of rational actions for players are $(_9C,_9C)_{1,2}$ and $(_9D,_9D)_{1,2}$.

Russia's dominant strategy of defect $_8S_2^2$ in game g_8 ends in Hostage game, g_{10}. In this game, Russia have two options: either not boycotting Turkey $_{10}C$ and not setting a condition for relationships normalization or using economic leverage and boycotting Turkey $_{10}D$. On the contrary, Turkey also has two actions: either cooperating $_{10}C$ and apologizing to Russia or showing indifference and defect $_{10}D$. The game Nash equilibrium is $(_{10}C,_{10}D)$. Game g_{10} is producer of dominant strategy of cooperation $_{10}S_1^1$ and dominated strategy of defect $_{10}S_1^2$ for player 1. Also, g_{10} is the producer of the dominant strategy of defect $_{10}S_2^2$ and dominated strategy of

cooperation $_{10}S_2^1$ for player 2. So, g_{10} is strategy maker of order (2,2). The pair of rational actions is $(_{10}C,_{10}D)_{1,2}$.

3.7 Reconstructing relations and future prospects

After Turkey's apology and Erdoğan's trip to Russia on August 8, 2016, he met Putin in Saint Petersburg. This historical visit improved relationships of the two countries. The issue of Syria political crisis, the Middle East circumstances and the role that Turkey can play among them, ensured Russians that they don't insist after Erdoğan's apology, go toward compromise and flexibility and normalize relationships.

But international circumstances show that there is still the probability of new controversy in Caucasus, Middle East, Black Sea and Mediterranean Sea. The experience of this crisis showed that European and Western countries particularly America direct the region accidents not based on Turkey's benefits but according to its own benefits. Also Russia found that role playing in the region was not possible without considering benefits of the other countries including Turkey. On the other hand, as the two countries economic system is dependent on each other, the interdependence increases the conflict costs.

Therefore, the two countries must attempt to consolidate relationships instead of creating tension. The pair of rational actions $(_9C,_9C)_{1,2}$ and $(_{10}C,_{10}D)_{1,2}$ of both players in games g_9 and g_{10} results in two Prisoner's Dilemma game g_{11}, respectively. In game g_{11}, both countries have two actions: either looking for relationships consolidation and cooperation $_{11}C$ or looking for their future benefits and defect $_{11}D$. Nash equilibrium of the game is $(_{11}D,_{11}D)_{1,2}$. In this game, dominant strategy for two players is defect and dominated strategy is cooperation. Thus, g_{11} is strategy maker of order (2,2).

Pair of actions $(_{11}C,_{11}C)_{1,2}$ is dominant Pareto compared to pair of actions $(_{11}D,_{11}D)_{1,2}$. This game shows that the two countries may in short term achieve little benefits through non-collaboration but in long term, it is beneficial for both countries that to abandon tension and to collaborate leads to obtaining more benefits. On the other hand, naturally countries always attempt to consider various choices for their foreign policy. No country limits its choice merely based on one scenario and one strategy. Particularly fluidity of the Middle East issues, very fast change of the regional and international players' situation, getting involved and alliances' formation,… all affect Turkey and Russia. On the other hand, the desire to find the fields of cooperation is greater. But obviously Russia and Turkey look for different choices, variety of foreign policy and increasing maneuver power and their role

making that obviously depends on changes process and the existing crisis course including Syria crisis and also the turbulent situation governing Middle East, both countries situation will change according to these circumstances and the course of events. In this regard, by looking to future of the two countries' relationships and future scenarios of Turkey and Russia's relationships, it can't be said that the two countries' relations and obviously it will not be possible to adjust these conditions and have ideal situation. Immediately after Erdoğan's trip to Russia, it is observed that Turkey started its mobility in Syria. But particularly, both countries have perceived that they have to reach a compromise point and consider somewhat each other's benefits. A very important issue is that Turkey found where the threats of this country are centralized.

Interference in the surrounding countries, collaboration with countries like Persian Gulf countries (that are the origin of issuing terrorists and radicalism currents) and also excessive reliance on America power and West were among the vulnerability points that were found in the recent variations of Turkey particularly after coup.

