On Erdős-Straus Conjecture for $\frac{3}{n}$

Abdul Hameed[1], Ali Dino Jumani[1], Israr Ahmed[1], Inayatullah Soomro[1], Abdul Majid[2], Ghulam Abbass[1], Abdul Naeem kalhoro[1].

[1]Department of Mathematics, Shah Abdul Latif University Khairpur, pakistan
[2]Department of Biochemistry, Shah Abdul Latif University Khairpur, pakistan

Abstract
Number theory is very fascinating field of the Mathematics. It has extensive usage in our daily life. Due to this it has attracted research community since the ancient times. Egyptians were very much interested in the Number theory. In particular, they made a great contribution towards solving fraction problems. In this study we focus on A conjecture due to Paul Erdos and E.G. Straus that the Diophantine equation $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ involving Egyptian fractions always can be solved. By using the concept of this conjecture we have found that the Erdos-Struass conjecture is also valid for $\frac{3}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$.

Key words:
Math, Mathematic, Egyptian fractions, Number theory

1. Introduction

Number theory has remained a very interesting subject because of its wider applicability in the different fields. This subject is concerned with the study of the numbers. The main applications are included in the field of computer science, numerical methods, information technology, communications, and cryptography. In particular, Number theory has played a pivotal role in helping to develop algorithms, such as a, RSA a public key cryptography which considers the fact that there is the difficulty to factor large numbers [1-10].

Diophantine equations are very important topic in the field of Number theory. In this topic only integer solutions are allowed. Diophantine equations are concerned with the Egyptian fraction [4].

The concept of Egyptian fractions has gained attraction of many people going back to the three thousand years and still continues to attract the interest of mathematicians today. This concept is basically about the fractions in which numerator is considered as 1 and the denominator appears to be a positive integer [11-16].

Egyptian, in their early time of Second Intermediate Period, never had a system to write the fractions like $\frac{n}{m}$ Instead they write fractions as sum of unit fractions like $\frac{1}{m}$, that is, $\frac{11}{30}$ is to be written as the sum of $\frac{1}{5}$ and $\frac{1}{6}$ fractions. These fraction has been attracting attention since the second Intermediate Period of Egypt, that is, 1650-1550 BC. Fraction of the type $\frac{2}{2k+1}$ decomposed as sum of two and three fractions has been found in Rhind Papyrus scripts.

As it is well established that every rational number can be expressed as a finite sum of different 4 unit $\frac{1}{mn}=\frac{1}{n+1}+\frac{1}{n(n+1)}$ $\forall\, k \in \mathbb{N}$ fractions. For this verification and the fraction can be written as the sum of two fraction namely as $\frac{2}{2k-1}=\frac{1}{k}+\frac{1}{k(2k-1)}$ and $\frac{4}{4k-1}=\frac{1}{k}+\frac{1}{k(4k-1)}$ $\forall\, k \in \mathbb{N}$ for even number n and odd numbers n of the form $n = 4k-1$ respectively.

A famous conjecture due to Paul Erdős and E. G. Straus, known as Erdős-Straus conjecture on Egyptian fraction states that given a positive integer $n \geq 2$ there exists $x,\ y,\ z \in \mathbb{N}$ such that $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$.

Here we say that is Erdős-Straus number and induces a decomposition of the fraction $\frac{4}{n}$ into the sum of three equivalent fractions. Sierpiński and Schinzel extended the decomposition of the fraction $\frac{m}{n}$ to a more general conjecture by replacing 4 by other fixed positive integer $m \geq 4$.

In this note we prove that the fraction $\frac{3}{n}$ can be decomposed into the sum of three Egyptian fractions, $\frac{3}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$. as well.

2. The Proof of the Erdős-Straus Conjecture For $\frac{3}{n}$

In this note we express $\frac{3}{n}$ as a sum of at most three distinct unit fractions as in the light of Erdos-Straus conjecture.

For all $n \in \{2,3,4,...\}$ and for some $x, y, z \in \{1,2,3,........\}$.

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Theorem 1. $\frac{3}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$, $\forall n \in \{2,3,4,\dots\}$

and $\exists\, x,y,z \in \{1,2,3,\dots\}$.

