An Analysis Framework for Fuzzy Time Series Forecasting Models

Muhammad Faisal†, Moataz Ahmed†
mfaisal@kfupm.edu.sa moataz@kfupm.edu.sa
†Info. & Computer Science Dept., King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia

Summary
Time series has been catching considerable attention due to its wide range of applications. Fuzzy logic concepts have been applied to the analysis of time series resulting in producing Fuzzy Time Series (FTS). The classical time series uses numbers whereas FTS uses fuzzy sets or linguistic values. FTS forecasting is effective when the inputs are linguistic characterized by imprecision in nature. Forecasting in the presence of multiple factors is very important and challenging at the same time. Many FTS forecasting models have been developed and presented in the literature. However, there are still some challenges and gaps that needs to be addressed. To identify these gaps, we developed an analysis framework to allow for a systematic evaluation of FTS forecasting models using a set of criteria. We analyzed prominent FTS forecasting models and identified a set of gaps yet to be addressed. The set of gaps is meant to serve as an eye-opener on issues to be addressed in future research.
***Key words*:**
Comparison, Forecasting, Framework, Fuzzy, Time Series.

1. Introduction

Many real-world problems in the fields of medical, economy, engineering, etc. involve time series and eventually requires its analysis and forecasting. To elaborate this one would like to see the future value(s) by seeing the previous values(s). For example, by seeing the patient's previous history, a practitioner or doctor tries to predict the future conditions in order to improve and better treat a patient. There are several traditional and classical methods and tools available in performing this task such as ARMA, ARIMA, etc. Complex methods and tools such as Recurrent Neural Networks, LSTM, etc. are also used for this purpose. Fuzzy Time Series (FTS) also finds its place here since it was first proposed around 26 years ago. It is quite difficult to compare and tell who the winner is amongst all due to the many factors involved. However, Fuzzy Time Series enjoys some advantage over others due to its readability, manageability, and simplicity by providing rules that can be easily understood by humans. Again, to emphasize these attributes are subjective and one can select any other approach depending on the problem he wants to solve. Initially in the original proposal of FTS it was capitalizing on the concept of fuzzy sets but has a drawback of more computation time. Later it was simplified using simple arithmetic and many researchers just consider the maximum membership of a fuzzy set. Although it was simplified and becomes faster and has been widely used by many, it is not fully utilizing the concept of fuzzy set theory and may cause some loss of information. The aim of this work is to develop a kind of comparison framework by going through some prominent research studies done on FTS from its beginning till now. It may not be possible here to cover all the studies as they are quite large in numbers. This comparison framework will help in identifying some key factors that are needed in fuzzy time series forecasting. Once again it cannot be all the factors and we will make focus on the most important ones. Using this framework, we will mention some of the gaps that need to be filled by doing some research on them. A similar kind of framework has been developed earlier in [1] for the prediction of Multiple Sclerosis (MS) progression.

The rest of the article is organized as follows. The next section briefly discusses the background needed to understand the comparison framework. It is very basic, and readers can skip if they are already familiar with these concepts. After that we will mention some of the previous work done in this field and try to highlight pros and cons of the approach wherever possible. In the next section we will present our comparison framework and explain some of the important attributes needed for FTS forecasting. Next, we analyze several of the FTS work done using our comparison framework. Finally, we conclude the paper with some future research directions.

2. Background

1.1 Time Series

Time Series is a sequence of equally or non-equally spaced data over a period of time. It can be annually, monthly, weekly, hourly, or any other possible time unit. Its forecasting is based on some previous values of observations. It can be formulated as given a set of measurements x1,

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x2, …., xt and predict xt+1. It is reasonable to use when we expect that the patterns of past can be expected to continue in the future. To analyze time series, it is normally decomposed into three parts namely trend, seasonality, and residual [2].
Time series forecasting models usually assume the existence of a degree of correlation between successive values of the series. It is not as easy as normal classification or regression in the sense that it adds some more complexity in terms of order or lags and has temporal dependence. It is considered more powerful than traditional models because the latter mainly focus on univariate data, one time-step forecast, some fixed temporal dependence, and assuming complete data without any missing values and linear relationship amongst the features.

