Synthesis of Adaptive State Observer for Delayed State and Input System with Unknown Inputs for Irrigation Canal

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Summary
In this paper, a synthesis approach of adaptive observer adjusted for linear continuous delayed state and input systems is proposed. The treated system is exposed to unknown input. The delay is assumed to be variable and the unknown input is assumed to be constant. The contribution of this paper is to suggest a new adaptive observer synthesis methodology for an irrigation canal which represents delayed state and input system when unknown inputs are present. The key step is to use an easy method for the design of this observer which is the coupling of a state observer and an improved adaptation law employed to estimate the unknown parameters and the states of the system as well. The conditions for the presence of this observer are cleared. By applying information on asymptotic stability, we can propose methodology for synthesizing a linear adaptive observer to ensure the global uniform asymptotic convergence of the error's estimation. According to Lyapunov stability approach, the stability conditions are resolved with linear matrix inequalities (LMIs) concept. Simulations results illustrate the design approach and the effectiveness of obtained stability conditions and also have a good robustness to unknown inputs. The method of observer design is applied to irrigation canal to prove the validity of the found results.

Key words:
Adaptive state observer -Lyapunov stabilty LMIs technique-Unknown input-Irrigation Canal

1. Introduction

The problem of state observation for linear time delay systems interested various researchers [1, 2, 3]. It is obvious that the state observer synthesis problem for linear systems including unknown inputs attracted many researches and many efforts have been putted to design a state observer during the past decades [4, 5, 6, 7, 8, 9, 10,11, 12, 13 14, 15, 16].

Many researches usually refer to conventional observers to consider the problem of delayed state and input systems with unknown inputs such as state observer or observer based feedback control but these observers work according to the fact that the system dynamic are known but we can't avoid unknown parameters in the system like unknown inputs or disturbance which make this conventional observers not exact. For that, we have to resort to adaptive observer to treat the problem of these systems with unknown inputs. It is necessary to design an observer that can guarantee the rightness of estimated states of linear systems in the existence of unknown parameters.

Over the last decade, adaptive observer design becomes an important and active field that attracted many researchers [32, 33, 34]. In General, the goal of adaptive observer is to estimate simultaneously the unmeasured states and the unknown parameters. These observers are usually useful in the adaptive control, isolation and fault detection ([27], [28]). We refer to an optimization based approach to treat the synthesis of adaptive observer [29] and the presence of the adaptive observer based on the feasibility of a set of linear matrix inequalities. These results are found to ensure the presence of some Lyapunov functions verifying certain conditions.

A rather small research has been concentrated on adaptive observer design for linear systems included state and input delay and unknown inputs as well. Effective design of adaptive observer consists on the ability to verify stability and come up with stabilizing gains of the observer and also the right choice of a stable adaptation law [17, 24, 25, 26].

It is well known there are numerous methods used in the last decade for the observer synthesis such as LMI method [10, 18], an integral inequality approach method [20] free weighting matrix method [19], while the last method [19] still a very hard method and leads to a long and heavy calculation resolution to solve it. For avoiding such complication and difficulty, new convex optimization resolution for the system's stability including time delays and unknown inputs and are formulated by referring to Lyapunov method pressed on the good choice of the Lyapunov functional and can be applicable in the state observer design. The stability equations of the observation error system are formulated according to LMIs technique.

Irrigation presents more than 70% of universal water consumption and huge waste of water happen in irrigation canals because of bad management. Water conservation may be ameliorating by the implementation of automatic control system. For that, in this paper we suggest a new and easy methodology of adaptive observer synthesis for irrigation canal system. Comparing to the obtained results in the literature [21, 22], this paper considers the design

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problem of unknown input observer for a more general category of linear time delay systems whose the delays are found in the state and also in the input. Adding to that, the treated linear time delay system contains more than one delay. We can also extend our method to non linear case [30, 31].

This paper is presented as following. In Section 2, we consider the problem under discussion. In Section 3, we suggest a method of adaptive observer: we treat the equations and conditions of stability of the suggested observer. Simulations examples are given in Section 4 and to prove rightness of our method we applied it to irrigation canal in Section 5 and we discuss the results in Section 6 and finally a conclusion is presented in last section.

