Graphical method for obtaining PID parameters for systems with time delay

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Summary
In this paper, a new method for the computation of the parameters of a PI controller is given. The proposed method consists of plotting the stability boundary locus in the plane then the computing stabilizing PI controllers. The given technique require the use of a Pade approximation but it is not necessary to sweep over the parameters also to resolve a set of inequalities a linear programming is not imperative. Thus, it has several important advantages over existing results. In order to evaluate the quality and performance of the new approach, the proposed method is compared with a graphical method called D-partition method. The new method allows obtaining a minimum time domain measures of the closed loop system such as maximum percent overshoot and setting time. Finally, two numerical examples are presented to demonstrate the effectiveness of the proposed method.

Key words:
System with constant time delay; PI regulator; D-partition method; New graphical method.

1. Introduction

The phenomena of delay appear naturally in the modeling of many dynamic systems, for example biological system, communication network, teleoperated system, hydrolic systems…
Indeed, the delay has always been considered one of the most difficult problems encountered in controlling systems. Its presence has a considerable influence on the behavior of a closed-loop system and can even be the cause of instability or unwanted oscillations. In this way, stability of time-delay systems has been attracting the attention of many researchers.
Otherwise, there are many of controllers that are used to obtain a stable system.
The PID controller is one of the most used regulators for controlling system with time delay and it is widely applied in the industry because of its simple structure.
So they are deemed to be satisfactory and robust for the big majority of processes. There exist many different methods to find appropriate controller parameters [1].
There are two types of methods used in automatic for the calculation of the parameters of the PID regulator: analytical method such as the Ziegler-Nichols method (Ziegler and Nichols, 1942), which compromises between regulation (disturbance rejection) and tracking behavior [2] and graphical method such as D-decomposition method.
The D-decomposition or D-partition method was used respectively in [3,4,5] and [6], to determine the stability regions of a PID controller and a first-order controller for a linear delay system.
The work, developed in this paper, focuses on the determining of the stability regions of such a PI regulator applied to a linear system with constant time delay using a new graphical method of tuning PI parameters. This paper is an extension of the results given in [7] in which a proposed method was used to calculate all the parameters of a PI controller to stabilize system without time delay.
Indeed, a new method of computation of stabilizing PI controller for a First Order Plus Dead Time (FOPDT) and second order system with time delay (SOPDT) is proposed based on the results obtained in the case of a regular system given in [7,8,9].
Therefore, a new method for computation of the parameters of a PI regulator for system with constant time delay is given.
A graphical technique for obtaining the stability region of a PI parameter is described in this article that requires the use of a first order Pade approximation for more precision in determining the stability region. This method is very interesting since it can cope with systems that are open loop stable or unstable and can occur best performance measures of the step response of the closed loop system.
This paper is organized as follows: The D-partition method will be introduced in section 2 to identify the stability domain in space of controller parameters [10, 11].
Then, a new method is given for the calculation of the parameters of a stabilizing PI controller.
An application of the presented methods to FOPDT and SOPDT is given in section 3.
In the last part, a comparison between the results obtained using two methods of optimizing the PI parameters is deducted.
This comparison is made based on three performance measures as: maximum percent overshoot, setting time and rise time. In the end, conclusion section offers some conclusion remarks.

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2. Graphical Methods for Obtaining Stabilizing PI Controller

2.1 D-partition Method

In this section, we are interested of how to determine the set of all stabilizing regions in the parameter space of the PI controller.
Consider the nth order single-input single output control system shown in Fig.1 where the transfer function is defined as follow:

$$G(\mathrm{s})=\frac{N(\mathrm{s})}{D(\mathrm{s})}e^{-\tau p}=\frac{b_m s^m+b_{m-1}s^{m-1}+\ldots+b_0}{a_n s^n+a_{n-1}s^{n-1}+\ldots+a_0}e^{-\tau p} \quad (1)$$

Where $\tau > 0$ is the time delay.