Therefore, it can be said that a kind of diversification in Turkey's foreign policy will direct the two countries collaboration grounds. Whilst Russia is also confronted with sanctions and economic pressures from West and all of these factors move the parties toward increasing collaboration and interactions simultaneously with disagreements. The system history is as follows:

$$H = \left\{ \phi, \left\{ g_1, (_1C,_1D)_{1,2}, _1S_2^2 \right\} \left\{ (_1C,_1D)_{1,2}, \{ g_2, (_2C,_2D)_{1,2} \} \right\} \left\{ _1S_2^2, \{ g_3,_3S_2^2 \} \right\} \right.$$
$$\left\{ (_2C,_2D)_{1,2}, \{ g_4, (_4D,_4D)_{1,2} \} \right\}, \left\{ _3S_2^2, \{ g_5, (_5D,_5D)_{1,2} \} \right\}$$
$$\left\{ (_4D,_4D)_{1,2}, \{ g_6, (_6D,_6D)_{1,2} \} \right\} \left\{ (_5D,_5D)_{1,2}, \{ g_7,_7S_2^2 \} \right\}$$
$$\left\{ (_6D,_6D)_{1,2}, _7S_2^2, \{ g_8,_8S_1^2,_8S_2^2 \} \right\} \left\{ _8S_1^2, \{ g_9, (_9C,_9D)_{1,2} \} \right\}$$
$$\left. \left\{ _8S_2^2, \{ g_{10}, (_{10}C,_{10}D)_{1,2} \} \right\} \left\{ (_9C,_9D)_{1,2}, (_{10}C,_{10}D)_{1,2}, \{ g_{11} \} \right\} \right\}$$
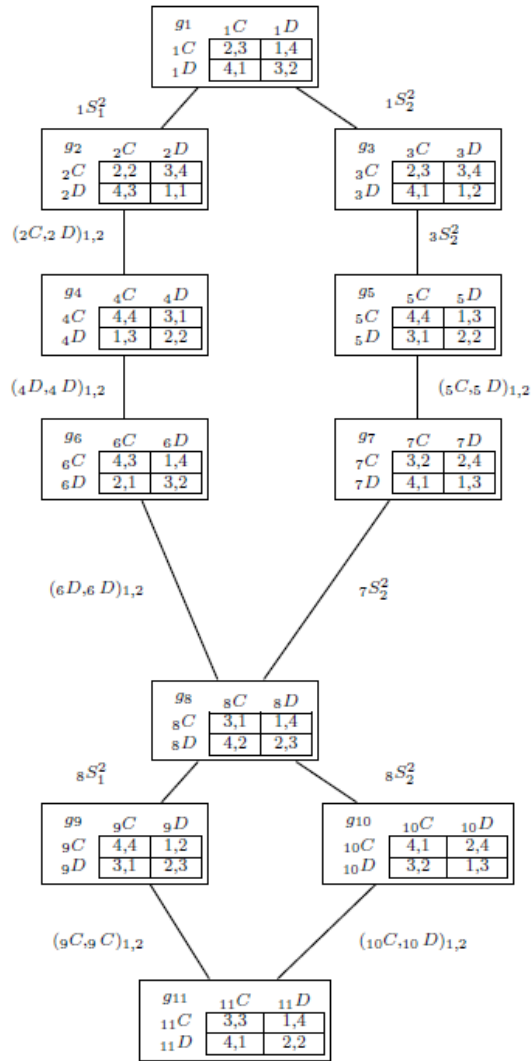
Fig. 2 The dynamic system of political relations games of the Turkey and Russia

4. Modeling economic relations

4.1 Cold War era

Unlike geopolitical relations which have been annoying historically, economic relations between Russia and Turkey mainly have been applied as a tool for providing and developing mutual political relations. During Cold War e, Soviet Union and Turkey have played the role of supplier and receiver of gas, respectively. But totally cooperation of two countries that resulted in more benefit has been defeated by non-collaboration. Therefore, the two countries entered two Prisoner's Dilemma games, g_1 (Fig.

