A Connected Graph with Non-concurrent Longest Cycles

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Abstract
The various researchers have introduced graphs with Gallia's property avoiding longest paths by each of its vertices. However in this Contribution, a graph is introduced with the property that each vertex is missed by some longest Cycles. The introduced graphs with Gallia's property have potential applications in the fields of computational geometry, networking and circuit designing.
Key words:
C_i^j, $\bar{C}_i^j$, P_i^j Hamiltonian cycle; longest cycle; Hypo-traceable; Hypo-Hamiltonian; Gallai's property;

1. Introduction

A cycle which contains all the vertices of G is called Hamiltonian cycle. A graph G which is non-Hamiltonian yet $G - v$ is Hamiltonian for all vertices v is known as a Hypo-Hamiltonian. The most renowned case of Hypo-Hamiltonian graph is Petersen graph. A path that takes a break without any redundancies, and does not need to begin and finish at the comparable vertex in graph G is said to be Hamiltonian path. A graph is said to be noticeable or traceable on the off chance that it has a Hamiltonian path. A graph G is a Hypo-traceable if graph G has no Hamiltonian path but deletion of any vertex has a Hamiltonian path for each v ϵ V.
The presence of hypo-Hamiltonian graphs and earlier the modernization of the hypo traceable graphs, in 1966 T. Gallai [13] asked whether there exist connected graphs with the property that every vertex is missed by some longest path. Just later, in 1969, Gallai's question was first responded by H. Walther [14], who introduced a planar graph on 25 vertices satisfying Gallai's property. Later H. Walther and H. Voss [1], and Tudor Zamfirescu [2], introduced such kind of graph with 12 vertices, and it was guessed that order 12 is the smaller possibility of such a graph. In the case of planar graphs, such type of a graph with lowest number of vertices *i-e* with 17 vertices, was provided by W. Schmitz [3]. A smallest non-planar graph of order 34 introduced by Thomassen [12]. The first 2-connected planar graph generated by Tudor Zamfirescu [4] with 82 vertces. The lowest famous example nowadays has 26 vertices [5], on the other hand the lowest example up to now has 32 vertices [4]. The graph G of order 20 with similar property has been introduced in earlier work [15].
During 1972, Tudor Zamfirescu [2] questioned related to the Gallai's property. let $\mathrm{P_i^j} = \infty$ ($\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j} = \infty$) if there is no any $i -$ connected graph (planar graph) such that individually set of j points remains disjoint from some longest path condition $\mathrm{P_i^j} \neq \infty$ ($\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j} \neq \infty$), let $\mathrm{P_i^j}$($\overline{\mathrm{P}}_\mathrm{i}^\mathrm{j}$) indicate the smallest number of vertices of an $i -$connected graph (planar graph) such that individually set of j selected vertices be there disjoint from some largest path. Analogously these cases are clearly C_i^j and $\bar{C}_i^j$ for longest circuits as a replacement for longest path. Further see in [6-12].

2. Results and Discussions

The purpose of this work is to show, that an example of $2 -$ connected graph G, of order 20 satisfying by Gallai's property.

Theorem 1: There is existing a graph of 20 vertices with the property that each vertex is missed by some longest Cycles.
Proof: Consider the graph G of figure 1, With 20 vertices, let W be a longest path in G, the longest cycle of G joining its end points have length $p(G) = 13$ avoiding v with empty intersection of all its longest cycles. Figure 2, shows longest cycles and all vertices are missed by each of them, where each vertex is avoided by some longest cycles. To confirm that all the vertices are avoiding individually by longest cycles, the results are shown in table 1.
Lemma: The graph G has no Hamiltonian cycle.

