Diophantine Quadruple with $D(9)$ property

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Abstract:
It is proved that the Diophantine $quadruple$, set of positive integer number with the property that the product of any two of them plus 9 is a perfect square, than generalization of the result is obtained.

Keywords:
diophantine quadruple, arithmetic progression , diophantine m-tuple

1. Introduction

Since the Greek Mathematician Diophantus of Alexandria studied the problem of Find four (positive rational) numbers such that the multiplication of any two of them adding by 1 is a perfect square He obtained solution $\left[\frac{1}{16},\frac{33}{16},\frac{17}{16},\frac{105}{16}\right]$ [1],[2] However The first Diophantine quadruple $D(1)$ have been founded by Fermat {1, 3, 8, 12}[3],like such that the multiplication of any two members and increased with n,whereas $n \in \mathrm{Z}-\{0\}$, as becomes result of perfect square, It was proved in 1969 by two mathematicians Baker and Davenport [5] that a fifth positive integer could be not be added to this set. However, Euler became capable in order to extend this set by adding the rational number $\frac{777480}{8288641}$,[6]. The question of presence of (integer) Diophantine quintuples was one of the oldest outstanding unsolved issue problems in the field of the Number Theory, in 2004 Andrej Dujella proved that most of or grate number of finite number of Diophantine quintuples exist. In 2016 the problem was finally resolved by He, Togbé and Ziegler.

Let us now consider the more general problem

.Let n be an integer. A set of positive integers $\{p_1, p_2\, p_3, \dots\ p_m\}$ is said to have the property of Diophantus of order n, symbolically$D(n)$, if the product of its any 2 distinct elements adding by n is a perfect square. Such a set is called a Diophantine m-tuple.[4],[10-16]

2. Results and Discussions

In this paper we constructed a new $D(9)$ property. That satisfying Diophantine Quadruple i-e the product of any two numbers from the set of four positive integers and increased by 9 is equal to perfect square

Now By using the above polynomial formula for integers $\{p,q,r,s\}$ then we can find infinite set of Diophantine Quadruples with $D(9)$ property.

$$\{n, 4n+12, 9n+18, 16n^3+80n^2+124n + 60\} \qquad (1)$$

$$\left\{p, 4p+12, 9p + 18, \frac{8p+20\sqrt{(pq+9)}\ \sqrt{(pr+9)}}{3}\right. \qquad (2)$$

Now proof

Table 1: Perfect Square

P	q	r	s
1	16	27	280
2	20	36	756
3	24	45	1584
4	28	54	2860
5	32	63	4680
6	36	72	7140

With the property $D(9)$ Now in the above Diophantine Quadruples $\{p_1, p_2, p_3, p_4\}, \{q_1, q_2, q_3, q_4\}$ and $\{r_1, r_2, r_3, r_4\}$ In the arithmetic progression further development while

$$p = 1,2,3 \dots..$$
$$p = 1, s = 2-1 = 1$$
$$p_n = p + (n-1)s$$
$$p_n = 1 + (n-1)1$$
$$p_n = 1 + n - 1 \qquad \mathrm{p_n = n, \forall n Z^+}$$

Likewise

$$q = 16, 20, 24, \dots.$$
$$p = 16, \qquad s = 20 - 16 = 4$$

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$$q_n = a + (n-1)s$$
$$q_n = 16 + (n-1)4$$
$$q_n = 16 + 4n - 4$$
$$q_n = 4n + 12 \qquad q_n = n, \forall n Z^+$$

Similarly

$$r = 27, 36, 45, \dots.$$
$$p = 27\ , s = 36 - 27 = 9$$
$$r_n = p + (n-1)s$$
$$r_n = 27 + (n-1)9$$
$$r_n = 27 + 9n - 9$$
$$r_n = 9n + 18 \qquad \mathrm{r_n = n, \forall nZ^+}$$

As there is no any common difference between

$$\{s_1, s_2, s_3, , s_4, s_5, s_6, s_7, s_8, s_9, s_{10}\} = \{280{,}2016{,}7216{,}18760{,}40392{,}76720{,}133216{,}216216{,}332920{,}491332\}$$