3). In this game, two countries have two actions: either cooperating to gain more benefit $_1C$ or making defect $_1D$. Nash equilibrium of the game g_1 is $(_1D,_1D)$. The pair of actions $(_1C,_1C)$ is Pareto dominant of game g_1. Thus, the pair of the rational actions of this game is $(_1C,_1C)_{1,2}$ and $(_1D,_1D)_{1,2}$. Both countries continuing the game can cooperate by choosing pair of actions $(_1C,_1C)_{1,2}$ or make defect by choosing the dominant strategy of non-cooperation or pair of actions $(_1D,_1D)_{1,2}$.

4.2 Start of Cooperation

Supplying natural gas from former Soviet Union to Turkey that started along with Trans-Balkans pipeline since 1987, showed a transformation in traditional relations of the two countries. The next stage of Turkey and Russia economic relations started by agreeing to build gas pipeline called Water Current that increased the gas amount that Russia supplied to Turkey. This agreement was implemented despite various turbulent in political relations between two countries. Main reasons for the renewed tensions were limitations that were applied by Turkey in transportation of ships through Black Sea straits and another reason was Turkey's particular aim to develop its influence in Central Asia and South Caucasus and attempt to conquer "hearts and minds" of Muslim residents of Russia land. Two factors have made a new round of economic interaction between Turkey and Russia: first, Turkey's new enthusiasm to exploit its geographical advantage and second, Turkey's dependency on foreign energy. Turkey saw itself as an international bridge and transit path of raw energy material of Russia, Caucasus, Caspian Sea and partly Arabic governments to Europe and Israel. Turkey's pair of rational actions $(_1C,_1C)$ in game g_1 resulted in Benevolent game, g_2. In this game, Turkey has two strategies: either in order to increase economic benefits and exploit geographical position attempt to normalize relations and develop collaborations with Russia $_2C$ or avoid developing economic cooperation $_2D$.

In return, Russia also has two actions: either trusting in Turkey to increase collaborations and create mutual dependency $_2C$ or not trusting and preventing dependency $_2D$. The game Nash equilibrium is $(_2C,_2D)$. Game g_2 is producer of dominant strategy of cooperation $_2S_1^1$ and dominated strategy of defect $_2S_1^2$ for player 1. Also, this game is producer of dominant strategy of defect $_2S_2^2$ and dominated strategy of cooperation

$_2S_2^1$ for player 2. Therefore, g_2 is strategy maker of order (2,2). Pair of rational actions for two players is $(_2C,_2D)_{1,2}$. Also Russia's pair of rational actions $(_1C,_1C)$ in game g_1 ends in Stag Hunt game, g_3. In this game, Russia has two actions: either adjusting its economic policies by increasing two parties benefits $_3C$ or considering the Turkey's enthusiasm, exploiting the arisen opportunity merely to increase national and regional benefits $_3D$. On the other hand, Turkey also has two actions. Turkey can look for more benefits by attracting Russia satisfaction $_3C$ or wouldn't trust in Russia $_3D$ and follow its own economic policies. This game isn't strategy maker. The players' pair of rational actions are $(_3C,_3C)_{1,2}$ and $(_3D,_3D)_{1,2}$.

4.3 Toward coordination

Russia and Turkey relations have had a positive and incremental trend since Justice and Development Party taking power in 2002. Namely, Turkey attempted to improve relations with Russia from 2002 . In 2002, Russia showed its interest to implement the second water current project. This plan included constructing the second pipeline that passed from Turkey toward countries located in the south and west borders of this country. In Putin's trip to Ankara as first president after 32 years, 11 new agreements were signed until the two parties trade reached more than 100 billion dollars every year. The increasing collaboration trend of the parties reached to the extent that in 2008, Russia was the greatest commercial partner of Turkey [3] and [21].
Germany was the greatest commercial partner of Ankara. Annual turnover between Turkey and Russia in 2008 was over 38 billion dollars. In this regard, the two countries have expressed their desire to increase mutual turnover to 100 billion dollars per year. In May 2010, during the Medvedev's visit to Turkey, Ankara and Moscow agreed that for tourism trips and one month inhabitancy, there is no need for visa. Also two countries constituted the high council of collaboration and common economic council.
Leaders of the two countries also singed an agreement to construct the first nuclear powerhouse in Turkey. Turkey is the most desired destination of Russian tourists such that in 2011, about 3.5 million Russian people traveled to Turkey [3] and [19}[. Turkey's dominant cooperation strategy in game g_2 ends to Harmony game, g_4. In this game, Turkey has two actions either to consolidate cooperation and develop economic relations with Russia and establish a economic common council $_4C$ or keep relations at the same level $_4D$. In return, Russia also has two actions

