A connected graph with non-concurrent Longest Paths

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Abstract
In this an example of graph is presented with the property that each vertex is missed by some longest Paths.

Keywords:
Hamiltonian path; longest path; Hypo-traceable; Gallai's property; $\bar{P}_i^j$

1. Introduction

A path that takes a break without any redundancies, and does not need to begin and finish at the comparable vertex in graph G is said to be Hamiltonian path. A graph is said to be noticeable on the off chance that it has a Hamiltonian path. A graph G is a hypotraceable if graph G has no Hamiltonian path ybut deletion of any vertex has a Hamiltonian path for each v ϵ V. A cycle which contains all the vertices of G is called Hamiltonian cycle. A graph which is non-Hamiltonian yet $G - v$ is Hamiltonian for all vertices v is known as a hypo-Hamiltonian. The most renowned case of Hypo-Hamiltonian graph is Petersen graph.

The presence of hypo-Hamiltonian graphs and earlier the modernization of the hypo traceable graphs, in 1966 T. Gallai [14] asked whether there exist connected graphs with the property that every vertex is missed by some longest path. Just later, in 1969, Gallai's question was first responded by H. Walther [1], who introduced a planar graph on 25 vertices satisfying Gallai's property. Later H. Walther and H. Voss [2], & Tudor Zamfirescu [3], introduced such kind of graph with 12 vertices, and it was guessed that order 12 is the smaller possibility of such a graph. In the case of planar graphs, such type of a graph with lowest number of vertices i-e with 17 vertices, was provided by W. Schmitz [4]. A smallest non-planar graph of order 34 introduced by Thomassen [13]. The first 2-connected planar graph generated by Tudor Zamfirescu [5] with 82 vertces. The lowest famous example nowadays has 26 vertices [6], on the other hand the lowest example up to now has order 32 [5].

In 1972, Tudor Zamfirescu [3] questioned related to the Gallai's property. let $\mathrm{P}_\mathrm{i}^\mathrm{j} = \infty$ $(\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j} = \infty)$ if there is no any $i -$connected graph (planar graph) such that individually set of j points remains disjoint from some longest path condition $\mathrm{P}_\mathrm{i}^\mathrm{j} \neq \infty$ $(\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j} \neq \infty)$, let $\mathrm{P}_\mathrm{i}^\mathrm{j}(\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j})$ indicate the smallest number of vertices of an $i -$connected graph (planar graph) such that individually set of j selected vertices be there disjoint from some largest path. Analogously these cases are clearly C_i^j and $\bar{C}_i^j$ for longest circuits as a replacement for longest pat

2. Results and Discussions

The purpose of this work is to show, that an example of a $1 -$ connected graph G, of orders 20 satisfying by Gallai's property.

Theorem 1: There is existing a graph of 20 vertices with the property that each vertex is missed by some longest Paths.

