Joint Estimation and Compensation of Channel and Phase Noise in MIMO-OFDM Systems

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Summary

In this paper we study the joint estimation and compensation of Channel Impulse Response (CIR) and Phase Noise process in MIMO-OFDM systems. The proposed scheme is a semi-blind (SB) channel estimation scheme. This scheme uses blind data and equally spaced pilots to estimate channel.

The joint channel and PN estimation is done iteratively. To assess the performance of our scheme we derive the Cramer-Rao Lower Bound (CRLB) for MIMO-OFDM with partial training. Simulation results show that proposed scheme performs well at low and moderate SNR.

Key words:

Channel Estimation, MIMO-OFDM, Phase Noise.

1. Introduction

MIMO-OFDM systems are becoming widely accepted in all modern communication standards. They find application in WiMax (IEEE 802.16), WLAN (IEEE 802.11n) and LTE schemes. Their biggest advantage being more spectrally efficient and maximizing the system through-put. However this comes at the cost of complex receiver design and complicated channel estimation. The traditional scheme is to transmit certain training sequences from transmit antenna and then to perform estimation of channel using these sequences. From the literature we know that these schemes require multiples of pilot symbols to achieve acceptable channel estimates. It certainly makes sense to explore semiblind (SB) schemes which use blind data in addition to known training sequences. In literature several SB channel estimation techniques have been considered. One of the most widely studied SB channel estimation technique is the Bases Expansion model (BEM). This technique has been studied in [1]–[3] for MIMO OFDM transmission systems under slightly different channel conditions. The BEM technique is very effective in fast-fading channel scenarios. In [4] the authors have used a SB scheme based on modified power method [5] to estimate channel and mitigate the effects of pilot contamination caused by non-orthogonality of pilots. In [6] the authors have proposed a SB Maximumlikelihood algorithm for channel estimation in full duplex MIMO-OFDM systems.

The idea of whitening rotation (WR) based channel estimation for general MIMO scheme has been considered in [7]. The authors have provided a comprehensive

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estimation error analysis for SB and training based schemes; this technique has also been considered for channel estimation in MIMO, MIMO MC-CDMA and MIMO-OFDM based systems in [8], [9] and [10] respectively.

In this paper we study a semi-blind channel estimation for MIMO-OFDM system in presence of phase noise (PN) process. The proposed solution is based on WR based model. A similar problem set has been considered in [11] for a SISO-OFDM system. The authors have devised a strategy to estimate the channel and track PN process using previously detected OFDM symbols in a decision directed manner. In [12] authors have also considered joint channel and PN compensation in time-variant (TV) channels. However authors have assumed perfect knowledge of TV channel and estimated only the common phase error (CPE) using pilot sequence.

In [13] the authors have proposed estimation of CIR and PN process in MIMO-OFDM system through Maximum aposteriori Probability estimation. The proposed iterative algorithm takes Tx and Rx PN into consideration and provides a good performance/complexity trade-off. However this system may not be suitable in the case of doubly selective channel.

The WR channel estimation scheme has been previously considered in [7], [9] for time invariant (TI) cases with the assumption that second order statistics of the channel are known A-priori; In this paper we do not make such assumptions, using a novel windowed covariance matrix we calculate the second order statistics of a (slowly) timevarying channel. Furthermore the PN process tracking is performed using uniformly scatter pilot samples. Simulation results show that the proposed scheme can estimate the CIR and PN process fairly accurately.

To the best of our knowledge no work till date has addressed joint CIR and PN process estimation in MIMO-OFDM systems through WR based estimation techniques. The commonly used TV channel estimation technique the BEM is very effective against high mobility channels but the main draw back of this scheme is that it is hard to separate the problem of channel estimation from other parameters such as PN and I/Q imbalance.

Notation: Boldface small letters represent vectors, while boldface capital letters represent matrices; the superscripts T and H represent respectively transpose and Hermitian

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operators; \bigcirc is the Kronecker product operator; diag (·) represents the construction of a diagonal matrix; blkdiag(X) vertically stacks diag (·) of each row of X; Special symbols X is obtained through blk operations over X; **1** represents a column vector of all one elements; **I** is the identity matrix from a vector; **F** is a DFT matrix of size N × N; **F** is then partitioned into sub-blocks, i.e. $\mathbf{F} = [\mathbf{T}|\mathbf{V}]$, where **T** is the L_H × N portion of **F**; Note that **F** is a unitary matrices.