2. Problem Formulation

We can write the delayed state and input system including unknown inputs as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=0}^{2} A_i x(t-\tau_i(t)) + \sum_{i=0}^{2} B_i u(t-\tau_i(t)) + Ww(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

With:

$x \in \Re^n$ is the system state.

$u \in \Re^m$ is the system input.

$w(t)$ is the unknown input or disturbance vector.

τ_i : Time delays verifying:

$0 \le \tau_i(t) \le h_i$, $\dot{\tau}_i(t) \le d_i < 1$ $(i = 1, 2)$, For all $t \ge 0$

$\tau_0(t) = 0$

$(i = 0, 1, 2)$, A_i, B_i, C and W are known constant matrices with their suitable dimensions.

In general, C can be suggested to be of full rank and W has a full column rank.

In this control system, we prove that the system state $x(t)$ can be rebuilded from the input of the system $u(t)$.

The obtained system (1) is delayed state and input system with unknown input.

In the next section, it is necessary to develop an adaptive state observer that rebuilds the state $x(t)$ by the use of the available output of the system $y(t)$ and the available input $u(t)$ in order that the estimation error is convergent asymptotically to zero value when the time delays, the unknown input are present in the system verifying:

$$\lim_{t\to\infty} [x(t) - \hat{x}(t)] = 0 \tag{2}$$

3. Adaptive Observer Design for Delayed State and Input Systems with Unknown Inputs

The aim of this section is to propose an adaptive state observer for a hydraulic channel model including state and input time delays and unknown inputs as well. We design a category of adaptive state observer with a simple structure which is dependent of the time delays and we verify that the estimation error is convergent asymptotically to zero value in the existence of unknown inputs. We provide also the conditions and the equations to check the asymptotic stability of the errors observation in the existence of the time delays and unknown parameters.

For the treated system described by equation (1), we suggest the next adaptive state observer:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=0}^{2} A_i \hat{x}(t-\tau_i(t)) + \sum_{i=0}^{2} B_i u(t-\tau_i(t)) + L(y(t) - \hat{y}(t)) + W\hat{w}(t) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{3}$$

With $\hat{x}(t) \in \Re^n$ is the vector of observer state, $\hat{y}(t) \in \Re^m$ is the vector of observer output; $L \in \Re^{n*m}$ is a constant matrix called the gain matrix of the observer and $\hat{w}(t)$ is the estimate of the unknown input $w(t)$.

The state observer in equation (3) is combined with the equation of unknown input estimate updating by the following adaptation law represented as:

$$\dot{\hat{w}}(t) = SQB_0^T W\tilde{x}(t) \tag{4}$$

Where:

$S \in \Re^{s*s}$ is a learning weight matrix that we have to set it interactively, $S = S^T > 0$, B_0 and W are constant matrices and $Q \in \Re^{m*s}$ is the adaptation law gain matrix.

The synthesis of the observer matrix parameters has to verify that the observation error in equation (5) is convergent to zero value defining:

$$\tilde{x}(t) = x(t) - \hat{x}(t) \tag{5}$$

We note:

$$\tilde{y}(t) = C\tilde{x}(t) \tag{6}$$

We introduce the following error's adaptation of the system:

$\tilde{w}(t) = w(t) - \hat{w}(t)$

Now, we introduce the next theorem that considers that the state estimate $\hat{x}(t)$ of adaptive state observer (3) with the estimate $\hat{w}(t)$ given in the adaptation law (4) is convergent asymptotically to the real state $x(t)$.

After that, we give a new methodology to calculate the adaptive state observer parameters using the linear matrix inequalities (LMIs).

For that, we can present the following Theorem.