either to take step along with coordination and developing relations $_4C$ or making defect $_4D$. g_4 is strategy maker of order (2,2). Nash equilibrium and pair of rational actions is $(_4C,_4C)_{1,2}$.

The pair of rational actions $(_3C,_3C)_{1,2}$ in game g_3 results in Coordination game, g_5. In this game, Russia has two actions either by considering two parties benefits increase economic turnover and establishing a joint economic council $_5C$ or avoiding creating more dependence $_5D$. In return, Turkey also has two actions either to take step in order to remove the existing obstacles and to facilitate cooperation $_5C$ or to take non-cooperation $_5D$. g_5 isn't strategy maker for two players. The players' pairs of rational actions are $(_5C,_5C)_{1,2}$ and $(_5D,_5D)_{1,2}$.

4.4 Reducing dependencies

In 2011, also the two countries reached important agreements on constructing new pipeline of Trans-Anatolia. The plans of Turkey's gas pipeline transfer and exploiting this position have a complicated nature. This issue made an opportunity for Turkey until increasing its maneuver power against Russia and Europe Union uses this position as a trump card in foreign policy. Some of Turkey's gas projects make western countries more dependent on Russia and make Russia as a main energy resource of Europe and some others reduce dependency of western countries on Turkey's gas resources. On the other hand, Russia politically using gas leverage in the past years has left a negative record of itself to the extent that Europe countries prefer Turkey to Russia. In addition, during about two decades, extensive attempts have been made to transfer energy resources of Persian Gulf area and Central Asia through Turkey. Different lines for energy transfer have been designed and implemented include Baku-Tbilisi-Ceyhan pipeline, gas pipeline of Baku-Tbilisi-Erzurum, Trans-Caspian pipeline, gas pipeline of Turkey-Greece-Italy and Nabucco pipeline.
Energy issue and competition for transfer led to economic relations and constituted over 65 percent of 35 billion dollar turnover in the two countries. The sanction of Iran oil and gas and reduction of Turkey's purchase from Iran, increased the rate of Turkey's energy import from Russia and made this country more dependent on Russia [2]. In this regard, Professor William Hill, historian of political relations of Turkey said at the time: "currently, interdependency has been created between two countries but it is likely that this interdependency will be weakened during the next 10 years as a loss for Turkey increasing Turkey's dependency on Russia particularly regarding energy"]1[. In examining the commercial relationships of the two countries, not only an interest to increase and

deepen this dependency is observed, but also both countries attempt to run away from the current situation and reduce dependency and that differences and disagreements of the two countries in political arena show that development of economic relations hasn't resulted in political closeness and opinion similarity. Attempts of the two countries s to reduce dependency are as follows:

- The south current pipeline plan that was suggested in 2012 for transferring gas from Black Sea to Bulgaria, Serbia and Europe.
- The plan of gas transfer from north in 2011 for transferring gas through Finland border to Germany, England and Netherland.
- The plan of oil pipeline of Bourgas Alecsander and Polis that was suggested since 2005 for transferring Russia oil through turning around Bosporus and Dardanelles straits from Russia to Bulgaria and Greece and Aegean Sea.
- Lack of interest to implement the plan of Samasun to Ceyhan pipeline that connected Turkey's north to its south.