2.1 Fuzzy Logic

In a normal classical set theory, an element can have a membership of either 1 or 0 meaning that either it is inside the set or totally outside of it. Opposed to it is a Fuzzy Set theory in which an element can have a partial membership between 0 and 1. It is very useful in perceiving the knowledge of human society where people normally do not talk with exact numbers. For example, considering the height it is normally expressed as tall, short, etc. and not with some exact number. Together with this linguistic nature, the presence of uncertainty in any application makes the fuzzy logic a suitable candidate for finding the solution for various kind of problems. It was first introduced by Zadeh [3] and after that applied with many variations to tons of applications. Fuzzification, Relations or rules, and Defuzzification are considered the main steps involved in finding the solution with fuzzy logic [4]. Along with many areas it has found its strong placing in the area of machine learning especially for prediction and forecasting. A number of machine learning techniques in the case of building energy consumption forecasting has been reviewed in [2]. Researchers have used pure fuzzy logic concepts along with different combinations to come up with different methodologies and algorithms. The main interest for us in this paper is the use of fuzzy logic in time series forecasting along with some evolutionary algorithm like genetic algorithm. The former is commonly referred in the literature as Fuzzy Time Series [5], [6] and the latter as Genetic Fuzzy Systems [7]. Both areas have load of references and research available.

3.1 Fuzzy Time Series

In order to elaborate and better understand the need of FTS let us try to look into an example. Suppose we want to evaluate the quality of food cooked by one or several chefs. This quality is very subjective and can take several linguistic values such as excellent, very good, good, reasonable, not so bad and so forth. These kinds of values are easier to comprehend as compared to assigning some discrete values within some interval. We can record any chef's performance for a day or week or any other duration to evaluate. Assuming they are not perfect theses values changes as the time changes. At the end we will have a time series in terms of linguistic values. Now most traditional and classical statistical methods for time series forecasting cannot handle this type of input and that is where we take advantage of FTS. This is one of the main reasons for FTS to be used to cope with the linguistic nature of the inputs. The added advantage FTS has is that the numerical inputs can be easily converted to some linguistic inputs and then forecasting can be done on those. Also, at the end of forecasting it is up to the application/user to report the result in fuzzified form or can be converted back again to some numerical value with various possible methods available. Dealing with the linguistic nature of inputs is one of the main reasons mentioned by [5] to proposed FTS. It also gets much attention due to the approximation capability [8] and being a function approximator [9]. Another reason for the use of FTS as described by [10] is the small amount of data but this need some thoughts and verification by doing experimentation and carefully comparing with other methods.

FTS has been applied to many fields/areas some of which are:

- University Enrollments [10], [5], [6], [11], [12]
- Stock Exchange [13], [14], [15]
- Exchange Rates [14], [15]
- Energy Consumption [2]
- Others (Car Accidents Mortalities [16], Temperature, Crop Production, No. of Patents, etc.)

FTS forecasting models are created using several parameters such as order, number of variables used, interval length etc. can be broadly categorize using several parameters as

- First Order vs High Order
- Univariate vs Multivariate
- Fixed Interval Lengths vs Variable Interval lengths
- Non-Weighted Fuzzy Relations vs Weighted Fuzzy Relations
- Crisp Output vs Fuzzy Output

Each of the above category has its own importance and some short comings based on its application and data used. Like for example univariate could be used when the time series is simple, and it is known that other factors are not involved in its forecasting. Higher orders are used when past history for some time is required such as in medical applications. It is best to ask from the expert in the field to know about the number of orders to use. Variable and increased interval lengths will normally produce a better forecasting but at the cost of computation time. It is worth to mention that all the above categories are normally used in some form of combination.

The following 5 to 7 basic steps are normally involved in any kind of FTS forecasting [5], [6], [11], [12], [13]

1) Define the universe of discourse and the intervals,
2) Define the fuzzy sets,
3) Fuzzify the data,
4) Establish fuzzy logical relationships (FLRs),
5) Establish fuzzy logical relationship groups (FLRGs),
6) Forecast,
7) Defuzzify the forecasting results.

