Wavelet Analysis for precipitation attributes

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Summary
The wavelet analysis is used for precipitation time series of Southwestern Australia to identify the several time scales attributes of winter precipitation during 1951-2013. Significant findings reveal that the periodic oscillation of precipitation variability and the suggested break point scattered changes at multi time scales along the data are explored. Findings for important periods, the pattern of precipitation variability in the future has also been investigated. The significant findings reveal that there are fixed periodicity of 6—10 years and 3—5 years for the winter time precipitation variability. The variability pattern of the winter precipitation is in consistent with that of the seasonal and inter seasonal precipitation and both of them have the important time segments of 2 years and 10 years. Our calculations suggest that in the actual period of 1 year that the precipitation quantity will be comparatively low, which lies in 1998 and abundant during 2000—2010 in the Southwestern Australia.

Key words:
Wavelet transformation, rainfall variability, Southwestern Australia, hydrology, cyclic pattern.

1. Introduction

It has been becoming important to study these variables to save our self from the climate hazards and to acquire maximum benefits. It is also very important for the water resource management and other socio-economic activities. Rainfall is an important climate factor, which has significant influence on socioeconomic activities and national plans [1]. Factors affecting the decrease in the precipitation in SWWA, existing since the dawn of last century, is open to research and questions. It is quite vague that whether natural variability, greenhouse effect or other anthropogenic influences namely, erosion of land and pollution partake in shortfall in the precipitation. Although, some authors of meteorological fraternity uphold the view of erosion of vegetation cover [2-5]. Being part of the Australia, the southwestern Australia is an agricultural region. Under the continental Mediterranean climate, wet winter and dry summer and its variable rainfall is concentrated in summer. Severe deficit of rainfall and maximum rainfall occur continuously and impose significant impacts on agriculture and economy. Investigations on the attributes and variable trends of the precipitation in the Southwestern Australia are important for weather-climate predictions and continuous development. Time scale analysis for signals and indication, now wavelet analysis is more accurate than Fourier analysis this is because of high resolution attributes, best for both spatial and temporal domains for analyzing indication at multi-step and time [6-7]. Furthermore, wavelet transformation also has the tendency of detecting the future attributes of signals. Wavelet analysis has been widely used for the studies of meteorological multi-time scales analysis. In the past studies, wavelet transform is frequently employed to diagnose the multi-time scales attributes of seasonal precipitation, and significant findings have been reported [8-10]. The Southwest Western Australia (SWWA) is the important state of Australia, there is a need to explore the mechanism of the seasonal rainfall attributes and changing patterns in the SWWA. The objective of this study is to investigates using the wavelet analysis, the patterns of seasonal precipitation variability on numerous time scales because this exploration mainly the outcomes of a strong seasonal relationship, rather than due to a change in trend or a long time series of rainfall. The analysis done in this study may provide useful insights to the issues that are related to water resource management and short-term climate predictions in SWWA and related areas.

2. Data

The Monthly MSLP data set (2.5 degrees latitude by 2.5 degrees longitude) was obtained from National Centers for Environmental Prediction (NCEP) [11]. The dataset was used to calculate the Indian Ocean subtropical high (IOSH) (82.5 to 137.5 °E – 10 to 50 °S) pressure based three objective indices (IOSHPS, IOSHLN and IOSHLT)

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for winter (May-August) for the period of 1951-2013. Monthly precipitation data set (0.05 degrees latitude by 0.05 degrees longitude), constructed by the Australian Bureau of Meteorology [12], used to investigate winter (May-August) rainfall (1951-2013) over SWWA, Australia. Total rainfall monthly observation data set for SWWA was obtained from the Australian Bureau of Meteorology (BOM) which used for correlation analysis. Data sets for composite maps (60 to 155 °E – 10 to 50 °S) for winter (May-August) SLP, vector wind, and vertical wind, and the ENSO [13], SOI and SAM indices were also acquired from NCEP. The monthly data for the IOD index [14] was obtained from Japan Agency for Marine-Earth Science and Technology (JAMSTEC).

3. Methods

In this section we describe each method in details to provide the better insight about the normalization of rainfall and W.L transformation.