Theorem: The adaptive observer for delayed state and input systems with unknown inputs is stable if we have asymmetric and positive definite matrices $W > 0,\quad P > 0,\quad N_1 > 0,\quad Q_1 > 0,\quad Y > 0,\quad I > 0,\quad R > 0$ $R = \eta I, \eta > 0$

Such that:

$$H = \begin{bmatrix} H_{11} & * & 0 & N_1^T A_1 & N_1^T A_2 \\ H_{22} & -\sigma(N_1 + N_1^T) & \sigma N_1^T W & \sigma N_1^T A_1 & \sigma N_1^T A_2 \\ 0 & \sigma W^T N_1 & 0 & 0 & 0 \\ A_1^T N_1 & \sigma A_1^T N_1 & 0 & 0 & 0 \\ A_2^T N_1 & \sigma A_2^T N_1 & 0 & 0 & 0 \end{bmatrix} < 0 \tag{7}$$

With:

$$H_{11} = N_1^T A_0 + A_0^T N_1 - YC - C^T Y^T + Q_1 + C^T RC \tag{8}$$

$$H_{22} = P + \sigma N_1^T - N_1 - \sigma YC \tag{9}$$

And

$$N_1^T W = W^T B Q^T \tag{10}$$

If the previous conditions are satisfied, the adaptive state observer gain matrix is mentioned as:

$$L = N_1^{-1} * Y \tag{11}$$

And the adaptation law is mentioned as:

$$\dot{\hat{w}}(t) = SQB_0^T W \tilde{x}(t)$$

Proof: From system model in equation (1) and observer model in equation (3), we present the following dynamic of state estimation error as:

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{\hat{x}}(t)$$

$$\sum_{i=0}^{2} A_i \tilde{x}(t - \tau_i(t)) - LC\tilde{x}(t) + W\tilde{w}(t) - \dot{\tilde{x}}(t) = 0 \tag{12}$$

The Lyapunov function is constructed as:

$$V(e,t) = \tilde{x}^T(t)P\tilde{x}(t) + \tilde{w}^T(t)S^{-1}\tilde{w}(t) + \int_0^t (\tilde{x}^T(s)Q_1\tilde{x}(s)ds + \tilde{y}^T(s)R\tilde{y}(s)ds) \tag{13}$$

Where:

$P = P^T > 0, S = S^T > 0$, $Q_1 = Q_1^T > 0$, $R = R^T = \alpha I > 0$

Deriving $V(e,t)$ for $t \in [0, +\infty]$ yields:

$$\begin{aligned} \dot{V}(e,t) &= \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t) + \dot{\tilde{w}}^T(t)S^{-1}\tilde{w}(t) + \tilde{w}^T(t)S^{-1}\dot{\tilde{w}}(t) \\ &+ \tilde{x}^T(t)Q_1\tilde{x}(t) + \tilde{y}^T(t)R\tilde{y}(t) \end{aligned} \tag{14}$$

From equation (12), we can get the following equation (15):

$$(\tilde{x}^T(t)N_1^T + \dot{\tilde{x}}^T(t)N_2^T)(\sum_{i=0}^{2} A_i\tilde{x}(t - \tau_i(t)) - LC\tilde{x}(t) + W\tilde{w}(t) - \dot{\tilde{x}}(t)) = 0$$

$$\dot{\tilde{w}}(t) = \dot{w}(t) - \dot{\hat{w}}(t) = -SQB^T W\tilde{x}(t) \tag{15}$$