Some combine steps 4 and 5 or they do not use FLRGs. Step 7 is only required if we want to defuzzify and get the crisp value for the output. The creation of FLRs also vary and some uses multiple same FLRs as only one and ignore others whereas some assigns weight to each FLR and does not ignore [12]. The assignment of weight can also differ like the most recent one gets the higher weight or the most frequent one gets the higher weight, etc.

3. Literature Review

Song and Chissom [5], [6] are considered as the pioneers of creating a concept of fuzzy time series. They introduce several definitions especially time-variant and time-invariant fuzzy time series in their publication of two-part series. They specify different steps and used the University of Alabama data set to report the forecasting results. This makes the basis and a benchmark to follow by many researchers. One of the main drawbacks in their approach is the use of some complex max-min operation and creation of fuzzy relation matrix which is quite tedious and takes time if the input space is large. It was only applied for first order time series.

Chen [11] has proposed the same method but with simpler arithmetic which is easier and also provides better forecast. They also used the same university data set and prove robustness of their approach as well. Chen [12] has also proposed the higher order fuzzy time series forecasting where the forecasting results are even better for the university data set. Using high order their approach provide less Mean Square Error (MSE) and has also low complexity.

Yu [13] has tried to improve the results by arguing that the previous work ignores the repeated relations. Hence, they proposed a weighted fuzzy time series by providing some weights to each of the repeated fuzzy relation. They used Taiwan Stock Exchange (TAIEX) data and Root Mean Square Error (RMSE) for the performance measure and compare their method with other approaches. Similarly, Wang et al [17] has proposed a weighted fuzzy time series on difference of data and use Fuzzy clustering. They also use a different method for forecasting at the final stage.

Tsaur et al [10] has suggested another way for creating fuzzy relation matrix. Singh [18] has proposed their own computational algorithm for forecasting only and keeping the other steps same as in other methods/approaches. Singh [19] again proposed a similar kind of algorithm for third order FTS and analyze the results with various fuzzy intervals ranging from 5 to 20.

Zhang and Zhu [20] used the K-Means clustering on the intervals and improved the forecasting for University of Alabama data set. Saxena et al [21] used percentage change and mean based partitioning to forecast the same university dataset. However, their method produces 21 intervals for this data set and the forecast accuracy is highly dependent on the data set.

Chen [22] has applied genetic algorithm to tune the intervals in high order forecasting. Similarly, Rezan et al [23] used genetic algorithm for the intervals and combine weighted approach with differential algorithm.

Liu [24] has argued on single point forecasting and propose to predict a trapezoidal number in order to improve the accuracy. This can prove helpful for the experts to make better decision. Rubio et al. [25] also improved the forecasting of various stock indices by using trapezoidal fuzzy numbers for forecast but their method of calculating the length of intervals is producing a very high number (e.g. 52) in one of the examples they mention. Chen [26] in a more recent work used PSO to find the optimal intervals on 3 datasets but did not specify how much length on average they get.

All discussed above are for the univariate case meaning that only single variable or feature has been used to forecast. In the real-world scenario, it is quite common that many factors or variables decide the trend of the time series. This is referred to as multivariate case and it would be better to utilize the other factors in forecasting problems. It is worth noting that the problem will become complex as the number of other factors increases. Chen et al [14] used two factors second order along with Particle Swarm Optimization for forecasting Taiwan Stock Exchange (TAIEX). Chen [15] again used same two factors but with probabilities of trends as equal, down, or up. A good reference of several types and categories of FTS can be found in the same paper. Chen [27] in another work used PSO to find optimal intervals and use weighted method to improve the forecasting of TAIEX and Exchange rates data sets. Some other multivariate papers are [28], [29], [30], [31], [32], [16], [33], [34], [35], [36], [37].A brief modelling approaches for FTS can be found here [38].

4. Comparison Framework

In this section we propose the following framework comprised of a set of attributes for the sake of assessing and comparing prominent techniques available in the literature. We try to categorize work done on FTS using some attributes which are explained below.

Year [Cited By]: This attribute reflects the currency of the work. It shows how much important is the topic that it is still active and innovative ideas are being proposed till now. Along with cited by numbers it shows the worth of the proposed work in that research paper.