Regarding Turkey, since at least 35 percent of Turkey's energy imports are from Russia, we see attempts of this country to reduce dependency on Russia energy and diversification of its energy suppliers, including :

- The plan of Baku-Tbilisi-Erzurum gas pipeline that will be operated in 2019.
- Baku-Tbilisi-Ceyhan oil pipeline that was operated in 2006.
- The contract of selling Azerbaijan gas to Turkey in 2011.
- Cooperation in Trans-Anatolian gas pipeline was suggested since 2014 and it will transfer Azerbaijan gas to Turkey and then to Europe.

According to what was said before, Russia has sought to preserve its superiority in commercial relations and on the other hand, Turkey has sought to develop other countries collaboration and reduce dependency on Russia. Considering the circumstances, Turkey preferred non-collaboration and reduction of dependency on Russia. Such behavior of Turkey resulted in Bully game, g_6. In this game, Turkey has two actions of continuing cooperation with Russia $_6C$ or extending relations with neighbors and reducing dependency on Russia $_6D$. On the contrary, Russia also has two actions of either preserving the level of cooperation $_6C$ or reducing commercial turnovers $_6D$. Game g_6 is producer of dominant strategy of non-cooperation $_6S_1^2$ and dominated strategy of cooperation $_6S_1^1$ for player 1. This game isn't strategy maker for player 2. Thus, g_6 is strategy maker of order (2,1). The only pair of rational actions is $(_6D,_6C)_{1,2}$.

The pair of rational actions $(_5D,_5D)_{1,2}$ in game g_5 ends to Hegemony game, g_7. In this game, Russia has two actions of either continuing interactions with the same approach of increasing cooperation $_7C$ or considering the occurring changes, seeking to change approach and preserve superiority by extending cooperation with other alliances $_7D$. In return, Turkey also has two actions of either giving priority to cooperation continuation and gradually changing its approach $_7C$ or proceeding for competition, confronting and finally making defect $_7D$. Game g_7 is producer of dominant strategy of cooperation $_7S_1^1$ and dominated strategy of defect $_7S_1^2$ for player 1. Also, this game is producer of dominant strategy of defect $_7S_2^2$ and dominated strategy of cooperation $_7S_2^1$ for player 2. So, g_7 is strategy maker of order (2,2). Nash equilibrium and pair of rational actions is $(_7C,_7D)_{1,2}$.

4.5 Making tension and reducing relations

The mentioned issues as well as Syria issue since 2011 and different and even conflicting approach of the parties toward the mentioned issue and overthrowing of Russia fighters by Turkey on November, 24, 2015 led Russia president to sign the law of Turkey sanction named the Law of "ensuring Russia national security and applying particular economic measures about Turkey" three days after this event. According to this law, importing some goods from Turkey was prohibited. Russia also canceled tourism to Turkey. In fact, political conflict of the two countries and following its change of the two countries strategies in political game presented in the previous section affected the economic game of the two countries and made that Russia changed its strategy in the economic game area and, in some sense, made their economic relations to be controversial to some extent. Turkey's dominant strategy of defect $_6S_1^2$ in game g_6 and Russia's dominant non-cooperation strategy $_7S_2^2$ in game g_7 ended to Conflict game of g_8. In this game, both countries have the two actions of continuing economic cooperation regardless of political tensions $_8C$ or reducing relations and economic interactions and defect $_8D$. This game is producer of dominant strategy of defect and dominated strategy of cooperation for two players. Thus, g_8 is strategy maker of order (2,2). The only pair of rational actions is $(_8D,_8D)_{1,2}$.

4.6 Reconstructing relations

The economic relations of Russia and Turkey after overthrowing of Russia fighter, were affected in the areas of tourism and exporting Turkey's food material to Russia. Also the security of Turkey energy, particularly in gas import section with import of 55 percent of its needed gas from Moscow, and also Turkey's great plans in which their budget amounted to ten billion dollars was compromised. These issues along with other factors forced Erdoğan to apologize Russia.

