Methods of Mathematical Modelling for Signal Distortion Diagnostics in the Information Channels

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Summary

The development of new mathematical modeling methods as applied to information channels is dictated by the need to diagnose signal distortions and correct them in order to increase information reliability. The main diagnostic problem is associated with the development of methods for detecting distortions and evaluating their parameters, which is impossible without first constructing a mathematical model of the information signal. The classical approach to diagnostics consists in analyzing the information signal on the receiving side and determining the frequency of occurrence of erroneous symbols and the modulation error coefficient. However, these parameters give an idea only about the degree of deterioration of the signal quality, but not about the possible reasons for its decrease, moreover, a long observation time is necessary to obtain a reliable estimate, i.e. deterioration in the quality of the information signal is recorded with a long delay. The scientific value of the research is a generalized mathematical model of the information signal encoded by the method of orthogonal frequency separation of quadrature amplitude modulated signal features is proposed and investigated.

Key words: mathematical modeling, information channels, signal distortions, information reliability, degree of deterioration, signal quality.

1. Introduction

The purpose of this research is to create effective mathematical models and methods for diagnosing distortions of the information signal, providing the possibility of its quick correction in high-speed information channels.[2,17] The work uses general methods of mathematical analysis, linear algebra and the theory of functions of a complex variable, mathematical statistics, as well as methods of approximation theory, computational mathematics, signal theory, and the theory of potential noise immunity[5,21].

The practical value of the research is the proposed algorithm and a set of programs synthesized based on it allow one to estimate the distortion parameters of information signals based on orthogonal frequency separation coding of quadrature amplitude modulated signal features, with lower computational costs compared to previously known algorithms, in particular in Wi-Fi wireless networks, WiMax, LTE, digital broadcasting networks of DVB-T,[3,11,28] DVB-T2 standards and others, characterized by the transmission of compressed information streams. Unlike other methods, the proposed method provides simultaneous estimation of the parameters of all typical distortions[30]. The algorithm developed on the basis of this method allows you to dynamically track parameters, which makes it possible to diagnose an information channel with early detection of distortions from small deviations of the signal with quick correction[19,30].

2. Research Methods and Framework

Initial state is considered the analysis of known methods for diagnosing distortions of an information signal based on coding by the method of orthogonal frequency separation of quadrature amplitude modulated signal signs in the presence of negative influences from the signal-forming blocks of the transmitting device is carried out[4,22,31]. As a result of the analysis, conclusions were drawn that when using classical methods to evaluate diagnosed parameters due to their algorithmic and computational complexity[13], it is impossible to achieve the following requirements for the diagnostic system:

1. Comprehensive identification of typical signal distortions.
2. Assessing the effect of various types of distortion on the transmission of an information signal.
3. Determining the state of the channel the time before the receiving side detects a failure[15,18].

The formulation of the main research task is given and the scientific tasks of the dissertation are described. In the next stage of the research, a mathematical model of the information signal encoded by the method of orthogonal frequency separation of quadrature amplitude modulated signal features[1,20,23], taking into account the features of the formation of the information stream, is developed. The model subsequently made it possible to propose a mathematical method for estimating the distortion parameters of an information signal for diagnostic systems of the operation of a transmitting device[6,27,29].

In fig. Figure 1 shows the procedure for generating and converting an information signal, which reflects the
The process of dividing a binary stream into a large number of substreams[7,25], each of which controls the operation of the encoder. The encoder, which is based on the quadrature-amplitude modulation algorithm, according to certain rules generates for each substream a sequence of numerical values of the Fourier coefficients, which in the next block are subjected to the inverse discrete Fourier transform procedure and as a result an output signal is generated[8,10,11]. The modulation order of substreams is selected depending on the required transmission rate, the linearity of the transmitting device and the requirements for noise immunity. To minimize the effects of intersymbol interference between individual characters, pauses called guard intervals are introduced[9,14,20].