3.1 Rainfall normalization

We consider each month as below (above) or normal by calculating each month precipitation at each station in the following manner:

$$Z = \frac{y_i - \bar{y}}{\sigma}$$

where, y_i is the monthly precipitation in the year i, $\bar{y}$ is the mean monthly precipitation and σ is the monthly standard deviation (S.D).

3.2 Wavelet Transformation

The foundation of W.L transform is that a output is compared with a relation β (*time*), often termed to as the mother wavelet. The mother wavelet can be in any function, real or complex, accompanying the admissibility condition expressed as [15, 16]. The W.L is a complex form of the wavelet function α(t) with the signal y(t). Orthogonal wavelets are related with functions β(t). This function can be complex form with the signal to bring an approximation coefficients Q. The discrete wavelet transform (DWT) can be expresses as

$$T_{p,q} = \int_{-\infty}^{\infty} y(t) \propto_{p,q} (t) dt \qquad (1)$$

By considering an orthonormal wavelet vectors α p,q(t) we can prepare the original. The estimated values of the output signal at the scale p and location q can be expressed as

$$Q_{p,q} = \int_{-\infty}^{\infty} y(t)\, \beta_{p,q} (t) dt \qquad (2)$$

But the discrete input signal is of limited length N. The output values that can be evaluated is 0 <p < P. Therefore, a discrete estimation of the signal can be expressed as described below equations (3), (4) and (5).

$$y_0(t) = y\, P(t) + \sum_{p=1}^{P} d_p(t) \qquad (3)$$

$$d_p(t) = \sum_{q=0}^{P-p} T_{p,q} \propto_{p,q} (t) \qquad (4)$$

$$y_p(t) = y_{p-1}(t) - d_p(t) \qquad (5)$$

4. Results

In this section we discuss the results obtained by W.L. Table 1 shows the winter precipitation decadal values over the entire temporal domain 1951—2013 in the SWWA. It suggests that the winter precipitation shows decreasing trend in the 1960s and 1980s, with negative slope, and highest in the last two decades, with increasing trend. The decadal winter precipitation in the early part of last decade is average, with rainfall anomaly 0.52 and anomaly percentage 2.75%.

The findings of the time variability of the average winter precipitation are presented in Fig. 1(a), which describes negative pattern of average winter precipitation with trend value -1 mm per 20 years during 1951—2013. Fig. 1(b) describes the periodicity in a sense of oscillation of winter precipitation at various time period over the last 61 years in which x-axis is the temporal domain, and the y-axis is functional value. The magnitude of output is expressed by wavelet coefficient value. To be more clear presentation of the findings, gridded contours are presented: with red (blue) color for positive (negative) anomalies expressing available (scattered) precipitation. The wavelet transformation findings provided the insight about the precipitation changing pattern are varying at various time scales and the complex pattern of precipitations at the low scales are overlapped in the higher scales.

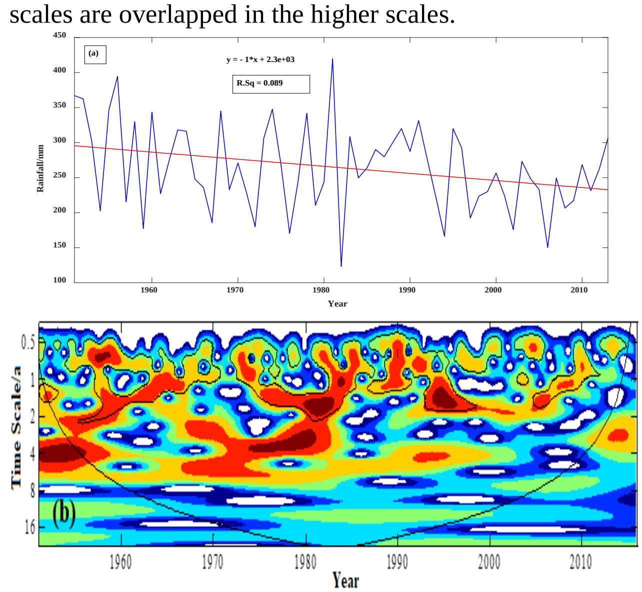


Fig. 1. (a) The seasonal variations of winter precipitation (b) Wavelet transformation

In in Table 2 we have presented the extended winter (MJJA) precipitation anomalous values over the period of 1951-2013 in the SWWA, the pattern describes that the extended winter precipitation is higher than normal from the 1970s to 1990s, with increasing trend, and negative observations during the 1950s and the 1960s, with decreasing pattern.