Adding equation (15) to equation (14), we can get,

$$\begin{aligned} \dot{V}(e,t) &= \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t) + \dot{\tilde{w}}^T(t)S^{-1}\tilde{w}(t) + \tilde{w}^T(t)S^{-1}\dot{\tilde{w}}(t) \\ &-(\tilde{x}^T(t)N_1^T + \dot{\tilde{x}}^T(t)N_2^T)(LC\tilde{x}(t) + \dot{\tilde{x}}(t)) \\ &-(LC\tilde{x}(t) + \dot{\tilde{x}}(t))^T(N_1\tilde{x}(t) + N_2\dot{\tilde{x}}(t)) \\ &+\tilde{w}^T(t)W^T(N_1\tilde{x}(t) + N_2\dot{\tilde{x}}(t)) \\ &+(\tilde{x}^T(t)N_1^T + \dot{\tilde{x}}^T(t)N_2^T)W\tilde{w}(t) \\ &+(\tilde{x}^T(t)N_1^T + \dot{\tilde{x}}^T(t)N_2^T)\sum_{i=0}^{2} A_i\tilde{x}(t - \tau_i(t)) \\ &+\sum_{i=0}^{2} \tilde{x}^T(t - \tau_i(t))A_i^T(N_1\tilde{x}(t) + N_2\dot{\tilde{x}}(t)) \\ &+\tilde{x}^T(t)Q_1\tilde{x}(t) + \tilde{y}^T(t)R\tilde{y}(t) \end{aligned} \tag{16}$$

Then, with using equation (4), we get:

$$\begin{aligned} \dot{V}(e,t) &= \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t) - \tilde{x}^T(t)W^T BQ^T\tilde{w}(t) \\ &+\tilde{w}^T(t)W^T(N_1\tilde{x}(t) + N_2\dot{\tilde{x}}(t)) - \tilde{w}^T(t)QB^T Wx(t) \\ &+\sum_{i=0}^{2} \tilde{x}^T(t - \tau_i(t))A_i^T(N_1\tilde{x}(t) + N_2\dot{\tilde{x}}(t)) \\ &+(\tilde{x}^T(t)N_1^T + \dot{\tilde{x}}^T(t)N_2^T)W\tilde{w}(t) \\ &+(\tilde{x}^T(t)N_1^T + \dot{\tilde{x}}^T(t)N_2^T)\sum_{i=0}^{2} A_i\tilde{x}(t - \tau_i(t)) \\ &-(LC\tilde{x}(t) + \dot{\tilde{x}}(t))^T(N_1\tilde{x}(t) + N_2\dot{\tilde{x}}(t)) \\ &+\tilde{x}^T(t)Q_1\tilde{x}(t) + \tilde{y}^T(t)R\tilde{y}(t) \\ &-(\tilde{x}^T(t)N_1^T + \dot{\tilde{x}}^T(t)N_2^T)(LC\tilde{x}(t) + \dot{\tilde{x}}(t)) \end{aligned} \tag{17}$$

The effect between of the unknown input error's estimation $\tilde{w}(t)$ and the state estimation error $\tilde{x}(t)$ is set to zero and gives:

$$\tilde{w}^T(t)W^T N_1\tilde{x}(t) - \tilde{x}^T(t)N_1^T W\tilde{w}(t) - \tilde{x}^T(t)W^T B_0 Q^T\tilde{w}(t) - \tilde{w}^T(t)QB_0^T W\tilde{x}(t) = 0 \tag{18}$$

Equation (18) leads to the following condition:

$$N_1^T W - W^T B_0 Q^T = 0 \tag{19.a}$$

Equation (17) is simplified as:

$$\dot{V}(e,t) = \dot{\tilde{x}}^T(t)P\tilde{x}(t) + \tilde{x}^T(t)P\dot{\tilde{x}}(t)$$

$$-(\tilde e^T(t)N_1^T + \dot{\tilde e}^T(t)N_2^T)(LC\tilde e(t) + \dot{\tilde e}(t))$$
$$+\tilde w^T(t)W^T N_2\dot{\tilde e}(t) + \dot{\tilde e}^T(t)N_2^T W\tilde w(t)$$
$$+(\tilde e^T(t)N_1^T + \dot{\tilde e}^T(t)N_2^T)\sum_{i=0}^{2} A_i\tilde e(t-\tau_i(t))$$
$$+\sum_{i=0}^{2}\tilde e^T(t-\tau_i(t))A_i^T(N_1\tilde e(t) + N_2\dot{\tilde e}(t))$$
$$-(LC\tilde e(t) + \dot{\tilde e}(t))^T(N_1\tilde e(t) + N_2\dot{\tilde e}(t))$$
$$+\tilde e^T(t)Q_1\tilde e(t) + \tilde y^T(t)R\tilde y(t) \qquad (19.b)$$