Building blocks: 1) serial data stream converter in parallel; 2) quadrature amplitude encoder; 3) a reverse Fourier coding unit; 4) block insertion of the protective interval; 5) a parallel to serial data converter; 6) a block for removing the guard interval; 7) direct Fourier coding unit; 8) quadrature amplitude decoder

3. Proposed Methodologies and Experimental Results

Physically, the signal conditioning procedure corresponds to the following sequence of actions. Information frames consisting of a finite sequence of bits divided into several parallel low-speed, in comparison with the original sequence[3,26], bit streams are received at the input of the transmitting device. In each of the parallel streams, in turn, a group of several bits is allocated that encode the amplitude and the initial phase (Fourier coefficients) of the harmonic signals on the corresponding stream so that all the received harmonic signals are orthogonal to each other to exclude their mutual influence[21]. The encoding process itself is called quadrature amplitude modulation of signal attributes (frequencies)[16]. Because a group of bits has a finite length, then the corresponding harmonic signal also has a finite duration in time. The set of orthogonal harmonic signals obtained as a result of a single coding procedure determines the orthogonally-frequency-separated symbol, and the amplitude-phase pair determines the quadrature-amplitude symbol or cell[31]. Because the input information stream is divided into frames, then a sequence of orthogonally-frequency-separated symbols forms transmission frames [8,24,26]. Before sending a set of orthogonal harmonic signals to a channel, frequency conversion operations are carried out to the operating range of the transmitter, as well as pre-insertion of the guard interval, in other words, a time pause between orthogonal-frequency separated symbols[21,26], the purpose of which is to combat multipath signal reflection in a physical channel: pause duration is set so that in the channel before sending the next character all re-reflections of the previous character

![Diagram](image-url)
have time to fade in order to exclude and reducing interference processes at the input of the receiver[21,27].

The developed mathematical model of an information signal based on orthogonal frequency separation encoding of quadrature amplitude modulated signal features differs from similar models[21] by taking into account the distribution of data by symbols and the presence of a protective interval in the generated signal and has the following form:

\[
    z(t) = \text{Re} \left[ \exp(2\pi j f t) \sum_{r=1}^{N} \sum_{s=-\infty}^{\infty} [C_{r,s} \times \Psi_{c,s}(t)] \right],
\]

\[
    h' = h - (H_{\text{max}} + H_{\text{min}})/2, \quad (2)
\]

\[
    T_s = T_u + T_g, \quad (3)
\]

Where \( N \) is the number of orthogonally-frequency-separated characters in the transmission frame; \( h \) - number of the value of the signal sign; \( H_{\text{min}} \) and \( H_{\text{max}} \) - respectively, the minimum and maximum values of the signal attribute (lower and upper boundaries); \( S \) - number of the orthogonal-frequency separated symbol, \( r \) - transmission frame number; \( T_g \) - duration of the guard interval; \( T_u \) - the duration of the useful part of the orthogonal-frequency-separated symbol; \( f \) - reference frequency of the transmitter; \( C_{r,s,h} \) - the value of the quadrature amplitude modulated cell \( h \) for the signal attribute \( S \) in the frame symbol \( r \).

The values of quadrature amplitude modulated cells \( C_{r,s,h} \) (Fig. 2) are complex numbers of the form

\[
    C_{r,s,h}(t) = \text{Re} \left\{ C_{r,s,h}(t) \right\} + j \text{Im} \left\{ C_{r,s,h}(t) \right\} = Q_{r,s,h}(t) + j I_{r,s,h}(t)
\]

where \( Q_{r,s,h}(t) \) and \( I_{r,s,h}(t) \) are the coordinates of the points of the modulation diagram, the algorithm for displaying which binary symbols of the transmitted stream is represented by a matrix of modulation coefficients \( M \):

\[
    M = \begin{pmatrix}
        (q_1, i_1) \\
        (q_2, i_2) \\
        \vdots \\
        (q_{2^m}, i_{2^m})
    \end{pmatrix}, \quad (4)
\]

\[
    (Q, I) = M_{\text{index}}^*, \quad (5)
\]

\[
    \text{index} = 1 + \sum_{i=1}^{m} b_i 2^{i-1}, \quad b_i \in \{0,1\}. \quad (6)
\]