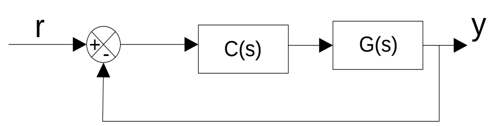

Fig. 1 Feedback control system

C(s) is a PI controller with the transfer function:

$$\mathrm{C(s)}=K_p+\frac{K_i}{s} \quad (2)$$

where K_p is the proportional parameter and K_i is the integral parameter.
So, our purpose is to calculate the parameters of the PI controller defined in the previous equation which guarantee the stabilization of the system (1) as well as the best performances in terms of maximum percent overshoot, rise time and setting time. But it is difficult to calculate the PI parameters analytically. Therefore, the calculation becomes easier and faster using a graphical method. In this way, the problem of determining the exact set of PI parameters is solved by using the D-decomposition which allows us to deduce the region of asymptotic stability in the plane of parameters (X, Y) [3,4,12].
D-partition is Classical method that represents a simple and efficient computational method of determining the asymptotic stability region in the parameter space [3].
This method allows us to obtain a set of PI gain values that stabilize our closed-loop system while transforming the characteristic polynomial $\Phi_1(\mathrm{s})$ of the system presented in figure.1 from the time domain into the frequency domain.
Either the characteristic polynomial of our closed-loop system as described below:

$$\Phi_1(\mathrm{s})=s\,\mathrm{D(s)}+(\mathrm{k}_p\,s+k_i)\,\mathrm{e}^{-\tau s} \quad (3)$$

By multiplying equation (3) by $\mathrm{e}^{\tau s}$, $\Phi_1(\mathrm{s})$ becomes as follow:

$$\Phi_1(\mathrm{s})=s\,\mathrm{D(s)}\,\mathrm{e}^{\tau s}+(\mathrm{k}_p\,s+k_i) \quad (4)$$

Then, by transforming the equation from the temporal domain to the frequency domain and equating the real and imaginary parts to zero we obtain the following equality:

$$\Phi_1(jw)=jw(\mathrm{R(w)}+\mathrm{jI(w)})(\cos(\tau\,\mathrm{w})+\mathrm{j}\sin(\tau\,\mathrm{w}))+(\mathrm{k}_p\;jw+k_i) \quad (5)$$

which can also be described in matrix form

$$\begin{bmatrix}-wI'(w) & R'(w)\\ wR'(w) & I'(w)\end{bmatrix}\begin{bmatrix}k_p\\ k_i\end{bmatrix}=\begin{bmatrix}wI(w)\cos(\tau w)-wR(w)\sin(\tau w)\\ wR(w)\cos(\tau w)-wI(w)\sin(\tau w)\end{bmatrix} \quad (6)$$

the matrix on the left is of singular type and can be determined as defined in [4] as follow in equation (7) and (8).

$$k_p(w)=\frac{\mathrm{R}'(\mathrm{w})\,\mathrm{k}_i-w\,I(\mathrm{w})\cos(\tau\,\mathrm{w})-w\,R(\mathrm{w})\sin(\tau\,\mathrm{w})}{w\,I'(w)} \quad (7)$$

$$k_i(w)=\frac{w\left[I(\mathrm{w})\sin(\tau\,\mathrm{w})-k_p R'(w)-R(\mathrm{w})\cos(\tau\,\mathrm{w})\right]}{I'(w)} \quad (8)$$

where

$$N(w)=\mathrm{I}(w)+jR(w) \quad (9)$$

and

$$\mathrm{D}(w)=\mathrm{I}'(w)+jR'(w) \quad (10)$$

So solving the equation (7) and (8) simultaneously, we can obtain the stability boundary locus, $l(k_p,k_i,w)$ in (k_p,k_i) plane.