Order: Lag or Window Size are some synonyms for the word "order". This is an important factor and have great impact on FTS forecasting. It specifies how many previous values of time series are needed to forecast the next value. In very simple models only the most recent previous value is used to predict the next value which are called First (F) Order models. If previous two values are used, then it is Second (S) Order model and so on. Normally after Third (Th) or more they are referred as High (H) Order models. It is not easy to answer how many lags are good for the prediction and one may need to see autocorrelation to have some idea about the number. Taking help from experts in the field or developing several models with different orders especially using some evolutionary algorithm like Genetic Algorithm (GA) can help in getting a good order for the FTS forecasting [9].

Resampling: By doing resampling one tries to change the frequency of time observations either by increasing (upsampling) or by decreasing (downsampling). For example, we may have time observations on a daily basis for whole year and we try to predict on monthly basis. Together with proper selection of order and use of resampling can improve the forecasting accuracy. Our attribute identifies in the form of Yes/No to specify whether an approach uses or apply any resampling or not in their work.

Intervals: It is widely known as Membership Functions (MFs). This criterion perhaps affects the prediction of FTS more than any other factor. The forecasting is highly sensitive especially on the number of intervals used. It must be carefully selected as too many intervals cause overfitting whereas too less can have underfitting. It also effects the interpretability of the system in case if there are too many intervals as it will not be easy for humans to comprehend them. Normally a range between 5 to 9 is considered to be more interpretable for humans [39]. An important consideration is also how the intervals are formed which can be fixed or variable. For simpler models fixed intervals are selected but normally variable length interval can improve the accuracy. Evolutionary algorithms can also help in finding the best length of intervals which is also called tuning the membership functions in the fuzzy literature. Our framework will display only numbers in case of fixed length intervals which can be more than one and show explicitly variable length intervals and shows wherever possible what method used to achieve this.

Shape: This criterion may not have a big impact on the forecasting accuracy but needs some attention as well. The mostly used shape is the triangular (T) one which is considered as most of the time default in FTS forecasting. However, several researchers have used trapezoidal (Tp) shape. Other possibilities could be Gaussian (G), Bell-shaped (B), etc. Again, one must take care in selecting the shape and justify it properly, e.g. selecting Gaussian to avoid abrupt discontinuity or considering the number of parameters needed in Gaussian which are 2 as compared to triangular that takes 3 parameters.

Factors: As can be seen from the previous section that most of the work on FTS is done by using single factor which is referred as Univariate (U). However, many recent papers try to tackle problems that involve more than one factor which is called Multivariate (M). Multivariate case is mostly applied on stock forecasting problems. Although increasing the number of factors does not necessarily improve the forecasting but if done carefully using some techniques like correlation analysis it can provide better results. Increasing too many factors can cause the problem of curse of dimensionality especially in the case of scarce data. Another care must be taken in selecting the number of factors is that while creating the rules it must not exceed between 7 to 9 antecedents as it would be difficult to understand by humans [9].

Heuristics: This criterion tries to identify whether any other heuristics procedure is used in addition to the normal fuzzy time series steps. Genetic Algorithms (GA), Simulated Annealing (SA), Ant Colony Optimization (ANT), Particle Swarm Optimization (PSO) are some examples of heuristics and evolutionary algorithms. If not stuck in local optimum they can provide a better solution than derivative-based methods. The global optimum can be achieved by careful design and selection of parameters and repeating multiple times to get more confidence on the outcome of results [8]. Researchers have used theses heuristics at several steps such as finding the interval lengths, membership function parameters, etc. Any weighted method (Wt) used such as in creating the rules or at any place is mentioned by using this same criterion.

Validation: In the form of Yes (Y) / No (N) it will mention whether any kind of validation technique is used to support the proposed approach or only the results of training dataset are reported. The validation can be division of whole dataset into training and testing with 70/30 ratio or any other suitable one. It can also be in the form of k-fold cross validation, boot strapping, etc.

Raw Data: This is Yes (Y) / No (N) attribute which tells us whether any pre-processing is done on the actual dataset or the same raw data is used for forecasting. This is especially useful when the dataset is using some validation and it does not perform well on the test dataset due to the use of only raw data. Pre-processing or use of some transformation like difference transform can provide much better forecasting results. Most researchers use the raw data like in the case of actual enrollment in the university [5], [6], [11], [12], but several others used features like the difference [17] or percentage change [21] etc.