2. Transmission Model

In this system we assume that all the receiver antennae are driven from the same local oscillator and therefore each received sequence is assumed to suffer from same PN process. The set of all received data sequences is defined as follows

$$\mathbf{R} = \mathbf{HS\Theta} + \mathbf{z} \tag{1}$$

where R is a N_RN × N matrix, with received vector at k-th subcarrier, with $\mathbf{y}(k) = [\mathbf{y}^{1}(k) \cdots \mathbf{y}^{N_{R}}(k)]^{T}$ at the diagonal of the matrix, **H** is N_RN × N_TN the channel frequency response (CFR) matrix where **H**(k) is the N_R × N_T transfer function related to k-th subcarriers arranged diagonally, S is the N_TN × N data matrix with $s(k) = [s^{1}(k) \cdots s^{N_{T}}(k)]^{T}$ at the diagonal of the matrix, set of pilot symbols transmitted through all the transmit antenna is defined as <u>Sp</u> = [s(1), ..., s(P)], **Θ** is the N × N circulant phase noise process matrix defined as **Θ** = **FEF**^H, where **E** is the diagonal matrix obtained from N × 1 PN process vector effecting the received sequence and **z** is N_RN × N matrix of complex additive noise. This equation can be rewritten as

$$Y = \mathcal{H}(I_{N_T} \odot T) \mathbb{S} \Theta + z \tag{2}$$

where **Y** is the $N_R \times N$ matrix with **y**(k) (as defined above) at the k-th column, \mathcal{H} is $N_R \times N_T \cdot L_h$ the channel impulse response matrix. The channel estimation problem for the model in (1) is to estimate $N_R N_T N$. However this channel estimation problem can be reduced to estimation of $N_R N_T L_h$ unknowns through (2) as $L_h \ll N$. The CIR matrix of the complete MIMO system is defined as

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \boldsymbol{h}^{1,1} & \dots & \boldsymbol{h}^{1,N_T} \\ \boldsymbol{h}^{2,1} & \dots & \boldsymbol{h}^{2,N_T} \\ \vdots & \ddots & \vdots \\ \boldsymbol{h}^{N_R,1} & \dots & \boldsymbol{h}^{N_R,N_T} \end{bmatrix}$$
(3)

where $\mathbf{h}^{r,t}$ is the 1 × L_h time-domain channel impulse response vector from t-th transmit antenna to r-th receive antenna, **T** is a $L_h \times N$ sub-matrix of **F**. The semi-blind channel estimation in MIMO-OFDM system require special training sequences embedded within the OFDM systems. In our implementation we assume equally spaced clusters of training sequences, the clusters of these training sequences are designed to be orthogonal to each other. The training sequence is designed sequence such that $S_p S_p^H = N_p \cdot I_{N_T}$, where Np is the number of pilots in an OFDM symbol. In our implementation the LO is assumed to have a phase locked loop (PLL) such model is commonly considered in receivers in [14]–[16] as PLLs operate with a low stable frequency, using multiplier and dividers a variety of frequencies can be achieved. The phase noise process is model as white noise process filtered through a lowpass filter.

$$P_{\emptyset}(f) = K_{\emptyset} \left[\frac{1 + \binom{f}{f_z}^2}{1 + \binom{f}{f_p}^2} \right]$$
(4)

The phase noise process is completely characterized by RMS phase deviation of VCO σ vco and phase lock loop bandwidth β_{PLL} .