2.2 New Graphical Method for Obtaining the Stability Domain Set for a PI Regulator

Consider the system defined in (1). The problem is to determinate the parameters of the PI regulator of the form (2) and which can stabilize the system described in figure (1) while guaranteeing the best performances.
So after substituting s=jw in the plant transfer function (1) and decomposing its numerator and its denominator polynomials into their even and odd parts as given in [7] in the case of regular system and in [13] to determine the stabilizing PID controllers for interval systems, the given system (1) with constant time delay becomes as follow:

$$G(\mathrm{s})=\frac{N_E(-\mathrm{w}^2)+\mathrm{jw}\,N_O(-\mathrm{w}^2)}{D_E(-\mathrm{w}^2)+\mathrm{jwD}_O(-\mathrm{w}^2)} \quad (11)$$

Then the closed loop characteristic polynomial of equation (3) can be written as follow:

$$\Delta(jw) = \begin{bmatrix} (k_i N_e - k_p w^2 N_0)\cos(w\tau) \\ +w(k_p N_e + k_i N_0)\sin(w\tau) - w^2 D_0 \end{bmatrix} + j\begin{bmatrix} w(k_i N_0 + k_p N_e)\cos(w\tau) \\ +wD_e - (k_i N_e - w^2 k_p N_0)\sin(w\tau) \end{bmatrix} = 0 \quad (12)$$

Then, equating the real and imaginary parts of (12), we obtain

$$k_p = \frac{-w\cos(\tau w)(D_E N_E + w^2 D_0 N_0) + w^2 \sin(\tau w)(N_O D_E + N_E D_O)}{wN_O{}^2 + w^3 N_O{}^2 (2\cos^2(\tau w) - 1)} \quad (13)$$

$$k_i = \frac{-wk_p N_e \cos(\tau w) + w^2 k_p N_0 \sin(\tau w) - wD_e}{wN_0 \cos(\tau w) - N_e \sin(\tau w)} \quad (14)$$

So solving these two equations, we can get both PI parameters (k_p, k_i) and then the stability region of the closed loop system defined in (12). The stability boundary locus will split up the parameter plane into stable or unstable region.
Hence, to make sure of the stability domain, you have to choose a test point inside each region, which holds the values of stabilizing k_p and k_i parameters.

3. Simulation

The PI controller has been used in many industrial control systems where delays occur mostly of the First-Order-Plus-Dead-Time (FOPDT) or Second-Order-Plus-Dead-Time (SOPDT) types.
Those types are the most common process models and have been extensively used in modeling and control of diverse process systems.
For that; to validate the approaches proposed above, we will focus in this section on those two types.

3.1 Exemple1: FOPDT system

In this example the proposed method is applied on a PT326 thermal process, which is a system used in many industrial systems such as furnaces, air conditioning, and so forth [14], where temperature control is reached through a combination of more than one means [15].
The flow of air flow through the conduit can be adjusted by a valve. There is an electrical resistance inside the tube, and by the effect Joule, the heat released by the resistance and transmitted by the convection to the circulating air. This process can be modeled as a linear delay system as described in the following equation and presented in Fig. 2.

$$G(s) = \frac{0.58e^{-0.56s}}{1.57s + 1} \quad (15)$$

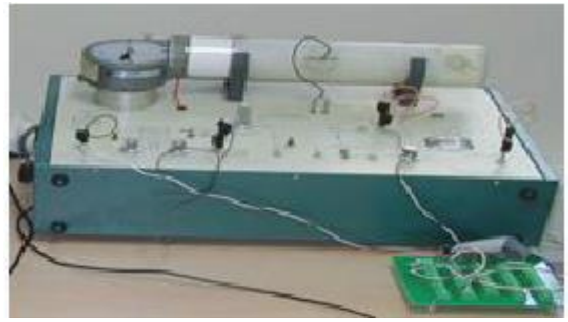

Fig. 2 Front panel of a process trainer PT-326

The main objective of this system is to maintain the temperature of the air at a desired level.
Hence, the need to calculate all stabilizing values of PI regulator, which make the closed loop characteristic polynomial for the system Hurwitz stable.
Then, a comparison with the D-partition method is also presented to demonstrate the effectiveness of the new graphical method and the advantages of each one.

