A Novel Approach to the Iterative Method WCIP for Fast and Accurate Analysis of the Quasi- Periodic Lumped Circuits

Mohamed Karim AZIZI Abdullah Alhumaidi Alotaibi
Medkarim.azizi@gmail.com a.alhumaidi@tu.edu.sa
Laboratory of Research of Electronics and Microwaves Dept of Physics Faculty of Sciences of Tunis, University of Tunis El Manar Tunis, Tunisia
Computer science program Dept of Mathematics Turabah University College, Taif University Taif, Saudi Arabia

Abstract
In this paper, the theory of a new approach to the wave concept iterative process (WCIP) iterative method implemented to analyze periodic lumped circuits is detailed. This method has several advantages compared with the commercial simulators of lumped circuits, namely easy tracing and simulation of broad periodic circuits within a short time, easy modeling and simulation of distributed circuits by extended equivalent models which are valid on the broadband of frequency, visualization of the distributions of the current and electric field, etc. These advantages are proven by the theoretical study and the simulation results. The validation of the proposed approach is made by a study of a directional coupler and a comparison between the results of simulations and those of the ADS simulator. A good agreement between both results is noticed.

Keywords
WCIP method, quasi-periodic lumped circuits, auxiliary sources, simplified /extended equivalent model, S parameters, electric field distribution, directional coupler.

1. Introduction

Many numerical methods are used for the electromagnetic computation of planar distributed circuits, such as Finite Element Method (FEM) [1], Finite-Difference Time-Domain (FDTD) [2] and Method of Moments (MOM) [3]. But, these methods have the disadvantages of large memory storage and long computation time. The WCIP method, which is used by many publishers over a period of two decades [4-5-6-7], is efficient in terms of its ease of use due to the absence of test functions as well as its fast computation time due to the use of Fast Fourier Transform (FFT) [8-9-]. We have reformulated the WCIP method [10-11] to analyze the periodic lumped circuits. The first steps of this new approach just concern the visualization of the distribution of the electric field and current density [12, 13]. So, we have extended this work to permit the analysis of lumped circuits with more parameters (Z, S parameters) [14-15].
Lumped elements (capacity, inductance and resistance) are used at low frequencies where the wavelength of the signal is sufficiently broad in comparison with the physical dimensions of the circuit built by these elements. In the lumped circuits, the current and voltage variables depend only on time while they depend on both time and space in the distributed circuits. The current and voltage of a long wavelength are uniform, i.e. quasi static through a short conductor in the circuit. This characteristic makes it possible for the behavior of the circuit to be modeled by lumped elements [16]. This circuit modeling becomes less precise as the frequency increases. In other words, the modeling of the distributed circuit using micro strip technology by means of equivalent electric models for the pre-design of several microwave applications is limited to reduced frequency bands. In fact, these models do not take into account the distributed nature of these circuits, i.e. space variable. Thus, to obtain a true behavior of these electric models on broadband, it is necessary that the dimensions of a basic cell of a periodic circuit [17] are sufficiently low when compared with the wavelength, which corresponds to the higher frequency of the interest band. With these dimensions, one can use the current as well as voltage and apply Kirchhoff's laws in the study of the electric circuits.
We can model the planar structures with distributed elements in an effective and precise manner on broadband frequency if each section of a transmission line is described by several L-C cells. The tracing of a structure with several lumped elements in X and Y directions is complicated if one uses commercial simulators like the Advanced Design System (ADS). With this new approach to the WCIP method [18], one can profit from the advantage of easy tracing of a broad periodic circuit in the form of a matrix of pixels with a reduced size, which can be simulated within a short time. Moreover, one can visualize the distributions of the current and electric field even if the adopted circuit includes lumped elements, which is not the case for the commercial simulators of lumped circuits.
The new approach to the WCIP method is based on the technique of the auxiliary sources where an impedance traversed by a current can be replaced by an auxiliary source and vice versa. The use of auxiliary sources enables us to have a general form of the admittance matrix which connects the currents to the voltages. This general form offers a flexibility with the change of the lumped elements of a given cell in the periodic circuit according to the desired application.