Using equation (3), we get the following equation:

$$\dot V(e,t) \le \dot{\tilde e}^T(t)P\tilde e(t) + \tilde e^T(t)P\dot{\tilde e}(t)$$
$$-(\tilde e^T(t)N_1^T + \dot{\tilde e}^T(t)N_2^T)(LC\tilde e(t) + \dot{\tilde e}(t))$$
$$+\tilde w^T(t)W^T N_2\dot{\tilde e}(t) + \dot{\tilde e}^T(t)N_2^T W\tilde w(t)$$
$$+(\tilde e^T(t)N_1^T + \dot{\tilde e}^T(t)N_2^T)(A_0\tilde e(t) + A_1\tilde e(t-d_1))$$
$$+(\tilde e^T(t)N_1^T + \dot{\tilde e}^T(t)N_2^T)A_2\tilde e(t-d_2)$$
$$+(\tilde e^T(t)A_0^T + \tilde e^T(t-d_1)A_1^T)(N_1\tilde e(t) + N_2\dot{\tilde e}(t))$$
$$+(\tilde e^T(t-d_2)A_2^T)(N_1\tilde e(t) + N_2\dot{\tilde e}(t))$$
$$-(LC\tilde e(t) + \dot{\tilde e}(t))^T(N_1\tilde e(t) + N_2\dot{\tilde e}(t))$$
$$+\tilde e^T(t)Q_1\tilde e(t) + \tilde y^T(t)R\tilde y(t) \qquad (20)$$

Equation (20) is presented as follows:

$$\dot V(e,t) = \tilde X^T(t)H\tilde X(t) < 0 \qquad (21)$$

With:

$$\tilde X(t) = \begin{bmatrix} \tilde e(t) & \dot{\tilde e}(t) & \tilde w(t) & \tilde e(t-d_1) & \tilde e(t-d_2) \end{bmatrix} \qquad (22)$$

And

$$H = \begin{bmatrix} H_{11} & * & 0 & N_1^T A_1 & N_1^T A_2 \\ H_{22} & -N_2^T - N_2 & N_2^T W & N_2^T A_1 & N_2^T A_2 \\ 0 & W^T N_2 & 0 & 0 & 0 \\ A_1^T N_1 & A_1^T N_2 & 0 & 0 & 0 \\ A_2^T N_1 & A_2^T N_2 & 0 & 0 & 0 \end{bmatrix} < 0 \qquad (23)$$

Where:

$$H_{11} = N_1^T(A_0 - LC) + (A_0 - LC)^T N_1 + Q_1 + C^T RC \qquad (24)$$
$$H_{22} = P + N_2^T(A_0 - LC) - N_1 \qquad (25)$$

After putting some substitutions:

$$N_1^T L = Y \qquad (26)$$
$$N_2 = \sigma N_1, \sigma > 0 \qquad (27)$$

Now, we can finish the proof in the theorem.

4. Illustrative Examples

To prove the use of our approach, in this section, we treat the next numerical example

Example1: delayed state and input system with unknown input is represented by the next linear system described in (28):

$$\begin{cases} \dot x(t) = A_0 x(t) + A_1 x(t-\tau_1) + A_2 x(t-\tau_2) + B_0 u(t) + B_1 u(t-\tau_1) + B_2 u(t-\tau_2) + Ww(t) \\ y(t) = Cx(t) \end{cases}$$

With:

$$A_0 = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix},\ A_1 = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix},\ A_2 = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix},$$

$$B_0 = B_1 = B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ C = \begin{bmatrix} 2 & 0 \end{bmatrix}\ W = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix},$$

$$0 \le \tau_i(t) \le h_i,\ \dot\tau_i(t) \le r_i < 1\ (i = 1, 2),$$
$$\tau_0(t) = 0$$
$$\dot\tau_u = \dot\tau_d = 0$$
$$\tau_u = 60s;\ \tau_d = 50s$$
$$\tau_1 = \tau_u;\ \tau_2 = \tau_d;\ r_1 = r_2 = 0$$

Because of the system structure, we consider only two delays (τ_1, τ_2) but the proposed method can be extended in case of more than two delays.