After Turkey apology, Erdoğan in August 18, 2016, visited Putin in Saint Petersburg, Russia. This historical visit improves relationships of the two countries. It can be said that both parties needed establishment of relations with each other to prevent loss and they had considerable overlap in different sections. The desire of Turkey to reconstruct and resume economic relations ended to Assurance game, g_9 . In this game, Turkey has the option of the two potential actions of either adjusting its policies to reconstruct relations $_9C$ or proceeding to revenge policies and making defect $_9D$. In return, Russia also has two actions of either giving positive answer to Turkey request in order to prevent economic losses $_9C$ or continuing unilateral sanctions and considering non-collaboration $_9D$. The game isn't strategy maker for two players. The only pair of rational actions is $(_9C,_9C)_{1,2}$.

Russia annoyance and anger with Turkey policies and dominant strategy of defect in game g_8 ended to Blackmailer game, g_{10} . In this game, Russia has the two possible actions of either cooperating $_{10}C$ and not applying pressure on Turkey or using economic relations as leverage to advance its objectives $_{10}D$. Turkey also has two actions of improvement seeking policy $_{10}C$ and defect $_{10}D$. g_{10} is producer of dominant strategy of cooperation and dominated strategy of defect for player 1. Also, this game is producer of dominant strategy of defect and dominated strategy of cooperation for player 2. Pair of rational actions for two players is $(_{10}C,_{10}D)_{1,2}$.

4.7 The future of bilateral relations

In economic area, Turkey economy is important for Russia from different aspects. Considering the region changes and the documents that parties signed in the past years, it shows the strategy in which the two countries must attempt to increase collaborations to the level of before tension period instead of non-collaboration. It is possible that two countries gain benefits by following their economic policies but they will achieve more benefits through economic collaborations.

On the other hand, U.S. sanctions aimed at targeting the two countries' economy in this year. Sanctions resulted in the devaluation of the currencies of these two countries (Lira and Ruble) against the dollar. So, the two countries can replace the "cooperation" instead of "competition" in various fields, to start a new phase in bilateral relations.

Based on the content expressed, the pair of rational actions $(_9C,_9C)_{1,2}$ and $(_{10}C,_{10}D)_{1,2}$ ends to Stag Hunt game g_{11} . In this game, the two countries have two actions, namely, either abandoning conflicts in order to increase national and regional benefits and doing collaboration $_{11}C$ or just looking for increasing their own benefits and preventing repeated interactions and extending cooperation $_{11}D$. Game g_{11} isn't strategy maker for none of the players and the pair of rational actions for the two players are $(_{11}C,_{11}C)_{1,2}$ and $(_{11}D,_{11}D)_{1,2}$.

The History of the system is as follows:

$$H = \left\{ \begin{array}{l} \phi, \{g_1,(_1C,_1C)_{1,2}\}, \{_1C,_1C)_{1,2},(g_2,_2S_1^1)\}, \{_1C,_1C)_{1,2},(g_3,(_3C,_3C)_{1,2})\}, \\ \{_2S_1^1,(g_4,(_4D,_4D)_{1,2})\}, \{_3C,_3C)_{1,2},(g_5,(_5D,_5D)_{1,2})\}, \\ \{_4D,_4D)_{1,2},(g_6,_6S_1^2)\}, \{_5D,_5D)_{1,2},(g_7,_7S_2^2)\}, \\ \{_6S_1^2,_7S_2^2,(g_8,_8S_1^2,_8S_2^2)\}, \{_8S_1^2,(g_9,(_9C,_9C)_{1,2})\}, \\ \{_8S_2^2,(g_{10},(_{10}C,_{10}D)_{1,2})\}, \{_9C,_9C)_{1,2},(_{10}C,_{10}D)_{1,2},(g_{11})\} \end{array} \right\}$$