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The organization of the paper starts with a description of the based theory of the new approach to the WCIP. In the second part, we consider the application of a directional coupler where a calculation of the inductance L and capacity C of the extended equivalent model (EEM) of the coupler used by the WCIP method is achieved in the same way as those of the equivalent simplified model (ESM). The approach is validated by the comparison of the simulation results with the ADS Software. The limited frequency of the ESM is proven by comparing it with the ADS simulation results of the distributed model.

2. The Theory of wcip method

2.1 L-C Transmission Line Model

An L-C transmission line network can take either one or two dimensional configurations, as shown by the Fig.1.

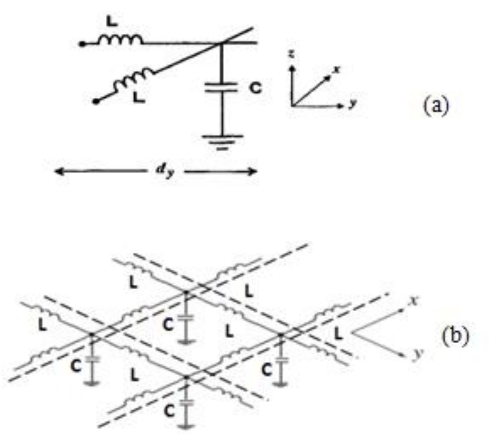

Fig.1. L-C based transmission line model: a) one dimensional configuration, b) two dimensional configuration.

2.1.1 Spectral domain:Calculation of the reflection spectral coefficient

The study of a periodic two-dimensional LC network can be simplified to that of a unit cell due to the periodic walls (dashed lines) which surround it. To obtain the general forms of the mathematical relations between the current and the voltage, we can replace each element (inductor or capacitor) in a unit cell by an auxiliary source (Ex, Ey, Ez), as it is shown in the Fig.2.

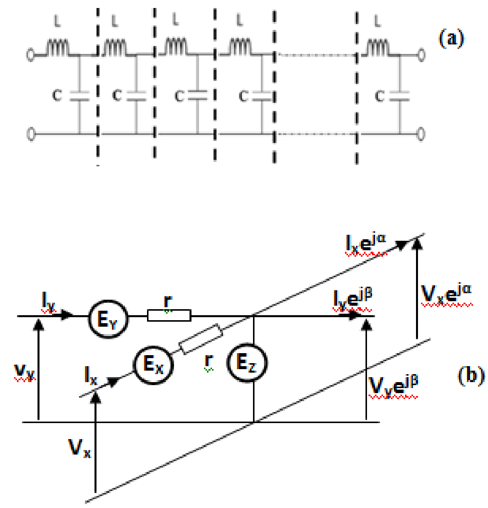

Fig.2. a) A unit cell in a two dimensional L-C network, b) Equivalent circuit using auxiliary sources

Based on Kirchhoff's laws and the Fouquet's theorem applied in the study of periodic structures, we can express the relation between the input and the output of a unit cell in terms of voltage and current by equations 1 to 4.

$$Ix + Iy + Iz - Ix.e^{j\alpha} - Iy.e^{j\beta} = 0 \quad (1)$$

$$Vx.e^{j\alpha} = Vy.e^{j\beta} = Ez \quad (2)$$

$$Vx + Ex - rIx - Ez = 0 \quad (3)$$

$$Vy + Ey - rIy - Ez = 0 \quad (4)$$

Where α and β are the spatial dephasing between two cells along the axis X and Y, respectively.
If Dx and Dy are the dimensions of the total two-dimensional circuit, α and β are then given by (5)

$$\alpha(m) = \frac{2\pi\, m}{M} \quad ; \quad \beta(n) = \frac{2\pi\, n}{N} \quad (5)$$

$$With \quad M = \frac{Dx}{dx}\ ; \quad N = \frac{Dy}{dy}\ ;\ M, N \in IN$$

Where (dx,dy) and (m,n) are the dimensions and the coordinates of a unit cell in the two-dimensional circuit, respectively.
Using the previous equations (1 to 4), we can easily express the voltage of each auxiliary source according to the other elements of the circuit, as follows (6) [14].