Solution:

For the simulation, we select the following parameters: $\eta = 0.2$ and $\sigma = 0.1$

With use of the LMI Toolbox in MATLAB, we can solve the inequality (21), we get:

$$P = \begin{bmatrix} 0.0096 & -0.0087 \\ -0.0087 & 0.0022 \end{bmatrix}\ Y = \begin{bmatrix} -0.2464 \\ -0.0045 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 0.0049 & 0 \\ 0 & -0.0031 \end{bmatrix}\ N_1 = \begin{bmatrix} 0.0011 & 0.0002 \\ 0.0002 & 0.0012 \end{bmatrix}$$

Now, we compute the observer gain parameters.

So that the observer gain matrix obtained using Equation (11) is:

$$L = \begin{bmatrix} -233.728 \\ 34.4314 \end{bmatrix}$$

Solving Condition described in Equation (19.a), we obtain the adaptation law gain matrix:

$$Q = \begin{bmatrix} 0.0011 & 0.0002 \\ 0.0002 & 0.0012 \end{bmatrix}$$

At last, we set the learning weight matrix as:

$$S = \begin{bmatrix} 15 & 0 \\ 0 & -1 \end{bmatrix}$$

From (3) and (4) and after getting the parameters of adaptive state observer for the treated systems, we can design an adaptive state observer with an easy structure for this numerical example.
It is clear from the above Theorem that the estimated states of the state adaptive observer converges asymptotically to the real state of the system of the time delay system with unknown inputs.
The simulation of the original system's states, the system estimated states and the observation error are represented in Fig 1 and Fig 2.

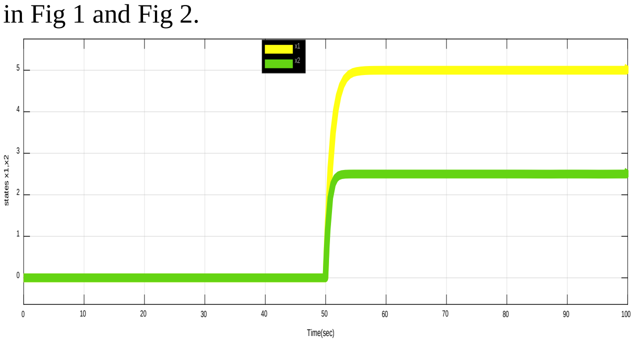

Fig. 1 The simulation of the original states of the system

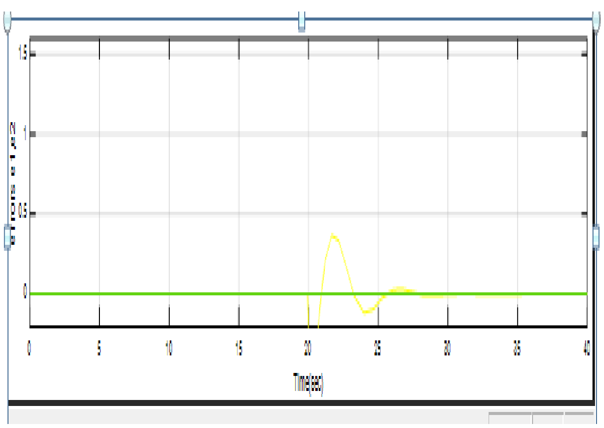

Fig. 2 The simulation of the error's observation

After taking the chosen parameters, the simulation's results of this example are seen in Fig 1and Fig 2.
Fig 1 shows the trajectories of system states. It is demonstrated from Fig 2 and from the trajectory of the error's observation that the estimated states track the original states and the stability of states is satisfied.
It can be confirmed from Fig 2 that the estimated states of the suggested adaptive state observer converges asymptotically to the real state of the time delay systems with unknown input illustrated in (1).