As you can see, the stream is divided into a sequence of bits of the form \( b_1, b_2, ..., b_m \), where \( m \) is the order of quadrature amplitude modulation. The indicated sequences form binary numbers, which, when converted to the decimal notation using the formula (6), give indices that reflect the line numbers in the matrix of modulation coefficients (4), according to which the values of the coordinates of the points in the diagram are located.

In accordance with algorithm (4) - (6), for each sequence of bits with a length \( m \) from the common stream, coordinate values are generated \( Q_{r,s,h}(t) \) and \( I_{r,s,h}(t) \).

The signal \( z^*(t) \) at the input of the receiving device has the form \( z^*(t) = z(t) + n(t) \) where \( n(t) \) is a function that describes the additive signal of interference and distortion in the communication channel at the input of which a useful signal acts \( z(t) \).

Conversion of the input signal at the receiving device:

\[
    C^*_{r,s,h}(t) = \exp(-2\pi j f t) \sum_{r,s,h} \left( z^*(t) \times [\Psi_{c,s,h}(t)]^* \right), \quad (7)
\]

where \( C^*_{r,s,h} \) in general terms can be represented as

\[
    C^*_{r,s,h} = C_{r,s,h} + n_{r,s,h}, \quad (8)
\]
\( n_{r,s,h} \) - Component of components \( n(t) \) superimposed on \( C_{r,s,h} \) the symbol \( S \) of the signal sign \( h \) of the frame \( r \) after conversion (8).

Thus, the proposed model (1) - (7) in the transmitter-channel-receiver system reflects the procedure for generating and converting the information signal, taking into account the distribution of data by symbols and the presence of a protective interval in the generated signal.

In the next stage of the research, a transformation matrix is derived that describes the distortions of the modulation diagram caused by a phase shift, amplitude mismatch, quadrature error, signal attenuation, interference distortion, phase jitter, and Gaussian noise; systems of equations are obtained whose solutions for a particular information signal give the parameters of these distortions.

Since \( C_{r,s,h}^{*} \) it is a complex number, its real and imaginary parts in (9) can be represented as a matrix of components:

\[
\begin{align*}
\text{Re}\{C_{r,s,h}\} &= \text{Re}\{C_{r,s,h}\} + \text{Re}\{n_{r,s,h}\}, \\
\text{Im}\{C_{r,s,h}\} &= \text{Im}\{C_{r,s,h}\} + \text{Im}\{n_{r,s,h}\}.
\end{align*}
\]  

(9)

Transformations of matrix (9) reflect the geometric distortions of the modulation diagram and are similar to the sequence of reflections, rotations, extensions, and shifts. In our case, due to the physical characteristics of information signals, reflections are impossible, since no interference can cause such a distortion. The remaining types of transformation \( \left( \begin{array}{c} \text{Re}\{C_{r,s,h}\} \\
\text{Im}\{C_{r,s,h}\} \end{array} \right) \rightarrow \left( \begin{array}{c} \text{Re}\{C_{r,s,h}^{*}\} \\
\text{Im}\{C_{r,s,h}^{*}\} \end{array} \right) \) correspond to phase shift, amplitude mismatch, quadrature error, and phase jitter (jitter). A typical channel also has inter symbol interference and Gaussian noise.