3. Joint Channel and PN Estimation

According to literature [7] it is well established that MIMO channel can be decomposed in two components $\mathcal{H} = \mathcal{W}Q^H$. The channel components can be determined through singular value decomposition (SVD) $\mathcal{H} = \mathcal{U}_h \Sigma_h \mathcal{V}_h^H$. Our objective is to minimize (5) by estimating the whitening and rotation matrices.

$$\widehat{\boldsymbol{Q}} = \arg\min_{\boldsymbol{O}} \left\| \boldsymbol{Y} - \boldsymbol{\mathcal{W}} \boldsymbol{\mathcal{Q}}^{H} \big(\boldsymbol{I}_{N_{T}} \odot \boldsymbol{T} \big) \boldsymbol{\mathbb{S}} \right\|_{F}^{2}$$
(5)

where $\|\cdot\|_F$ is the Frobenius norm, \mathcal{W} is the whitening matrix of size $N_R \times N_T L_h$ while Q is the rotation matrix of size $N_T L_h \times N_T L_h$; The estimates of Q are obtained through the cluster of pilot training symbols, while estimates of whitening matrix are obtained through the blind data symbols.

$$\widehat{\boldsymbol{Q}} = \boldsymbol{\mathcal{V}}_{Q} \boldsymbol{\mathcal{U}}_{Q}^{H} \quad \text{with} \\ \boldsymbol{\mathcal{U}}_{Q} \boldsymbol{\Sigma}_{Q} \boldsymbol{\mathcal{V}}_{Q}^{H} = SVD(\boldsymbol{\mathcal{W}} \boldsymbol{Y}_{p} \boldsymbol{X}_{p}^{H}).$$
(6)

where \mathbf{X}_{P} is the N_TL_h ×N_P set of pilot sequences, while \mathbf{Y}_{P}

is the $N_R \times N_P$ received set of pilot sequences. This is very important to note that the inter-carrier interference (ICI) arising from PN process is not considered while estimating the channel from known pilot sequences therefore there is a certain limitation to estimation accuracy.

A. Estimation of whitening matrix

The knowledge of whitening matrix is really essential to estimation of the overall channel model; We are lucky in this case the estimation of $\mathcal W$ is not effected by the PN process as illustrated below:

$$\hat{\psi}_{yy} = \frac{1}{\Delta N_b} \sum_{i=-\Delta/2}^{\Delta/2-1} \sum_{n=1}^{N_b} y_i(n) y_i^H(n)$$
(7)

where N_b is number of blind data in each OFDM symbol, Δ is the length of OFDM symbol window of observation for covariance matrix calculation, which implies that several previous and next OFDM symbols are involved in estimation of channel, for symbols at the edges of frame this window is truncated. The size of window Δ can be selected to enhance the quality of covariance matrix, using large window for time-invariant channel and smaller windows for time-variant channel. It is not difficult to see that PN does not effect the calculation of covariance matrix due to its inherent properties. The whitening matrix can be calculated as

$$\boldsymbol{\mathcal{W}} = \boldsymbol{\mathcal{U}}_{W} \boldsymbol{\Sigma}_{W}^{1/2} \quad \text{where} \\ \boldsymbol{\mathcal{U}}_{W} \boldsymbol{\Sigma}_{W} \boldsymbol{\mathcal{V}}_{W}^{H} = SVD\left(\frac{1}{\sigma_{s}^{2}} \left[\hat{\boldsymbol{\psi}}_{yy} - \sigma_{v}^{2}\boldsymbol{I}\right]\right)$$
(8)

B. Joint Estimation of rotation matrix and PN Process

Mitigation of CIR in the presence of PN process has been studied widely e.g. in [14], [16], [17]. All of the above schemes use OFDM training symbols. In contrast, the proposed estimation scheme considered in this work exploits the equi-spaced pilots and the statistical properties of the received data sequence.

We use an iterative scheme to estimate the channel and PN process. The process begins with the assumption that the PN process is absent i.e. $\mathbf{E} = \mathbf{I}$. The first estimates of rotation matrix Q^{H} are obtained as

$$\widehat{\boldsymbol{Q}} = \boldsymbol{V}_{Q} \boldsymbol{U}_{Q}^{H}$$
where $\boldsymbol{\mathcal{U}}_{Q} \boldsymbol{\Sigma}_{Q} \boldsymbol{\mathcal{V}}_{Q}^{H} = SVD \left(\widehat{\boldsymbol{\mathcal{W}}}^{H} \boldsymbol{Y}_{p} \boldsymbol{X}_{p}^{H} \right)$
(9)

The estimates of $\widehat{\mathcal{H}}$ are obtained from the above mentioned scheme and then it is used to estimate the PN process E by minimizing the following cost function

$$\hat{\boldsymbol{e}} = \arg\min_{\boldsymbol{e}} \|\boldsymbol{y}_p - \mathbb{A}\boldsymbol{e}\|^2 \tag{10}$$