5. Application to Irrigation Canal

Due to bad management in irrigation canals, a significant loss of water is generated.
For that, we need to implant an automatic system which can supervise any kind of disturbance, fault or malfunction in sensor or actuator (regulation gates). These parameters are assumed to be unknown and the irrigation canals represents a delayed state and input system with unknown inputs which can form the system to be treated.
For that, in this part we will consider our application which is presented also in [15] to prove the utilization of our techniques.
For a canal pool, the system is touched by backwater and partly is in uniform regime, the part touched by uniform flow is suggested to have a pure delay.
The considered inputs represent the upstream and the downstream relative discharges and the outputs represent the upstream and the downstream relative water depths.
The model of the considered canal is presented

by: $A_0 = 10^{-3}\begin{bmatrix} -0.4084 & 0 \\ 0 & -0.2787 \end{bmatrix}$,

$A_1 = 10^{-3}\begin{bmatrix} 0 & -0.2432 \\ 0 & 0 \end{bmatrix}$, $A_2 = 10^{-3}\begin{bmatrix} 0 & 0 \\ -0.4675 & 0 \end{bmatrix}$,

$B_0 = 10^{-3}\begin{bmatrix} 0.55 & 0 \\ 0 & -0.42 \end{bmatrix}$,

$B_1 = 10^{-3}\begin{bmatrix} 0 & -0.3714 \\ 0 & 0 \end{bmatrix}$, $B_2 = 10^{-3}\begin{bmatrix} 0 & 0 \\ 0.632 & 0 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$W = B_0$

$0 \le \tau_i(t) \le h_i, \dot{\tau}_i(t) \le r_i < 1 \ (i = 1, 2)$,

$\tau_0(t) = 0$

$\tau_1 = \tau_u;\ \tau_2 = \tau_d;\ r_1 = r_2 = 0$

With:

τ_u : delay related to the upstream water level.

τ_d : delay related to the downstream water level.

We simulate this algorithm to solve the problem of hydraulic channel model included time delays and unknown inputs.

Solution:

For the presented canal, the delays are constant.
$(\dot{\tau}_u = \dot{\tau}_d = 0)\ \tau_u = 846.5s;\ \tau_d = 707.5s.$

For the simulation, we select the following parameters:
$\eta = 0.2$ and $\sigma = 0.1$

With use of the LMI Toolbox in MATLAB, we can solve the inequality (21), we get:

$$P = \begin{bmatrix} 1.8983 & -0.1358 \\ -0.1358 & 2.4643 \end{bmatrix};\ Y = \begin{bmatrix} 0.0999 & 0.0009 \\ 0.0009 & 0.1009 \end{bmatrix};$$

$$Q_1 = \begin{bmatrix} 0.0075 & 0.0003 \\ 0.0003 & 0.0058 \end{bmatrix},\ N_1 = \begin{bmatrix} 2.1205 & -0.1508 \\ -0.1508 & 2.7497 \end{bmatrix},$$

So that the observer gain matrix obtained using equation (11) is:

$$L = \begin{bmatrix} 0.0473 & 0.003 \\ 0.0029 & 0.0369 \end{bmatrix}$$ Solving condition described in Equation (19.a), we obtain the adaptation law gain matrix:

$$Q = \begin{bmatrix} 385.5399 & 35.9055 \\ -27.4188 & -654.681 \end{bmatrix}$$

Now, setting the learning weight matrix as:

$$S = \begin{bmatrix} 15 & 0 \\ 0 & 2 \end{bmatrix}$$

From equations (3) and (4) and after getting the adaptive state observer parameters for the irrigation canal, we can design an adaptive state observer with an easy structure for this simulation example.

We can assume from Theorem that the estimated states of the state adaptive observer are convergent asymptotically to the real state of the system of the time delay system with unknown inputs.

The simulation of the original system states and the observation error are illustrated in Fig 3 and Fig 4.

As one can observe from Fig 3 that the original states have slight chattering due to unknown input or disturbance of the system and have an appropriates amplitudes. It is illustrated from Fig 4 that the error's observation are performing well and are convergent to zero in precise time which confirm the robustness of the suggested observer in the existence of unknown input or disturbance.