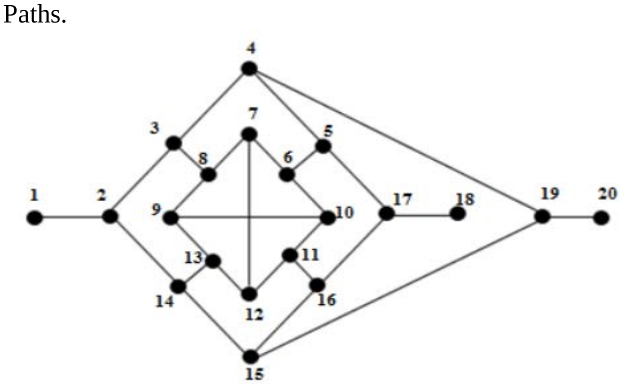

Fig. 1

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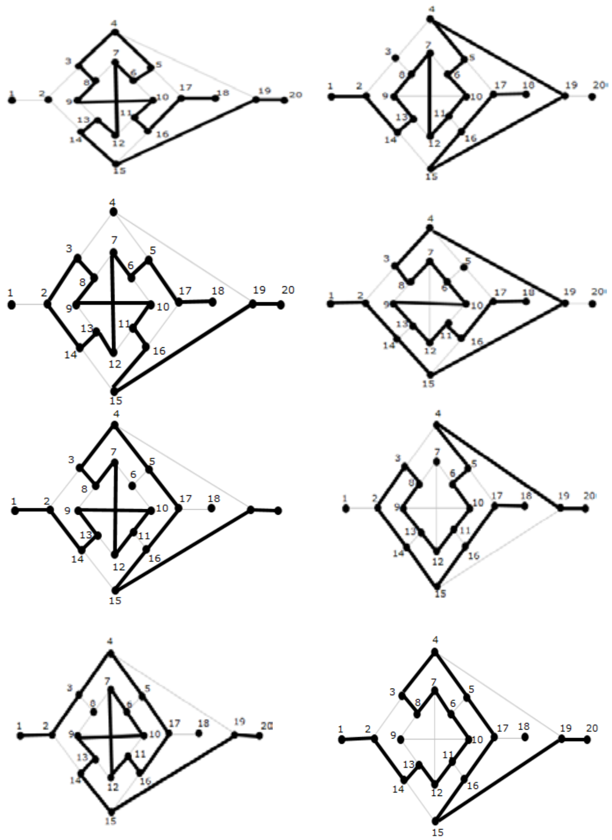
Proof: Consider the graph G of figure 1. With 20 vertices, let W be a longest path in G, the longest paths of G joining two of its end points have length $p(G) = 18$ avoiding v with $W \cap V = \emptyset$ of all its longest Paths. The paths shown Figure 2, below underlined vertices from 1 to 20, where each vertex is avoided by some longest paths.

Lemma: The graph G has no Hamiltonian path.



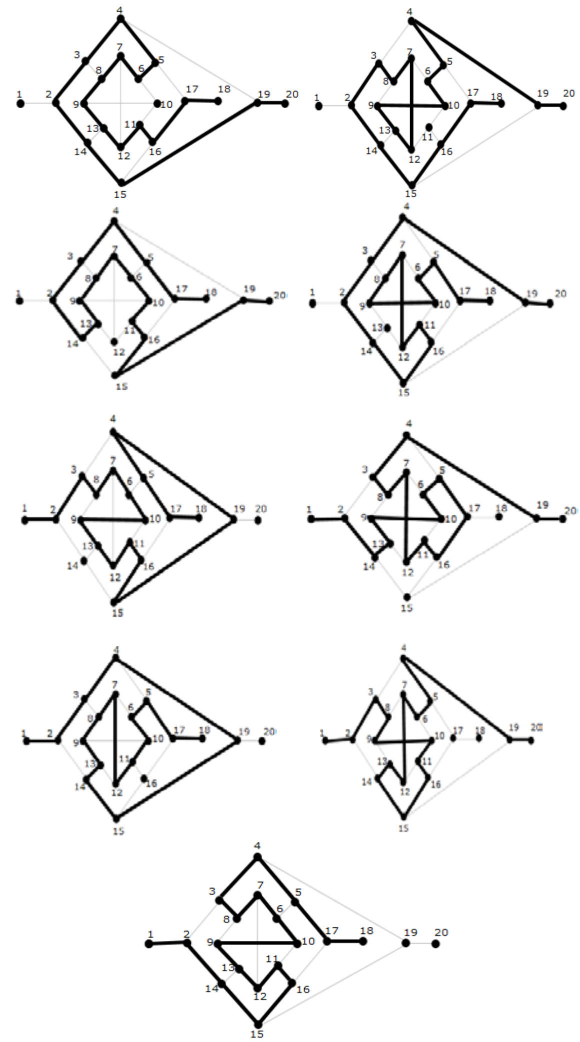


Fig. 1

The above results show that we have developed 1-connected graph in which each vertices is missed by some longest path. Highlighted paths shown in figure 2, Confirm our claim that there exists connected graph satisfying Gallai's property.

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