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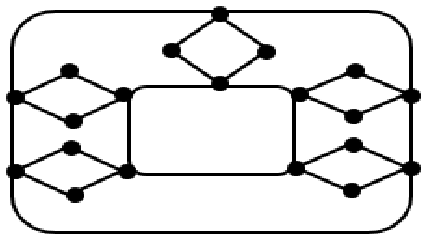

Fig. 1 Planar graph with 20 vertices

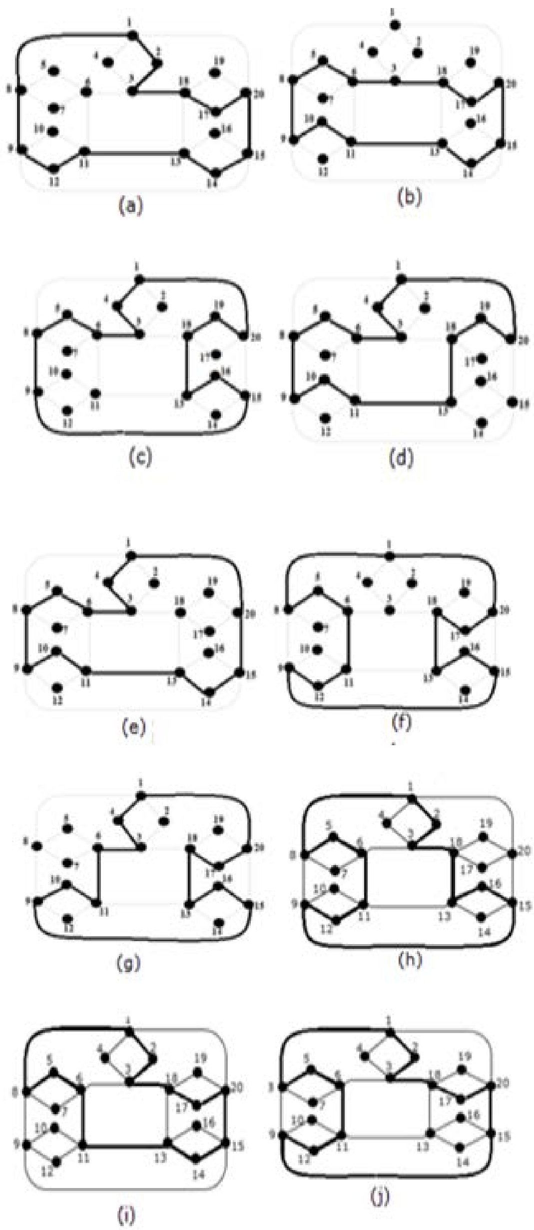

Fig. 2 Longest cycles

Table 1: Results for vertices missed by longest cycles.

Fig.	Longest Cycles	Missed vertices
(a)	1, 2, 3, 18, 17, 20, 15, 14, 13, 11, 12, 9, 8	4, 19, 16, 10, 5, 6, 7
(b)	8, 5, 6, 3, 18, 17, 20, 15, 14, 13, 11, 10, 9	1, 2, 4, 19, 16, 12, 7
(c)	1, 20, 19, 18, 13, 16, 15, 9, 8, 5, 6, 3, 4	2, 17, 14, 7, 10, 11, 12
(d)	1, 20, 19, 18, 13, 11, 10, 9, 8, 5, 6, 3, 4	2, 17, 14, 15, 16, 12, 7
(e)	1, 2, 15, 14, 13, 11, 10, 9, 8, 5, 6, 3, 4	2, 17, 18, 19, 16, 12, 7
(f)	1, 20, 17, 18, 13, 16, 15, 9, 12, 11, 6, 5, 8	2, 3, 4, 7, 10, 19, 14
(g)	1, 4, 3, 6, 11, 10, 9, 15, 16, 13, 18, 17, 20	5, 7, 8, 2, 12, 19, 14
(h)	1, 8, 5, 6, 11, 12, 9, 15, 16, 13, 18, 3, 2	3, 4, 17, 18, 20, 7, 10
(i)	1, 2, 3, 18, 17, 20, 15, 14, 13, 11, 6, 5, 8	4, 19, 16, 9, 10, 12, 7
(j)	1, 2, 3, 18, 17, 20, 15, 9, 12, 11, 6, 5, 8	4, 19, 13, 14, 16, 7, 10

The results shown in a table 1 for the date obtained for longest cycles and missed vertices for graphs of fig a to j. Given results show that we have developed two-connected graph in which each vertices is missed by some longest Cycle, computational results given as in table 1 verifies and confirm our claim that there exists connected graph satisfying Gallai's property

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