A phase shift is a determined phase error, which is an angle rotation of the modulation diagram around its axis \( \theta_{\text{offset}} \):

\[
\begin{align*}
\text{Re}\{C_{r,s,h}^{*}\} &= \cos\theta_{\text{offset}} \text{Re}\{C_{r,s,h}\} - \sin\theta_{\text{offset}} \text{Im}\{C_{r,s,h}\}, \\
\text{Im}\{C_{r,s,h}^{*}\} &= \sin\theta_{\text{offset}} \text{Re}\{C_{r,s,h}\} + \cos\theta_{\text{offset}} \text{Im}\{C_{r,s,h}\}.
\end{align*}
\]  

(10)

The physical cause of the phase shift is the error in determining the phase of the carrier in the transmitter.

The inconsistency of the amplitudes is described by introducing a gain for the real channel, different from the corresponding gain of the imaginary channel:

\[
\begin{align*}
\text{Re}\{C_{r,s,h}^{*}\} &= k_{E} \text{Re}\{C_{r,s,h}\}, \\
\text{Im}\{C_{r,s,h}^{*}\} &= 0 \text{Im}\{C_{r,s,h}\}.
\end{align*}
\]  

(11)

The physical reason for the mismatch of amplitudes is the mismatch of amplifiers of the material and imaginary components of the signal.

The quadrature error is represented as the result of multiplication by a matrix causing the slope of the modulation diagram:

\[
\begin{align*}
\text{Re}\{C_{r,s,h}^{*}\} &= k_{S} \text{Re}\{C_{r,s,h}\}, \\
\text{Im}\{C_{r,s,h}^{*}\} &= 0 \text{Im}\{C_{r,s,h}\}.
\end{align*}
\]  

(12)

Where \( k_{S} \) is the angle of deviation from the orthogonality of the material and imaginary components of the channel characteristics. The cause of the error is a failure in the phase-shifting unit of the transmitting device.

Attenuation of the signal:

\[
\begin{align*}
\text{Re}\{C_{r,s,h}^{*}\} &= K \text{Re}\{C_{r,s,h}\}, \\
\text{Im}\{C_{r,s,h}^{*}\} &= K \text{Im}\{C_{r,s,h}\}.
\end{align*}
\]  

(13)

Where \( K \) is the attenuation coefficient of the signal. The physical cause of signal attenuation is the natural process of signal attenuation.

Component of components \( n_{r,s,h} \) broken into two parts, one of which ( \( n_{r,s} \) ) associated with inter-symbol interference \( S \) in a frame symbol \( r \), other \( n_{h} \) - additive Gaussian noise for signal attribute value number \( h \).

Considering that interference distortions are caused by a false signal, which leads to a shift in the symbols of the modulation diagram, they are modeled by a vector of a false signal with amplitude \( A \) and phase \( \phi \) depending on the moment of measurement and the frequency difference between the false and useful signals.

Then the matrix \( \left( \begin{array}{c} \text{Re}\{C_{r,s,h}\} \\
\text{Im}\{C_{r,s,h}\} \end{array} \right) \) will take the form

\[
\begin{align*}
\text{Re}\{n_{r,s,h}\} &= A \cos\phi \text{Re}\{n_{r,s}\} + A \sin\phi \text{Im}\{n_{r,s}\}, \\
\text{Im}\{n_{r,s,h}\} &= A \sin\phi \text{Re}\{n_{r,s}\} - A \cos\phi \text{Im}\{n_{r,s}\}.
\end{align*}
\]  

(14)

The cause of interference distortion is the multipath propagation of a signal in a channel, leading to multiple signal reflections.

Jitter of the phase, in contrast to the phase shift, the inconsistency of the amplitudes, the quadrature error and the attenuation of the signal, is a random error that causes the rotation of the modulation diagram by an angle \( \theta_{i} \), which is a random variable having a Gaussian distribution with a zero mean value and dispersion \( \sigma_{i}^{2} \):

\[
\theta_{i} \sim N(0, \sigma_{i}^{2}).
\]
The cause of phase jitter is the instability of the clock, which manifests itself in fluctuations in the duration of synchronizing pulses. Summarizing (9) with (10) - (15) taken into account, we obtain