Proof. For all $\forall n\,\{2,3,4,\dots\}$ and for some

$x,y,z \in \{1,2,3........\}$

$$\frac{3}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \Rightarrow 3xyz = n(xy+xz+yz)$$

for n =2 and x =1, y = 4, z = 4

$$\frac{3}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

$\forall n \in \{3,5,7,11,\dots\}$, *for* some x,y,z $\in \{1,2,3,\dots\}$

$$\left[\frac{3}{n}=\frac{1}{n}+\frac{1}{\frac{n+1}{2}}+\frac{1}{\frac{n(n+1)}{2}}\right] \text{when } x = n \wedge y=\frac{n+1}{2} \wedge z=\frac{n(n+1)}{2}$$

From (1) it follows that $n \in \{2, 3, 5, 7\dots\}$ and for some m, $x, y, z, \in \{1, 2, 3 \dots\}$

$$n=\frac{(3m-1)xz}{\mathrm{m}(\mathrm{x}+z)} \wedge nm = y \wedge \gcd\ (\mathrm{n,3m\text{-}1}) \geq 1 \quad (2)$$

$\forall n \in \{4,8,12,16,\dots\}$, *for* some x,y,z $\in \{1,2,3,\dots\}$

$$\left[\frac{3}{n}=\frac{1}{n}+\frac{1}{\frac{3n}{4}}+\frac{1}{\frac{6n}{4}}\right] \text{when } x = n \wedge y=\frac{3n}{4} \wedge z=\frac{6n}{4}$$

On the strength of (2) n =3 and for some $x, y, z, \in \{1, 2, 3 \dots\}$

$$\left[nm = y, \wedge m=1 \wedge x=1 \wedge n=\frac{(3m-1)xz}{m(x+z)}=\frac{2xz}{x+z}\right] \Rightarrow$$

$$\left(n=3=\frac{2xz}{x+z} \wedge x=k+z\right) \Rightarrow$$

$$2z^2+(2k-6)z-3k=0 \Rightarrow$$

$$\Delta=(2k)^2+6^2=10^2 \Rightarrow k=4$$

Therefore

$$\left(z=2 \wedge x=4+z=6=2n \wedge z=2n\right)$$

on the strength of [1] for all n $\in \{3,6,9,12\dots\}$ and for some x,y,z $\in \{1,2,3,\dots.\}$

$$\left[\frac{3}{n}=\frac{1}{2n}+\frac{1}{n}+\frac{1}{\frac{2n}{3}}\right] \text{when } x = 2n \wedge y = n \wedge z=\frac{2n}{3}$$

$\forall n \in \{6,9,12,15,18,24,\dots\}$ *and for some* $x,y,z \in \{1,2,3,\dots\}$

$$\left[\frac{3}{n}=\frac{1}{n}+\frac{1}{\frac{4n}{2}}+\frac{1}{\frac{4n}{6}}\right] \text{when } x = n \wedge y=\frac{4n}{2} \wedge z=\frac{4n}{6}$$

$\forall n \in \{10,15,20,25,\dots\}$ *and for some* $x,y,z \in \{1,2,3,\dots\}$

$$\left[\frac{3}{n}=\frac{1}{n}+\frac{1}{\frac{4n}{5}}+\frac{1}{\frac{7n}{5}}\right] \text{when } x = n \wedge y=\frac{4n}{5} \wedge z=\frac{7n}{5}$$

$\forall n \in \{7,14,21,\dots.\}$ *and for some* $x,y,z \in \{1,2,3,\dots\}$

$$\left[\frac{3}{n}=\frac{1}{n}+\frac{1}{\frac{4n}{7}}+\frac{1}{\frac{28n}{7}}\right] \text{when } x = n,\ y=\frac{4n}{7}, z=\frac{28n}{7}$$

$\{2\}U\{3,5,7,11,13,\dots\}U\{4,8,12,16,20,\dots\}U\{6,9,12,15,18,\dots\}U$
$\{10,15,20,25,30,\dots\}U\{7,14,21,28,35,\dots\}U\{3,6,9,12,\dots\}=\{2,3,4,5,6,7,8,\dots\}$

Obviously $\{\phi\} \neq \phi$.

Corollary 1. For all n $\in \{2,3,5,7,\dots\}$ *and for* some m,x,y,z $\in \{1,2,3,\dots.\}$

$$\left[n=\frac{(3m-1)xz}{m(x+z)} \wedge nm = y \wedge \gcd(n,3m-1) \geq 1\right].$$

Which is a required proof of Erdős-Straus Conjecture for $\frac{3}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$.

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