g_1

$_1C$	$_1C$	$_1D$
$_1C$	3,3	1,4
$_1D$	4,1	2,2

$(_1C,_1C)_{1,2}$ $(_1C,_1C)_{1,2}$

g_2

$_2C$	$_2C$	$_2D$
$_2C$	4,2	3,4
$_2D$	1,1	2,3

g_3

$_3C$	$_3C$	$_3D$
$_3C$	4,4	1,3
$_3D$	3,1	2,2

$_2S_1^1$ $(_3C,_3C)_{1,2}$

g_4

$_4C$	$_4C$	$_4D$
$_4C$	4,4	3,2
$_4D$	2,3	1,1

g_5

$_5C$	$_5C$	$_5D$
$_5C$	4,4	1,1
$_5D$	2,2	3,3

$(_4D,_4D)_1$ $(_5D,_5D)_{1,2}$

g_6

$_6C$	$_6C$	$_6D$
$_6C$	2,3	1,4
$_6D$	4,2	3,1

g_7

$_7C$	$_7C$	$_7D$
$_7C$	4,3	2,4
$_7D$	3,1	1,2

$_6S_1^2$ $_7S_2^2$

g_8

$_8C$	$_8C$	$_8D$
$_8C$	2,3	1,4
$_8D$	4,1	3,2

$_8S_1^2$ $_8S_2^2$

g_9

$_9C$	$_9C$	$_9D$
$_9C$	4,4	1,2
$_9D$	3,1	3,2

g_{10}

$_{10}C$	$_{10}C$	$_{10}D$
$_{10}C$	4,2	2,4
$_{10}D$	3,1	1,3

$(_9C,_9C)_{1,2}$ $(_{10}C,_{10}D)_{1,2}$

g_{11}

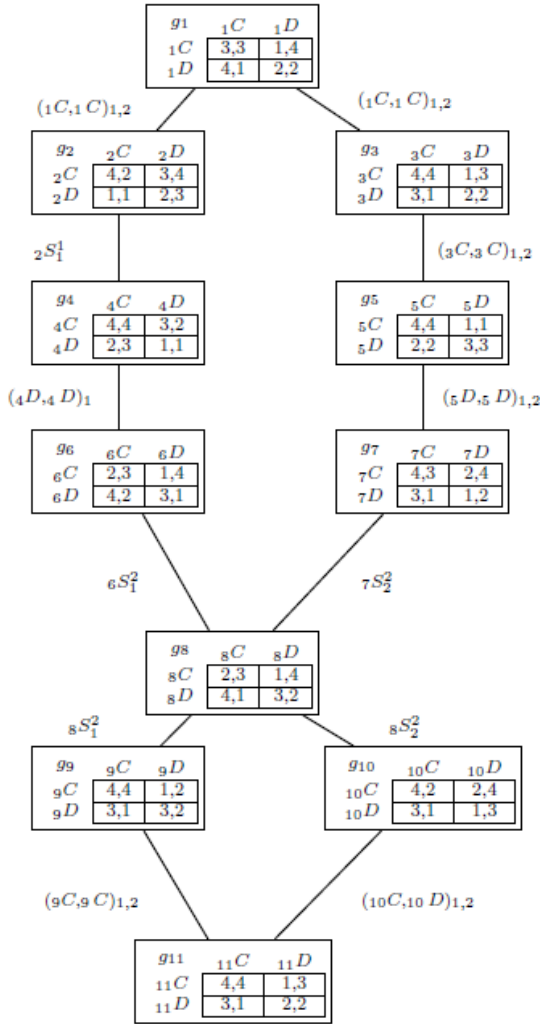| $_{11}C$	$_{11}C$	$_{11}D$
$_{11}C$	4,4	1,3
$_{11}D$	3,1	2,2

Fig. 3 Dynamic system of economic games of Russia and Turkey

5. Conclusion

In the contemporary time, the complexity of relationships in internal and international environment of governments has resulted in more entanglement of political and economic relations. So, study of political and economic strategies concurrently is necessary. Only the strategy can direct countries toward cooperation and constructive interaction. Making commercial relations isn't result of economic considerations but in some cases, political aims and national security also form economic relations vice versa. Therefore, we need a system that can model these relations over time and analyze their relationships and positive and negative effects. Study of dynamic systems of political and economic games of Russia and Turkey and modeling the games that have been formed between the two countries in political and economic areas and their chosen strategy illustrate the point that negative effect of political relations and security considerations on economic relationships of the two countries has been higher than positive effect.

But, conversely positive effect of economic relations on political relations between the two countries has been higher than negative effects. Since the two countries are under American economic sanctions, economic relations can be a factor for strengthening and expanding relations between the two countries. So aside from competition, the positive impact of economic relations between the two countries will lead to political union.

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