Then to generalize the d we find the common difference between

$$\{280{,}2016{,}7216{,}18760{,}40392, 76720{,}133216, 216216{,}332920{,}491332\}$$
$$\{s_1, s_2, s_3, , s_4, s_5, s_6, s_7, s_8, s_9, s_{10}\} =$$

Table 2: Differences

280	1736	3464	2880	864
2016	5200	6344	3744	864
7216	11544	10088	4608	864
18760	21632	14696	5472	864
40392	36328	20168	6336	864
76720	56496	26504	7200	864
133216	83000	33704	8064	
216216	116704	41768		
332920	158472			
491392				

Since the typical distinction is actually 864, right now all of us uses the actual cubic series or even third level polynomial $an^3 + bn^2 + c + d$ to simplify the s

$$an^3 + bn^2 + +d \qquad (i)$$

By putting $n = 1$ in equation (i) we get

$$a + b + c += 280 \qquad (j)$$

By putting $n = 2$ in eq: (i) we get

$$8a + 4b + 2c += 756 \qquad (k)$$

By putting $n = 3$ in eq: (i) we get

$$\mathrm{27a + 9b + 3c += 1584} \qquad \mathrm{(l)}$$

By putting $n = 4$ in eq: (i) we get

$$64a + 16b + 4c += 2860 \qquad (m)$$

Subtracting equation (k) from (j), (l) from (k) and (m) from (l) we get equation $(n), (o)$ and (p)

$$7a + 3b + c = 476 \qquad (n)$$

$$19a + 5b + c = 228 \qquad (o)$$

$$37a + 7b + c = 1276 \qquad (p)$$

Again subtracting equation (o) $from$ (n) and (p) from (o) we get

$$12a + 2b = 352 \qquad (q)$$

$$18a + 2b = 448 \qquad (r)$$

Again subtracting equation (q) $from\ eq. (r)$ we get

$\boldsymbol{a = 16}$

By substituting $a = 16$ $in\ eq.$ (r) we get

$\boldsymbol{b = 80}$

By substituting $a = 16$ and $b = 80$ $in\ eq{:}\ (p)$ $then\ we$ get

$\mathbf{c = 124}$

By substituting $a = 16\ , b = 80$ and $c = 124$ $in\ eq{:}\ (m)$ $then\ we$ get

$\boldsymbol{d = 60}$

By substituting $a = 16\ , b = 80\ ,\ c = 124$ and $d = 60$ $in\ eq{:}\ (i)$
$then\ we$ get

$$s = 16n^3 + 80n^2 + 124n + 60$$

We have generalized the integers p, q, r and s such as

$$\mathrm{p = n, \forall n \in Z^+}$$
$$q = 4n + 12$$
$$r = 9n + 18$$
$$s = 16n^3 + 80n^2 + 124n + 60$$

Now 3 is multiply and divide in (s)

$$s = \frac{3(16p^3 + 80p^2 + 124p + 60)}{3}$$

$$s = \frac{48p^3 + 240p^2 + 372p + 180}{3}$$
$$s = \frac{16p^2 + 64p + 60(3p + 3)}{3}$$
$$s = \frac{16p^2 + 24p + 40p + 60(3p + 3)}{3}$$
$$s = \frac{8p + 20(2p + 3)(3p + 3)}{3}$$
$$s = \frac{8p + 20\sqrt{(2p + 3)^2(3p + 3)^2}}{3}$$
$$s = \frac{8a + 20\sqrt{(4p^2 + 12p + 9)(9p^2 + 18p + 9)}}{3}$$
$$s = \frac{8p + 20\sqrt{(p(4p + 12) + 9} \sqrt{(p(9p + 18) + 9}}{3}$$

Now $by\ putting\ the\ value\ q = 4p + 12\ and\ r = 9p + 18$

$$we\ get\ the\ value\ of\ s$$
$$s = \frac{8p + 20\sqrt{(pq + 9)} \sqrt{(pr + 9)}}{3}$$

Now By using the above polynomial formula for integers $\{p, q, r, s\}$ then we can find infinite set of Diophantine Quadruples with $D(9)$ property.