According to simulation results, we can assume that the designed adaptive observer is effective for the stability of the treated time delay system.

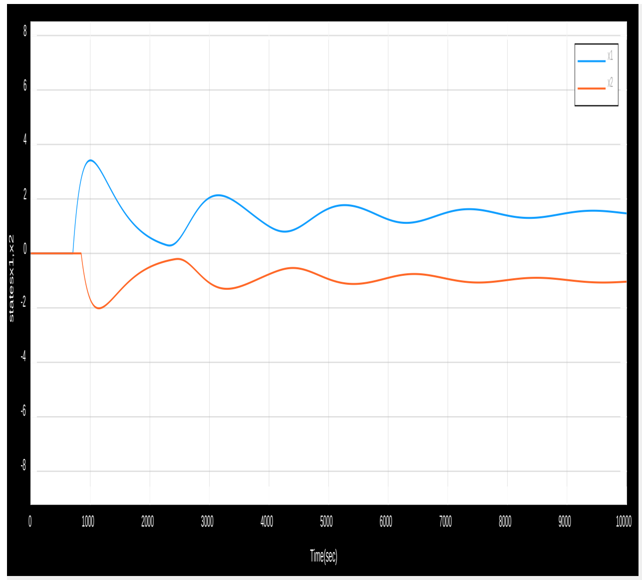

Fig.3 the simulation of the original states of the system

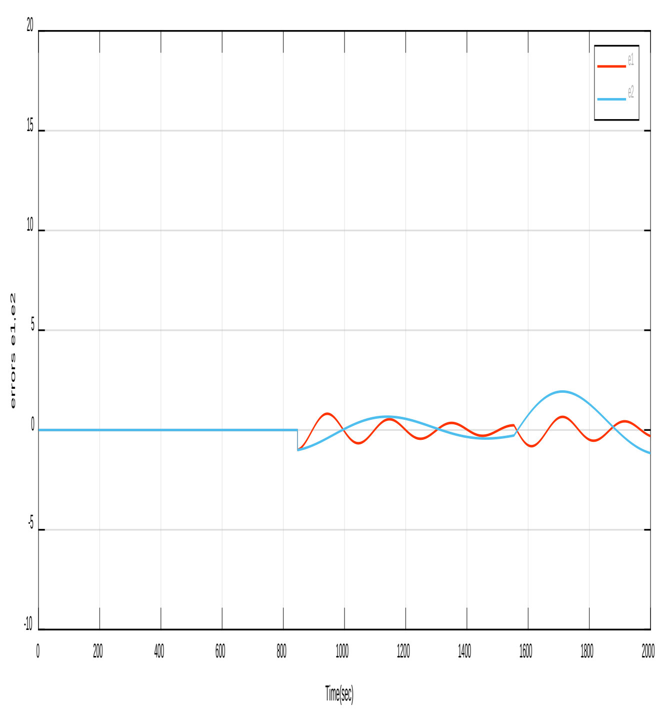

Fig. 4 the simulation of the error's observation

6. Discussion

This example has been treated in [14]. In [14], there was a very long process to compute the observer gain matrix. However, the observer dynamics can't be selected while the suggested algorithm gives more freedom degrees to select the observer gain. While in our work, we give a simple algorithm to get the adaptive observer gain that satisfy the asymptotic convergence of the error's estimation. The chattering effect is expected from the presence of unknown input and state and input delays.

7. Conclusion

In this paper, we propose an adaptive observer for time delay systems with unknown inputs.

State estimation and asymptotic convergence are both shown to be ensured if the LMI's are satisfied. Irrigation canal problem included time delays and unknown inputs have been considered. The synthesis of adaptive observer for time delay system has been treated and cast into a framework of a convex optimization. The suggested methodology of design of adaptive observer is easy to use because it does not include any parameter to be chosen before. The results of the simulations of the application to irrigation canal are given to prove the rightness and the validity of the synthesized adaptive observer. The main properties of the suggested observer are proven in simulation across an example dealing with the application to irrigation canal

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