\[
\begin{align*}
\left( \begin{array}{c}
\Re\{C_{r,s,h}\} \\
\Im\{C_{r,s,h}\}
\end{array} \right) &= \left( \begin{array}{cc}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{array} \right) \left( \begin{array}{c}
\Re\{C_{r,s}\} \\
\Im\{C_{r,s}\}
\end{array} \right) \\
&= \left( \begin{array}{cc}
\cos \theta_{\text{offset}} & -\sin \theta_{\text{offset}} \\
\sin \theta_{\text{offset}} & \cos \theta_{\text{offset}}
\end{array} \right) \left( \begin{array}{c}
\Re\{C_{r,s}\} \\
\Im\{C_{r,s}\}
\end{array} \right) \\
&= \left( \begin{array}{c}
\Re\{C_{r,s}\} + k_{e} \Re\{C_{r,s}\} \\
\Im\{C_{r,s}\} + k_{e} \Im\{C_{r,s}\}
\end{array} \right)
\end{align*}
\]

(15)

The table shows the modulation order diagrams \( m = 4 \) obtained by numerical simulation based on equation (16). The unknown parameters are estimated in (16) by analyzing the statistical moments of the sample of received symbols \( C_{r,s,h}^{*} \).

Simplifying (16) for the case of small phase angles, when \( \sin \theta_{\text{offset}} \approx \theta_{\text{offset}} \) and \( \cos \theta_{\text{offset}} \approx 1 \), excluding phase jitter, we have

\[
\begin{align*}
\left( \begin{array}{c}
\Re\{C_{r,s}\} \\
\Im\{C_{r,s}\}
\end{array} \right) &= K \left( \begin{array}{cc}
1 & 0 \\
0 & 1
\end{array} \right) \left( \begin{array}{c}
\Re\{n\} \\
\Im\{n\}
\end{array} \right) \\
&= \left( \begin{array}{c}
\Re\{n\} + k_{e} \Re\{n\} \\
\Im\{n\} + k_{e} \Im\{n\}
\end{array} \right)
\end{align*}
\]

(17)

<table>
<thead>
<tr>
<th>Distortions</th>
<th>Modulation Charts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase shift</td>
<td></td>
</tr>
<tr>
<td>( \theta_{\text{offset}} = 0.1884 )</td>
<td></td>
</tr>
<tr>
<td>Amplitude mismatch</td>
<td></td>
</tr>
<tr>
<td>( k_{e} = 0.80 )</td>
<td></td>
</tr>
<tr>
<td>Quadrature error</td>
<td></td>
</tr>
<tr>
<td>( k_{e} = 0.40 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude mismatch</td>
<td></td>
</tr>
<tr>
<td>( k_{e} = 0.60 )</td>
<td></td>
</tr>
<tr>
<td>Quadrature error</td>
<td></td>
</tr>
<tr>
<td>( k_{e} = 0.42 )</td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude mismatch</td>
<td></td>
</tr>
<tr>
<td>( k_{e} = 0.80 )</td>
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<td>Modulation Charts</td>
</tr>
<tr>
<td>---------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Jitter phase</td>
<td><img src="image1" alt="Distortion Chart" /></td>
</tr>
<tr>
<td></td>
<td>$\sigma_i^2 = 0.042$</td>
</tr>
<tr>
<td>Interference distortion</td>
<td><img src="image4" alt="Distortion Chart" /></td>
</tr>
<tr>
<td></td>
<td>$A = 0.5$, $\phi = 0.5652$</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td><img src="image7" alt="Distortion Chart" /></td>
</tr>
<tr>
<td></td>
<td>$n_h = 0.20$</td>
</tr>
<tr>
<td>Attenuation</td>
<td><img src="image10" alt="Distortion Chart" /></td>
</tr>
<tr>
<td></td>
<td>$K = 0.83$</td>
</tr>
</tbody>
</table>

Note. Marker “+” - signal without superimposed distortion; marker “×” - the same signal, but with introduced distortion; dashed lines limit the areas of error-free recognition of signal symbols; angles $\theta_i$, $\theta_{offset}$, $k_S$ and $\phi$ in radians; odds $K$, $k_E$, $A$ and $n_h$ normalized.