$$\begin{vmatrix} Ix \\ Iy \\ Iz \end{vmatrix} = \frac{1}{r} \begin{vmatrix} 1 & 0 & a \\ 0 & 1 & b \\ a^* & b^* & |a|^2 + |b|^2 \end{vmatrix} \begin{vmatrix} Ex \\ Ey \\ Ez \end{vmatrix} \qquad (6)$$

$$a = e^{-j\alpha}(1 - e^{j\alpha}) \; ; \; b = e^{-j\beta}(1 - e^{j\beta})$$

After the calculation of the Eigen values and the Eigen vectors of the matrix in equation (6), we can rewrite the admittance matrix, as follows (7) [14].

$$\bar{Y} = \frac{1}{r}(YY^{+} + (1 + |a|^2 + |b|^2)ZZ^{+}) \qquad (7)$$

With Y+ and Z+ are the transpose of the complex conjugate of the vectors Y and Z in an orthonormal base.

The spectral reflection coefficient is then given by the following equation (8)

$$\Gamma = \frac{1 - Z_0 \bar{Y}}{1 + Z_0 \bar{Y}} \quad ; Z_0 \; \textit{is the characteristic impedance}$$

(8)

When r tends towards 0 [14]:

$$\Gamma = 1 - 2YY^{+} - 2ZZ^{+} \qquad (9)$$

The relation between an incidents waves A and a reflected waves B in the spectral domain can be given by (10)

$$B = \Gamma . A \qquad (10)$$

2.1.2 Spatial Domain: Calculation of The Reflection Spatial Coefficient S

For each cell located by the coordinates (i, j) in a two-dimensional structure, we can express the incident spatial wave A as a function of the spatial reflection coefficient S and the reflected wave B along the three axes X, Y and Z, as follows (11).

$$\begin{pmatrix} A_x(i,j) \\ A_y(i,j) \\ A_z(i,j) \end{pmatrix} = \begin{pmatrix} A_{0x}(i,j) \\ A_{0y}(i,j) \\ A_{0z}(i,j) \end{pmatrix} + \begin{pmatrix} S_x(i,j) & 0 & 0 \\ 0 & S_y(i,j) & 0 \\ 0 & 0 & S_z(i,j) \end{pmatrix} \begin{pmatrix} B_x(i,j) \\ B_y(i,j) \\ B_z(i,j) \end{pmatrix}$$

The spatial reflection coefficient S along the three axes can be expressed by one of the three expressions given in equation (12) according to whether the auxiliary source is replaced by a short circuit, an open circuit or an impedance Z.

$$S_{x,y,z} = \begin{cases} -1 & \textit{the short circuit case} \\ +1 & \textit{the open circuit case} \\ \dfrac{Z - Z_0}{Z + Z_0} & \textit{an impedance Z case} \end{cases} \qquad (12)$$

Where Z0 is the characteristic impedance of the transmission line.