Dataset: This criterion will display what dataset(s) is/are used in the paper and the following abbreviations are used for some datasets that are mostly used: University of Albama Enrollments (UA), Any stock exchange data such as TAIEX, TAIFEX (StEx), Exchange Rates (ER), Car Accident Mortalities (Car), Spot Gold (Gold). If any of the dataset is rarely found such as patent data etc. it will be categorized as Others (O).

Performance Measure: Many performance measures exist in the literature in order to validate and support the proposed work. It can be Mean Square Error (MSE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), etc. Most of the work on FTS are reporting MSE or RMSE. Although for comparison purpose they might be good but again it is always better to report in some percentage form as only the whole number used may deceive the reader and could not reflect the actual performance of the approach. This criterion will help to identify and reminds the researcher to properly select the performance measures.

Constraints: Table 1 summarize the analysis of prominent techniques found in the literature. The table by no means provides a complete survey on FTS but rather some old classical and pioneering efforts are mentioned along with some recent trends and research on this topic. Readers can refer to a recent survey paper [40] where they reviewed journals of Elsevier for the past twenty five years on the topic of FTS.

Table 1: Comparison Framework

Author[Ref.]	Year [Cited By]	Order	Resampling	Intervals	Shape	Factors	Heuristics	Validation	Raw Data	Dataset	Performance Measure
(Song and Chissom) [5], [6]	1993, 1994 [1140, 918]	F	N	7	T, Tp	U		N	Y	UA	Error %
(S.-M. Chen) [11]	1996 [1101]	F	N	7	T	U		N	Y	UA	Error %
(S. M. Chen) [12]	2002 [472]	H	N	7	T	U		N	Y	UA	MSE
(Tsaur, O Yang, and Wang) [10]	2005 [127]	F	N	7	T, Tp	U		N	Y	UA	Error %
(Yu) [13]	2005 [399]	F	N	105	Tp	U	Wt	Y	Y	StEx	RMSE
(S.-M. Chen and Chung) [22]	2006 [102]	H	N	7	Tp	U	GA	N	Y	UA	MSE
(S. R. Singh) [18]	2007 [160]	F	N	7	T	U		N	Y	UA, O	MSE
(Liu) [24]	2007 [68]	F	N	7	Tp	U		N	Y	UA, O	MAPE, MSE

(S. R. Singh) [19]	2008 [81]	Th	N	5-20	T	U		N	Y	UA	MSE, Error %
(J.-W. Wang and Liu) [17]	2010 [10]	F	N	7	T	U	Wt, Fuzzy Cluster	N	N	UA	MSE, MAPE
(Zhang and Zhu) [20]	2012 [10]	F	N	7	T	U	K-Means	N	Y	UA	MSE
(Saxena) [21]	2012 [44]	F	N	21	T	U		N	N	UA	MSE, Error %
(Rezan et al.) [23]	2013 [4]	F	N	5-20	Tp	U	GA	N	Y	UA	RMSE
(Uslu V.R. et al.)[41]	2013 [31]	F	N	16, 17	T	U	Wt, GA	N	Y	UA, Car, Gold	RMSE, MAPE
(S.-M. Chen et al.) [14]	2013 [71]	S	Y	180	T	Two	PSO	Y	Y	StEx , ER	MSE, RMSE
(Chen et al.) [42]	2013 [131]	S	Y	10	T	U	PSO, SVM	Y	N	StEx	RMSE
(Selim and Elanany) [16]	2013 [2]	H	Y	3-15	T	M	GA	N	Y	UA, Car	MAPE, MSE
(S. M. Chen and Chen) [15]	2015 [61]	S	Y	100	T	Two	Probabilities	Y	Y	StEx , ER	MSE, RMSE
(S. M. Chen and Phuong) [27]	2017 [27]	S	Y	Variable (using PSO)	T	Two	Wt, PSO	Y	Y	StEx , ER	MSE, RMSE
(Rubio, Bermúdez, and Vercher) [25]	2017 [32]	F	N	52 [SD(data)/ 10]	Tp	U	Wt	Y	Y	StEx	RMSE
(S. M. Chen, Zou, and Gunawan) [26]	2019 [-]	F	N	Variable (using PSO)	T	U	PSO	N	Y	UA, Car, Gold	RMSE, MAPE