Where **e** is the N×1 vector of the PN process, $\mathbf{y}_{\mathbf{p}}$ is N_RN_p×1 vector obtained by rewriting the received sequence; A =blkdiag($\mathcal{H}(l_{N_T} \odot \mathbf{T}) S_p$), where \mathbf{S}_p is N×N_T matrix with zeros on non-pilot subcarriers. The estimation of PN process is based on the small phase noise assumption i.e. $e^{j\phi[n]} \simeq 1 + j\phi[n]$; therefore setting a constraint on the real component of PN process we can obtain the imaginary component of PN process as

$$Img\{\hat{e}\}_{i} = \frac{j}{2} \left((\mathbb{A}^{H}\mathbb{A})^{T} (\mathbb{A}^{H}\mathbb{A}) \right)^{-1} (\mathbb{A}^{H}\mathbb{A})^{T} \left(\mathbb{A}^{T} \boldsymbol{y}_{p}^{*} - \mathbb{A}^{*} \boldsymbol{y}_{p} \right)$$
(11)

The details of the iterative LS procedure to estimate the CIR matrix \mathcal{H} and the PN process **e** are outlined in Algorithm 1. The proposed estimator is a biased but as our simulation results show that the performance of the proposed scheme is acceptable for moderate and high values of σ_{VCO} within 2 iterations.

Algorithm 1 Joint PN and Rotation Matrix Estimation. An iterative algorithm for estimating the PN process and rotation matrix using pilot symbols.

1: Input: $\mathbf{Y}_p, \hat{\mathcal{W}}$ 2: Output: $\hat{\mathcal{Q}}$ and $\Im{\{\hat{\mathbf{e}}\}_i}$ 3: initialization: $\hat{e}_0 = [1, \dots, 1]^T$ 4: calculate: $\hat{\mathcal{Q}}_0 = \mathcal{V}_Q \mathcal{U}_Q^H$ where $\mathcal{U}_{Q} \Sigma_{Q} \mathcal{V}_{Q}^{H} = \text{SVD} \left(\hat{\mathcal{W}}^{H} Y_{p} \left(\mathbf{X}_{p} \hat{\Theta}_{0} \right)^{H} \right)$ $\hat{\mathcal{H}}_{0} = \hat{\mathcal{W}} \mathcal{Q}_{0}^{H}$ $\hat{\Theta}_{0} = \text{Fdiag} \left(\hat{e}_{0} \right) \text{F}^{H}$

5: repeat

- Calculate $\mathbf{A}_{i} = \hat{\mathcal{H}}_{i} (\mathbf{I}_{N_{T}} \odot \mathbf{T}) \mathbb{S}_{p} \hat{\Theta}_{i-1} \mathbf{F}^{H}$ 6:
- Rewrite the system as $\mathbf{y}_p = \mathbb{A}_i \mathbf{e}_i$ 7: where $\mathbb{A} = \text{blkdiag}(\mathbf{A}), \mathbf{y}_p = [y_p^1, \dots, y_p^{N_r}]^T$ is set of all observed pilots.
- Obtain LS estimates $\Im\{\hat{\mathbf{e}}_i\} = \frac{j}{2} \left((\mathbb{A}^H \mathbb{A})^T (\mathbb{A}^H \mathbb{A}) \right)^{-1} (\mathbb{A}^H \mathbb{A})^T \left[\mathbb{A}^T \mathbf{y}_p^* \mathbb{A}^* \mathbf{y}_p \right]$ 8:
- $\hat{\Theta}_i = \operatorname{Fdiag}(\hat{e}_i) \mathbf{F}^H$ 9:
- Calculate \hat{Q}_i using 4: 10:
- i=i+111:
- 12: until No significant improvement in cost function $\|\mathbf{y}_p - \hat{\mathcal{H}}_{i-1} (\mathbf{I}_{N_T} \odot \mathbf{T}) \mathbb{S}_p \hat{\mathbf{\Theta}}_i \|_F^2.$