We consider the example of a transmission line section to show the different types of cells (Sr: source cell; t: transmission cell; c: load cell), as depicted in Fig.3

Sr	t	t	t	t	t	t	t	t	t	t	t	t	c

Fig.3. A section of a transmission line with different types of cells

The spatial reflection coefficient S on each cell (i,j) in a two dimensional periodic circuit is given by (13).

$$S_{x,y,z}(i,j) = Hs(i,j).Vals_{x,y,z} + Ht(i,j).Valt_{x,y,z} + Hc(i,j).Valc\;(i,j)_{x,y,z}$$

(13)

Where the functions Hs, Ht, Hc and Vals, Valt, Valc are given by the equations 13.1 to 13.3 and the equations 14.1 to 14.3, respectively

$$Hs_{(i,j)} = \begin{cases} 1: \textit{presence of a source cell} \\ 0: \textit{else} \end{cases}$$

$$Hc_{(i,j)} = \begin{cases} 1: \textit{presence of a load cell} \\ 0: \textit{else} \end{cases}$$

$$Ht_{(i,j)} = \begin{cases} 1: \textit{presence of a transmission cell} \\ 0: \textit{else} \end{cases}$$

The auxiliary sources presented in Fig.2 are replaced by their corresponding impedances according to the type of the cell. The expression of the spatial reflection coefficient of each auxiliary source (impedance) is given by equations 14.1, 14.2 and 14.3 according to the type of the cell (a source cell, a transmission cell or a load cell), as shown by Fig.4, Fig.5 and Fig.6, respectively.

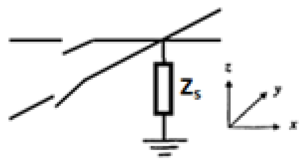

Fig.4 A source cell

$$\begin{cases} Vals_x = 1 \\ Vals_y = 1 \\ Vals_z = \dfrac{Z_s - Z_0}{Z_s + Z_0} \end{cases} \quad (14.1)$$

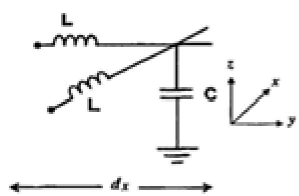

Fig.5. A transmission cell

$$\begin{cases} Valt_x = \dfrac{jlw - Z_0}{jlw + Z_0} \\ Valt_y = \dfrac{jlw - Z_0}{jlw + Z_0} \\ Valt_z = \dfrac{(1/jcw) - Z_0}{(1/jcw) + Z_0} \end{cases} \quad (14.2)$$

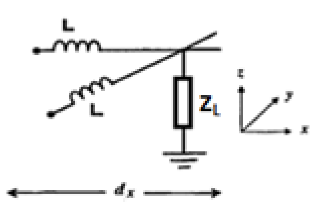

Fig.6 A load cell

$$\begin{cases} Valc_x = \dfrac{jlw - Z_0}{jlw + Z_0} \\ Valc_y = \dfrac{jlw - Z_0}{jlw + Z_0} \\ Valc_z = \dfrac{Z_L - Z_0}{Z_L + Z_0} \end{cases} \quad (14.3)$$

We can rewrite the equation (13) in a matrix form as (15)

$$\begin{pmatrix} S_X \\ S_Y \\ S_Z \end{pmatrix}_{(i,j)} = \begin{pmatrix} Vals_x & Valt_x & Valc_x \\ Vals_y & Valt_y & Valc_y \\ Vals_z & Valt_z & Valc_z \end{pmatrix} \begin{pmatrix} Hs \\ Ht \\ Hc \end{pmatrix}_{(i,j)} \quad (15)$$

2.1.3 The Itertive Process

The iterative process can then be given by the following equations (16)

$$\begin{cases} A = SB + A_0 \;\; \textit{in the spatial domain} \\ B = \Gamma A \;\; \textit{in the spectral domain} \end{cases}$$

(16)

A_0 : the incident wave excited by the feeding source.

The running of the iterative process finishes after a certain number of iterations representing its convergence, as explained by Fig.7 and the following equations (17).