- **D-partition Result**

Therefore, using the D-partition method described in the previous section and deducted respectively in equation (7) and (8) we obtain the values of stabilizing k_p and k_i parameters as follow:

$$k_p = \frac{1.57\, w\, sin(0.56w) - cos(0.56w)}{0.58} \quad (16)$$

$$k_i = \frac{= (1.57w^2 cos(0.56\,w) + w\, sin(0.56\,w)}{0.58} \quad (17)$$

The stability region for given problem is shown in Fig. 3:

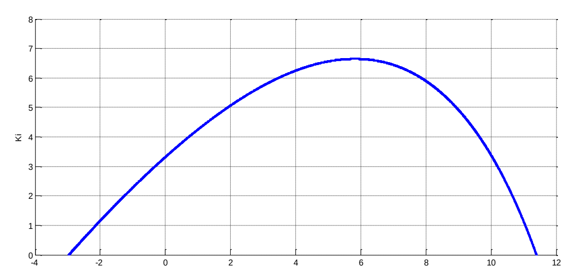

Fig. 3 Region of stability in the plan (k_p, k_i) with the D-partion method

By choosing a test point belonging to the stability region as shown above such that Kp= 4 and ki= 2, we get the response of our system to a step as shown in the following figure.

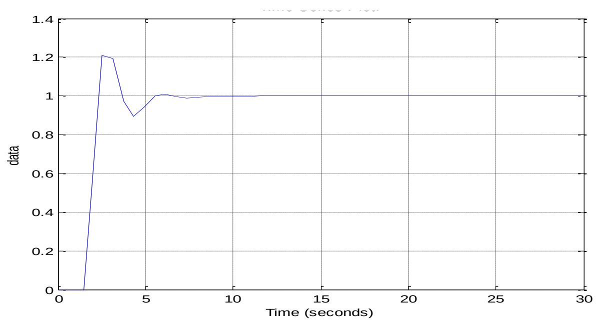

Fig. 4 Step response of the closed loop system.

- **Graphical Method Result**

Consider the first order transfer function with time delay given in (14).
From Eq (13) and (14), we obtain the following expressions

$$k_p = \frac{(0{,}528\,w^2\, sin(0.56\,w) - 0.58w\,cos(0.56\,w))}{1 - 0{,}3364\,w\,cos^2(0.56\,w)} \quad (18)$$

$$k_i = \frac{w + k_p 0.58\ w\cos(0.56w)}{0.58\sin(0.56w)} \quad (19)$$

Then with applying a first order Pade approximation, we obtain a simplified expression of the PI parameters as described in the following equations and presented in Fig. 5.

$$k_p = \frac{6.60647w^2 - 2.07118ki}{0.58w^2} \quad (20)$$

$$k_i = \frac{-1.57w^4 + 17.254w^2}{4.289 + 0.58w^2} \quad (21)$$

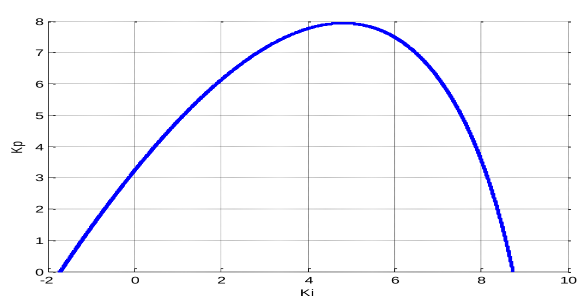

Fig. 5 Region of stability in the plan (k_p, k_i) with the proposed method

Choose a test point belonging to the stability region such that Kp= 2.819 and ki= 1.8839, we get the step response of our system as shown in Fig. 6.

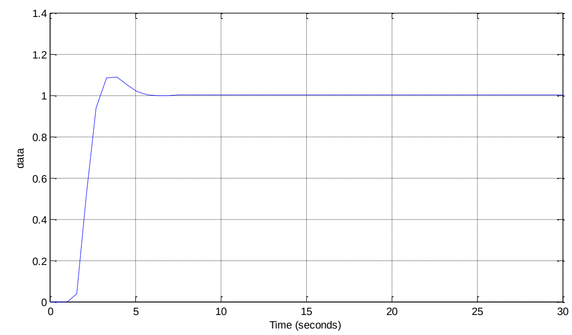

Fig. 6 Step response of the closed loop system.