4. Estimation Performance Analysis

In this section we would like to present lower bound for CIR estimation performance in the presence of phase noise. This cost function is for a MISO component of the system which can be easily generalized. The log likelihood function for the sequence observed at r-th antenna is defined as

$$\ln p(\mathbf{y}^{r}|\overline{\mathbf{h}}^{r,t})$$

$$= \ln(\pi^{NN_{R}} det(\mathbf{K}_{v})) - \left(\mathbf{y}^{r} - \sum_{t=1}^{N_{T}} \mathbf{\Theta} \mathbf{S}^{t} \mathbf{T}^{T} \overline{\mathbf{h}}^{r,t}\right)^{H} K_{v}^{-1}$$

$$= \left(\mathbf{y}^{r} - \sum_{t=1}^{N_{T}} \mathbf{\Theta} \mathbf{S}^{t} \mathbf{T}^{T} \overline{\mathbf{h}}^{r,t}\right)$$
(12).

where \mathbf{y}^r is the N ×1 vector of received data on the r-th antenna, \mathbf{S}^t is the diagonal matrix of data transmitted over tth antenna and $\mathbf{h}^{r,t}$ is the L_h × 1 CIR vector. In our analysis we have considered the CPE as a part of the channel and the rest of the (ICI) components are taken as additive noise therefore the covariance matrix can be defined as $\mathbf{K}_v = (\sum_{k=1}^{N-1} \sigma_{\theta}^2 [k] \sigma_s^2 \sigma_h^2 + \sigma_v^2) \mathbf{I}_{L_h}$, where $\sigma_{\theta}^2 [k]$, $\mathbf{k} = 1 \cdots N - 1$ is the variance of ICI terms, this can be calculated if the statistical properties of the PN process are known. The received signal at r-th antenna is defined as

$$y^{r} = \theta[0]H^{r,t} + \sum_{t=1}^{N_{T}} \sum_{\substack{i=0\\i\neq j}}^{N-1} S^{t} H^{r,t} \Theta(i,j) + v^{r}$$
(13)

where the first term is the useful component of the received signal contaminated by CPE, the second term is the data dependent interference component while the last component is additive white Gaussian noise. The variance of the data dependent interference component may be defined as:

$$\sigma_{ICI}^{2} = \sum_{t=1}^{N_{T}} E\left\{ \left| \sum_{\substack{i=0\\i\neq j}}^{N-1} \Theta(i,j) \boldsymbol{r}^{r,t} \right|^{2} \right\}$$
(14)

since the PN process and received signal are independent of each other it's possible to simplify expression further as:

$$\sigma_{ICI}^{2} = \sum_{t=1}^{N_{T}} E\left\{ \left| \sum_{\substack{i=0\\i\neq j}}^{N-1} \Theta(i,j) \right|^{2} \right\} E\{ |\boldsymbol{r}^{r,t}|^{2} \},$$

which simplifies down to:

$$\sigma_{ICI}^2 = N_T \sigma_s^2 \sigma_h^2 [1 - \sigma_\theta^2[0]] \tag{15}$$

which includes the effect of all higher order terms of PN process PSD. The (r, t)-th component for Fisher information matrix can be calculated as in [18]

$$\mathrm{FIM}^{r,t} = E\left\{ \left(\frac{\partial L(\cdot)}{\partial h^{*r,t}}\right) \left(\left(\frac{\partial L(\cdot)}{\partial h^{*r,t}}\right) \right)^H \right\},\,$$

Calculating the Fisher information matrix and some straight forward derivation we find

$$E\left\{\left\|\boldsymbol{\mathcal{H}}-\widehat{\boldsymbol{\mathcal{H}}}\right\|_{F}^{2}\right\} \geq \frac{N_{T}N_{R}L_{h}}{N_{p}}\left(\frac{\sigma_{h}^{2}\sigma_{\mathrm{ICI}}^{2}+\sigma_{v}^{2}}{\sigma_{s}^{2}}\right)$$
(16)

When considering a TI channel the CFR of a MIMO system can be rewritten as \mathcal{H} where k-th diagonal block represents the transfer function for k-th subcarrier (as defined (2)). The lower bound error analysis for LTV channel is more complicated. The following measure of channel estimation error for TV channels has been considered in simulations

$$MSE(\widehat{\boldsymbol{\mathcal{H}}}) = \frac{1}{N} \sum_{n=1}^{N} \left\| \widehat{\boldsymbol{\mathcal{H}}} - \boldsymbol{\mathcal{H}}_n \right\|_F^2$$
(17)