Figure 2(a) describes that extended winter precipitation showing a negative pattern, the overall trend during the period of 1951—2013 is 37.15 mm per 10 years, which is agreement with the observational analysis of extended winter precipitation variability. Fig. 2(b) describes the time frequency plots of wavelet transformation in the SWWA for the period of 1951-2013. The axes are defined in Fig. 2(b) are similar as in previous Fig. 1(b). The wavelet transformation reveals that at the higher time values of 6—14 years, the extended winter precipitation varies from higher to lowest, experiences two rounds of abundant- scattered precipitation phases, and the point of scattered variation is almost 1985. The results have significant correlation with the higher precipitation observations over decadal time scale. At 3—7 years' time level, extended winter rainfall follows different phases of scattered precipitation stages

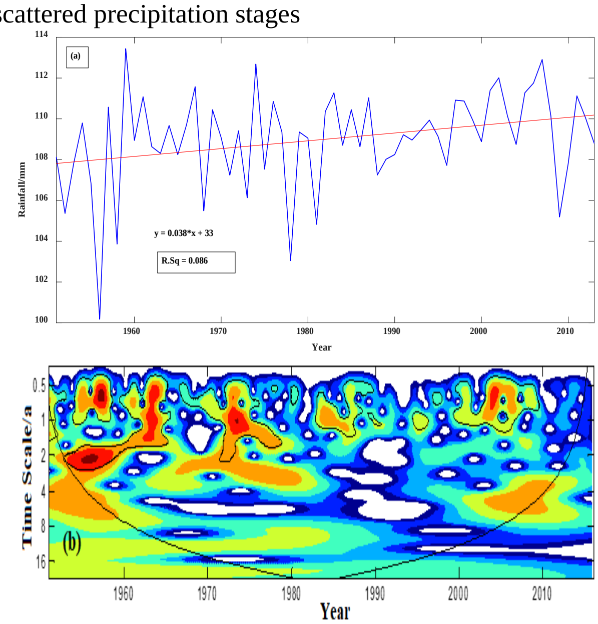
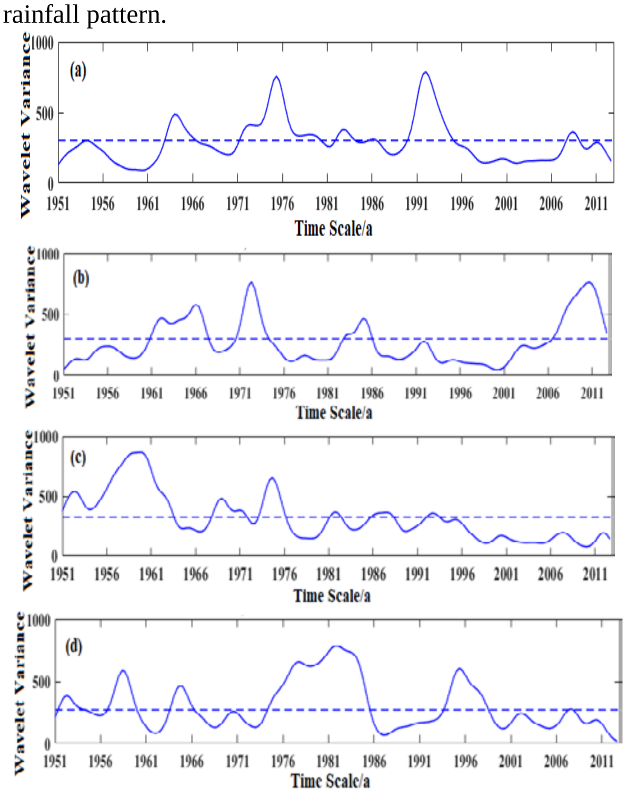


Fig. 2. (a) The seasonal variations of spring precipitation (b) Wavelet transformation