In table 1, a comparison is done between the performance of the D-partition result and the proposed method.

Table 1: Margin specifications

method	(k_p, k_i)	maximum percent overshoot	setting time (s)	rise time (s)
D-partition	KP=4 and Ki =2	21	2.35	0.7
New Graphical method	Kp= 2.45 and ki= 1.7	18.6	2.76	0.6

3.2 Exemple2: SOPDT system

Consider the second order transfer function of the control system with time delay where $\tau = 4$ [16].

$$G(s) = \frac{e^{-4s}}{s^2 + 0.2s + 1} \quad (22)$$

- **D-partition Result**

First, using the D-partition method described in the previous section and deducted respectively in equation (7) and (8) we obtain the values of stabilizing k_p and k_i parameters as follow:

$$k_p = (w^2 - 1)\cos(4w) + 0.2w\sin(4w) \quad (23)$$

$$k_i = w\sin(4w)(1 - w^2) + 0.2w^2\cos(4w) \quad (24)$$

Solving those two equations of PI regulator for $w \in [0, 0.75]$ we can obtain the stability domain in the parameter plane (k_p, k_i) as shown in Fig. 7. However the results is not correct and precise because if we choose a test point within the enclosed region such as $k_p = -0.2$ and $k_i = 0.15$ we will get a transfer function in closed loop with two poles that are not with negative real part.
As a result, those two parameters do not belong to the stability region of the PI regulator.

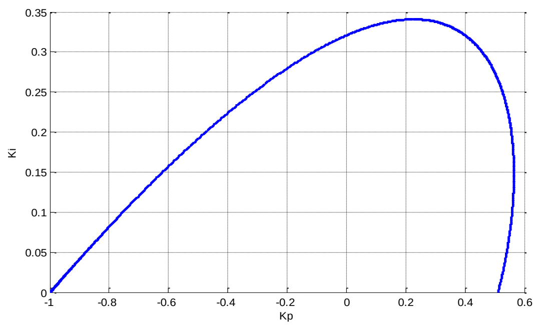

Fig. 7 Region of stability in the plan (k_p, k_i) with the D-partion method.

However, taking $k_p = 0.2$ and $k_i = 0.25$ we will obtain a stable step response of the closed loop system as presented in the following figure.

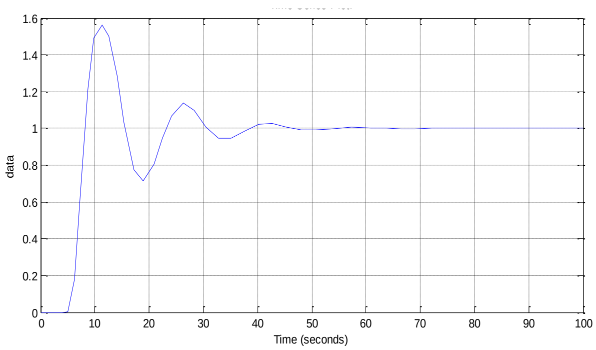

Fig. 8 Step response of the closed loop system.

Therefore, in this case the D-partion method is not certain and may give us an unstable system.

- **Graphical method result**

Applying the new method on the system defined in (22) we obtain the two expressions of the parameters (Kp, ki) as follows

$$k_p = 0.2w\sin(4w) - \cos(4w)(1 - w^2) \quad (25)$$

$$k_i = \frac{-w^3 + w + k_p w\cos(4w)}{\sin(4w)} \quad (26)$$

Then by taking first order Pade approximation for e^{-4s} in equation (22), we obtain:

$$G(s) = \frac{-s + 0.5}{s^3 + 0.7s^2 + 1.1s + 0.5} \quad (27)$$

and the PI parameters are computed as follow:

$$k_p = \frac{-w^4 + 1.1w^2 - 0.5k_i}{w^2} \quad (28)$$

$$k_i = \frac{-1.2w^4 + 0.55w^2}{w^2 + 0.25} \quad (29)$$

So, by solving those two equations of PI regulator for $w \in [-0.7, 0.7]$, the stability region can be shown in Fig. 9.