Thus, we obtained a transformation matrix describing the distortions of the modulation diagram caused by a phase shift, amplitude mismatch, quadrature error, signal attenuation, interference distortion, phase jitter, and Gaussian noise, and we also obtained the systems of equations underlying the diagnostic method, the solutions of which for a particular signal give the parameters of the specified distortion.
4. Results and Practical Implementations

This stage presents the results of the practical application of the method for estimating the distortion parameters of an information signal. The functional-logical block diagram of the algorithm for estimating the distortion parameters of the developed software package is presented in Fig. 3.

![Functional logical block diagram of the algorithm](image)

Fig. 3  Functional logical block diagram of the algorithm

In Fig. 6 presents the results of a comparative analysis of methods for estimating the distortion parameters of an information signal. The curves show the computational resources (time) spent on the evaluation procedure depending on the probability of distortion of the transmitted symbols in the information channel.

![Dependencies of computational time costs on the probability of symbol distortion](image)

Fig. 6  Dependencies of computational time costs on the probability of symbol distortion for methods based on: 1) a neural network; 2) the theory of filtration; 3) parametric Proni method; 4) transformation matrix

An analysis of the curves shows that the computational cost of the proposed method is much less than that of the other methods. This, in turn, means that it becomes possible, by performing a cyclically (continuously) evaluation procedure, to detect distortions by small deviations of the received information signal relative to the transmitted one, that is, to make early detection of distortions. Moreover, the computational cost depends...
weakly on the probability of distortion of the transmitted characters, that is, on the frequency of occurrence of erroneous characters.

In fig. 7 presents the results of a comparison of methods for their corrective ability. In the steady state, the curves of the dependence of the frequency of occurrence of erroneous symbols on time with a change in the signal-to-noise ratio are in the corridor (dashed lines) formed by the upper and lower boundaries of the frequencies of occurrence of erroneous symbols for a particular method. Analysis of the curves shows that in the steady state of the proposed method, the value of the frequency of occurrence of erroneous characters is significantly lower than that of other methods.

![Graph showing results of comparison of methods for their corrective ability](image)

**Fig. 7** Dependence of the frequency of occurrence of erroneous symbols on time for various signal-to-noise ratios in the range from 20 to 40 for methods based on: 1) neural network; 2) the theory of filtration; 3) the parametric method of Proni; 4) transformation matrix.

### 5. Conclusion

When performing this study, the following main results were obtained:

- A mathematical model of the information signal based on coding by the method of orthogonal frequency separation of quadrature-amplitude modulated signal features, taking into account the characteristics of the information flow, is proposed. The model reflects the procedure for generating and converting an information signal in a transmitter-channel-receiver system of an information channel.

- A transformation matrix is obtained that describes the effect of phase shift, amplitude mismatch, quadrature error, signal attenuation (attenuation), interference distortion, phase jitter (jitter) and Gaussian noise on an information signal encoded by the method of orthogonal frequency separation of quadrature amplitude modulated signal signs.

- A method is proposed and a numerical algorithm is developed that allows one to estimate the distortion parameters of an information signal encoded by the method of orthogonal frequency separation of quadrature amplitude modulated signal signs. The method allows you to find the numerical values of the typical distortion parameters.

- A complex of problem-oriented programs has been developed that implements the proposed method for estimating the distortion parameters of information signal encoded by the method of orthogonal frequency separation of quadrature amplitude modulated signal features.

- Numerical modeling and analysis of the results obtained. The effectiveness of the proposed model and method for the rapid detection and evaluation of the parameters of typical distortions of the information signal in the transmitting channels is shown.
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References


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