3. Main theorm

$$\{n, 4n + 12, 9n + 18, 16n^3 + 80n^2 + 124n + 60\} \qquad (x)$$

$$\{p, 4p + 12, 9p + 18, \frac{8p + 20\sqrt{(pq + 9)} \sqrt{(pr + 9)}}{3} \qquad (y)$$

However We can say that over each set (x) as well as (y) tend to be direct generalization along with $D(9)$ property, much more all of us may additional talk about within present research which how to locate the actual generalized models systematically along with $D(9)$ property, also shown table 3
$where\ p = 1,2,3, \dots \dots$

Table 3: Generalized sets systematically with D(9) property

p	q	r	S	$\sqrt{pq+9}$	$\sqrt{pr+9}$	$\sqrt{ps+9}$	$\sqrt{qr+9}$	$\sqrt{qs+9}$	$\sqrt{rs+9}$
1	16	27	280	5	6	17	21	67	87
2	20	36	756	7	9	39	27	123	165
3	24	45	1584	9	12	69	33	195	267
4	28	54	2860	11	15	107	39	283	393
5	32	63	4680	13	18	153	45	387	543
6	36	72	7140	15	21	207	51	507	717
7	40	81	10336	17	24	269	57	643	915
8	44	90	14364	19	27	339	63	795	1137
9	48	99	19320	21	30	417	69	963	1383
10	52	108	25300	23	33	503	75	1147	1653

4. Conclusion

The generalized Diophantine Quadruple with $D(9)$ property, the Diophantine Quadruple with $D(9)$ properties have been successfully proved with a few formulas some polynomial formulas for the positive integers {p, q, r, s } by an arithmetic progression are to be generalized ,it is concluded that there are an infinite Diophantine quadruple $D(9)$ property , also some systematically generalized sets are to be found with the valuable features aforementioned, and expect that our Diophantine Quadruple ,With $D(9)$ properties will be useful for applications in areas such Cryptography.

References

[1] Diophantus of Alexandria. Arithmetics and the Book of Polygonal Numbers. (I. G. Bashmakova, Ed.) (Nauka, 1974) (in Russian), pp. 103–104, 232

[2] Britton J. Novikov PS. Nérazréšmost' problémysopráznnosti v tériigrupp.IzvéstiáAkadémiiNauk SSSR, Séria mat., vol. 18, pp. 485–524. 1958.

[3] Kedlaya K. Solving constrained Pell equations. Mathematics of Computation of the, American Mathematical Society, 1998,67(222): 833-842

[4] Dujella A. There are only finitely many Diophantine quintuples. Journal fur die Reine und Angewandte Mathematik, 2004: 183-214.

[5] L. E. Dickson. History of the Theory of Numbers, Vol. 2.(Chelsea, 1966), pp. 513-520.

[6] Euler L. Nova subsidia pro resolutione formulae axx+ 1= yy. Opuscula analytica, 1783,1: 310-328.

[7] Franušić Z. Diophantine quadruples in quadratic fields: Prirodoslovno matematički fakultet, Sveučilište u Zagrebu, 2005.

[8] Platt DJ, Trudgian TS. Diophantine quintuples containing triples of the first kind. Periodica Mathematica Hungarica, 2016,72(2): 235-242

[9] He B, Togbè A, Ziegler V. There is no Diophantine quintuple. arXiv preprint arXiv:161004020, 2016.

[10] Brown E. Sets in which [J]. Mathematics of Computation, 1985,45(172): 613-620.

[11] Muriefah FA, Al-Rashed A. Some Diophantine quadruples in the ring Z [$\sqrt{-2}$]. Math Commun, 2004,9: 1-8.
[12] Assaf E, Gueron S. Characterization of regular Diophantine quadruples. Elemente der Mathematik, 2001,56(2): 71-81.
[13] Bashmakova I. Diophantus of Alexandria, Arithmetics and the Book of Polygonal Numbers. Nauka, Moscow. 1974.
[14] Dujella A. Some polynomial formulas for Diophantine quadruples. Grazer Mathematische Berichte, 1996,328: 25-30.
[15] Dujella A. Some estimates of the number of Diophantine quadruples. Publ Math Debrecen, 1998,53: 177-189.
[16] Franušić Z. Diophantine quadruples in the ring $Z[\sqrt{2}]$. Mathematical Communications, 2004,9(2): 141-148.