5. Channel Equalization & Data Detection

The overall channel transfer function matrix \mathcal{H} can be created by setting channel frequency response submatrices at the diagonals. Once the channel has been estimated the equalization process is pretty straight-forward. The soft-decision of received sequences are obtained using the following structure.

$$\widehat{\mathbf{S}} = \left(\widehat{\mathbf{H}}^{H}\widehat{\mathbf{H}} + \sigma_{v}^{2}\mathbf{I}\right)^{-1}\widehat{\mathbf{H}}^{H}\widecheck{\mathbf{R}}$$
(18)

Where $\mathbf{\tilde{R}}$ is the received sequence after PN compensation i.e. $\mathbf{\tilde{R}} = \mathbf{Y} \mathbf{\hat{\Theta}}^H$ and $\mathbf{\hat{H}}$ is the channel estimation matrix through (5) and (8) as described in Algorithm 1. If the channel is a TI channel the off-diagonal components of the CFR matrix will be exactly 0. However if the channel is varying within an OFDM symbol more and more offdiagonal components will be non-zero leading to additional ICI. This can certainly degrade the performance of any system since estimation of TV channel in MIMO systems is computationally complex and contemporary channel estimation schemes will have to estimate a large number of unknowns to reliably detect each OFDM symbol. The proposed scheme provides adequate results for slow TV channels.

6. Simulation Results

This article considers a typical MIMO-OFDM system with spatial multiplexing. The number of subcarriers in each

OFDM symbol is N = 128, the guard interval is assumed to be longer than the CIR; each block of the OFDM symbols contains 20 OFDM symbols; Uncoded 64-QAM modulated data has been considered; the system bandwidth is $1/T_s =$ 2MHz at a carrier frequency of 2.40GHz. There exist 4 sets of orthogonal pilots clusters (i.e. Np = 4 × 8) uniformly distributed in OFDM symbol.



Fig. 1 channel estimation bound and simulated performance of our proposed scheme in the presence of PN process, Loop Bandwidth β =20kHz, RMS phase noise variance $\sigma_{\phi} = 4^{\circ}$, 15,000 symbols simulated, OFDM symbol size N=128, Number of pilot symbols $N_P = 32$, $N_T = 2$, $N_P = 8$, Rayleigh channel Lh=4 taps, covariance windows size Δ =7.

The local oscillator is assumed to have a PLL, the loop bandwidth of PLL and its RMS phase deviation is assumed to $\beta_{PLL} = 20$ kHz and $\sigma_{PN} = 4 \circ$. We consider number of transmit antenna NT = 2 and number of receive antenna N_R = 8.

The channel impulse response of each MIMO channel is considered to be statistically independent of each other and assumed to have length $L_h = 4$ taps. Two different scenarios for channel process are considered for the first part of this work the channel is assumed to be time invariant during the block and for the second part of the results we assume that channel is a slowly time varying i.e. we consider a the time-frequency product $f_dT = 0.01$ which corresponds to speed of 65km/hr.

The MSE performance of the proposed scheme in the presence of PN process is illustrated in Fig. 1. The CIR estimation performance of the TI case is very close to CRB at low and medium SNR (both in the presence and absence of PN process). The $MSE(\widehat{\mathcal{H}})$ for mobility scenario is poorer as the number of required channel coefficients increases considerably. The uncoded SER performance of

the proposed scheme after channel and PN process compensation is illustrated in Fig. 2. For the non-mobility



Fig. 2 SER performance after channel estimation in the presence of PN process with $\sigma_{PN} = 4^\circ$, β PLL=20kHz, Rayleigh channel Lh=4 taps, NT=2, NR=8, N=128 and Np=32, 64-QAM constellation and 15,000 symbols with covariance windows size Δ =7.

7. Conclusions

In this paper we have studied semi-blind estimation of frequency selective Rayleigh MIMO channel in the presence of phase noise. We have devised a simple system which uses blind data and carefully designed pilots to estimate channel and phase noise process iteratively. The issue performance bounds on channel estimation error in the presence of PN process has also been addressed.

Simulation results show that the proposed solutions can effectively estimate CIR in the presence of PN process.

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