See Attributes explanation for the abbreviations used in the table

5. Discussion and Analysis

In this section we will try to analyze different approaches using our comparison framework. Some most prominent and recent literature on FTS is selected as it is not possible to cover all of them. Some attribute values are empty either because of inapplicability or it was not mentioned in the proposed work. The analysis of the above table discloses some interesting and eye-opening revelations which we will summarize below.

The initial work mainly focuses on first order models but as the time passes second and higher order models were also tackled. First order models are easier and can provide short term forecasting quite well but there is a need to work more on higher order models in order to improve. It may also be helpful especially in the medical applications where physicians or doctors would like to see the immediate previous history and also interested in some old past history. Together with resampling it can be further enhanced which may not necessarily be fixed resampling rather some evolutionary algorithms can be used to do this step. The table is pointing that much work must be done for the resampling.

Although more intervals or fuzzy sets make the prediction better, but the concept of interpretability is lost in doing that as it will be difficult for the experts in respective areas to understand and comprehend it. The human can better understand from 3 to at most 9 levels but anything beyond that would be quite difficult to capture in their mind. When the paper/research uses like 100 intervals it loses one of the main powers of fuzzy logic which is interpretability. The shape of MFs used is either triangular or trapezoidal and very few have tried gaussian or some other shape in FTS forecasting.

The number of factors used for prediction is a crucial decision but may reveal more information and better performance. There seems to be a need for more multivariate experiments together with the use of autocorrelation and other statistical methods. In addition to classical evolutionary algorithms like GA, PSO etc. their variations and advancement needs to be experimented. Other new and recent optimization and evolutionary techniques could be employed.

Most of the approaches are using the whole data to create rules which is like a lookup table and most probably an overfitting to the data. Moreover, there is no training and testing data and most of the results reported are for the whole data. This is especially the case when the dataset is small. Also, the use of only raw data can cause a problem for the testing data when it is outside of the range of training data. Many have used toy examples and have less amount of data to verify the results properly. There is a need to apply these on some real big data sets and to compare with other approaches especially on testing data set. The reporting of results must also be done in some percentage form and not by using some absolute numbers. Although not visible from the table but a comparison with other non-fuzzy approaches is also missing in most of the work.

6. Conclusion and Future Work

In this research an attempt was made to systematically evaluate and compare prominent FTS forecasting approaches by proposing and using a comparison framework. Several challenges and gaps were identified, and some suggestions were proposed wherever possible. The framework can be improved by considering more relevant attributes in future work. Considerations will be given to factors such as resampling methods, fuzzy output, comparison with other non-fuzzy approaches, etc. The impact of the number of intervals and the number of rules on the interpretability will be experimentally investigated as well. Models capable of offering an acceptable tradeoff between accuracy and interpretability will be investigated.

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Authors Profile

Muhammad Faisal is a PhD student in Computer Science & Engineering department at King Fahd University of Petroleum and Minerals (KFUPM). He completed his MS in Info. & Computer Science from KFUPM in 2001 and earned his BE (Computer Systems Engg.) from NED University, Karachi, Pakistan. He worked at Hafr Al-Batin Community College as a Lecturer for around 10 years in Computer Science & Engg. department. His research interests are Machine Learning, Fuzzy Logic, BioMedicine, Computer Vision, and Image processing.

Moataz Ahmed received his PhD in computer science from George Mason University in 1997. Dr. Ahmed is currently a faculty member with the Information and Computer Science Department, King Fahd University of Petroleum and Minerals, Saudi Arabia. He also severs as Adjunct/Guest Professor in a number of universities in the US and Italy. During his career, he worked as a software architect in several software houses. His research interest includes artificial intelligence and machine learning; and automated software engineering, especially, artificial intelligence-based software testing, software reuse, and cost estimation. He has supervised a number of theses and published a number of scientific papers in refereed journals and conferences in these areas.