To further gain insight about the important era of extended winter precipitation, we calculated the variance of the seasonal rainfall time series. The known mathematical model to calculate wavelet variance is given by (6)

$$Wg(v) = \int_{-\infty}^{\infty} |W_k(v,\tau)|^2 d\tau \qquad (6)$$

Where, $Wg(v)$ is wavelet variance, $Wk\ (v,\tau\ )$ is wavelet coefficient. The variance factor of W.L describes the distributive pattern of energy with range value ***a***, could be determine the magnitude of variability of temporal record. The time evaluation scale that has a maximum observation is the actual time scale which can exhibit the actual period of the data. As we have shown by equation (6), the variance factor of W.L precipitation record was prepared and the actual segment were calculated (Fig. 3). Fig. 3(a) describes that there are three peaks in the W.L variance graph of winter rainfall, adjacent to 3, 5 and 7 years' time slices. Consider the first peak in a correspondence to 3 years' time scales, is the actual principal period, which shows that the cyclic oscillation of 3 years is the most strengthened. The second important period of winter precipitation variability is 2 years and the third is 6 years. It can be seen that there are three peaks in the W.L variance graph of winter precipitation, corresponding to 3 and 9 years' time level (Fig. 3(b)). The first peak, against to 2 year time level, is the utmost actual era, which mentioned that the cyclic pattern of 2 years are very significant. The next important period is 9 years. For spring precipitation variability (Fig. 3(c)), the important cycles are 3, 1 and 5 years respectively. These findings reveals of significance and 5, 4 and 2 years respectively for autumn precipitation variability (Fig. 3(d)). As we have shown in Figure 3(e), we found two peaks in the W.L variance graphs of annual precipitation, in lieu of 2 and 9 years' time scales. The important periods of annual precipitation trends are 3 and 9 years in sequence of significance level, they are similar with the autumn rainfall pattern.



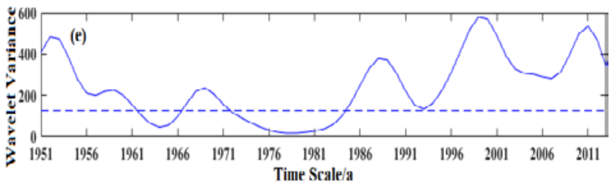


Fig. 3. (a) Variance of Wavelet Autumn (a), spring (b), summer (c), winter (d) and annual rainfall (e).

5. Conclusion

This section of the study provides conclusion and discussion of the about the analysis of rainfall attributes obtained using the W.L transformation over the study region The time series analysis which we have performed for non-stationary, multilevel time slices could be significant in water resource management and agricultural development by analyzing the runoff response to climatic change. We found that yearly precipitation and seasonal rainfall both of them exhibits the several time segments attributes. The cyclic pattern of 6—10 years is superseding for the complete data, and the cyclic pattern for 5 to 8 years is highly visible. The trend analysis of precipitation pattern at the lowest time segment is overlapped in that at the higher time steps. The variability pattern of the winter precipitation is in out of phase with that of the yearly precipitation, the most significant temporal levels of 2 and 9 years. All seasonal and annual precipitations have the important periods of 4 years. The variability pattern and direction of the autumn precipitation is completely aligned with the yearly precipitation. We conclude that the highest the winter precipitation, SWWA receives the maximum rainfall on annual basis, in contrast, SWWA receives the less rainfall in winter, and it experiences the severe deficit on annual basis. The wavelet analysis explores the attributes of temporal level counting and demonstrates the detailed symmetries of precipitation, examine the cyclic pattern which are deeply absorbed in data, determine the fluctuating attributes of precipitation and calculate the overall trend of data. The W.L analysis can be an important diagnostic procedure of investigating the climate multi time scales properties and predict synoptic level climate variability.

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Table 1. Winter precipitation results using W. L during 1951—2013

	1950s	1960s	1970s	1980s	1990s	2000	2010
Scores/mm	−5.21	4.15	−10.32	0.81	2.45	3.51	1.45
Scores percentage (%)	−12.29	5.84	−17.85	2.32	19.98	17.88	20.47

Table 2 Extended winter precipitation scores during 1951—2013

	1950s	1960s	1970s	1980s	1990s	2000	2010
Scores/mm	44.18	25.72	22.1	−37.05	−31.31	−22.15	−15.31
Scores percentage (%)	11.59	6.75	5.80	−9.72	−8.22	−9.32	−9.45