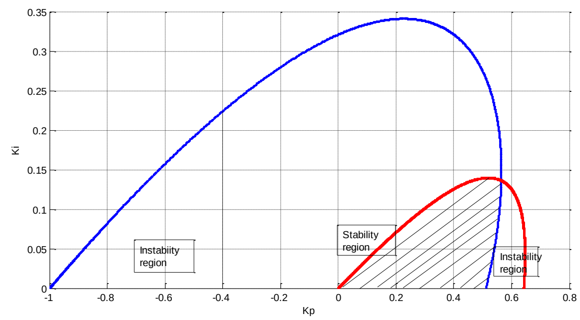

Fig. 9 Region of stability in the plan (k_p, k_i) with our proposed method

The large part represents the stability region obtained with the new method. This part contains unstable PI parameters such that for Kp = - 0.8 and ki = 0.05, and for example for kp = 0.15 and Ki = 0.17.

So, taking $k_p = 0.15$ and $k_i = 0.17$, we will obtain the following characteristic polynomial that prove the inefficiency of the obtained result:

$$\Delta(s) = s^4 + 0.7s^3 + 0.95s^2 + 0.005s + 0.085 \quad (30)$$

As a result, the use of a first order approximation of pade allowed us to limit the stability domain.

In conclusion, the stability region for our example is the hachured part shown in the previous figure.

Therefore, we can conclude that the first order approximation of Pade helped us in the accuracy of the stability domain.

The intersection of the two determined regions, gives a robust controller with the desired specifications in terms of response time, rise time and overshoot.

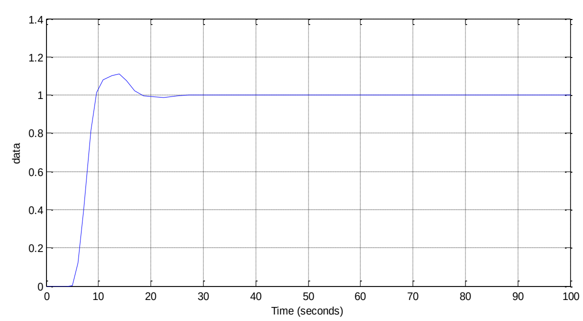

Fig. 10 Step response of the closed loop system.

Fig. 10 shows the evolution of our output as function time.

In this figure, we choose a couple (k_p, k_i) that belongs to the common domain of stabilizing parameters.
In table 2, a comparison is done between the both methods.

Table 2: results obtained by both methods

method	(k_p, k_i)	maximum percent overshoot	setting time (s)	rise time (s)
D-partition	KP=0.2 and Ki =0.25	56.4	7.8	1.6
new graphical method	KP=0.2 and Ki =0.06	11.2	9.2	2.96

As can be seen, by summarizing, the main characteristics of the proposed methods and applied to First-Order-Plus-Dead-Time (FOPDT) and Second-Order-Plus-Dead-Time (SOPDT) systems clearly show the superiority of the new method in terms of maximum percent overshoot and setting time.

4. Conclusion

In this paper, a new graphical method is given for calculation all stabilizing values of a PI controller, which is applied to a system with time delay [17]. The given method consists in plotting the position of the stability limit in the PI parameters plane, and then add a first order pade approximation to better precision of the stability region [7,18]. Three criteria are used to characterize and evaluate the performance of a system, such as overshoot, setting time and rise time. Those specifications allow us to specify the most robust stabilized domain in the plane (k_p, k_i) . Comparing with the D-partition methods, the examples given early prove the superiority of the proposed method. Application of the new graphical method to uncertain time delay system is under exam.

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