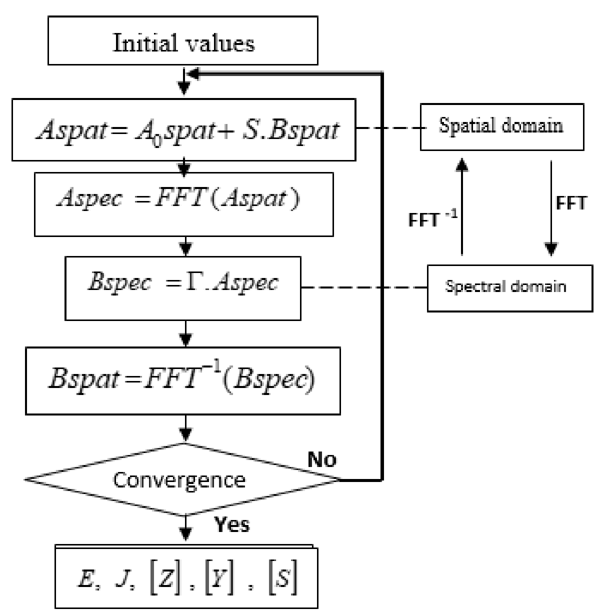

Fig. 7 The iterative process

The abbreviations used in the iterative process diagram: (spat: spatial; spec: spectral; FFT: Fast Fourier Transform; FFT^{-1}: the inverse Fast Fourier Transform).

$$\begin{cases} B^1 = \Gamma . A_0 \quad 1^{rst}\ iteration \\ A^1 = S.B^1 + A_0 \\ B^2 = \Gamma . A^1 \quad 2^{nd}\ iteration \\ A^2 = S.B^2 + A_0 \\ . \\ .B^n = \Gamma . A^{n-1} \quad n^{th} iteration \\ A^n = S.B^n + A_0 \end{cases} \qquad (17)$$

2.1.4 Expressions of current density and electric field

The amplitudes of the three auxiliary sources Ex, Ey and Ez represent the values of the electric fields according to three directions X, Y and Z, respectively.
For a cell with (i, j) coordinates in a periodic circuit, the electric field and current density are expressed as follows (18)

$$\begin{cases} E_{x,y,z} = \sqrt{Z0}(A_{x,y,z}(i,j) + B_{x,y,z}(i,j)) \\ J_{x,y,z} = \frac{1}{\sqrt{Z0}}(A_{x,y,z}(i,j) - B_{x,y,z}(i,j)) \end{cases} \qquad (18)$$

Knowing E and J, we can deduce the impedance parameters of a two ports network as (19).

$$Z = \sum_{x,y} \left(\frac{E_{(x,y)}}{J_{(x,y)}} \right) \qquad (19)$$

Thus the scattering parameters are given by (20)

$$\bar{S} = [1 - Z][1 + Z]^{-1} \qquad (20)$$

3. The Theory of the novel wcip approach

3.1 Study of a directional coupler under the WCIP method

An L-C transmission line model is a cascade of an L-C unit cell with a serial inductor and a shunt capacitor network. The distributed model of a directional coupler and its discretization in unit cells are shown in Fig.8.

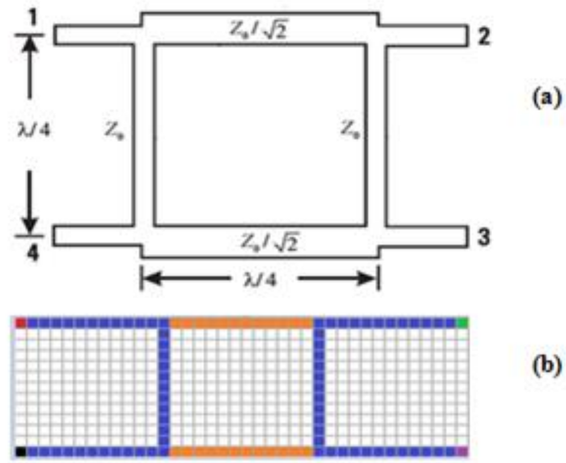

Fig.8. (a) The distributed model of a directional coupler, (b) Discretized model in several unit cells

The L-C extended equivalent model (EEM) of the directional coupler is depicted in Fig.9.

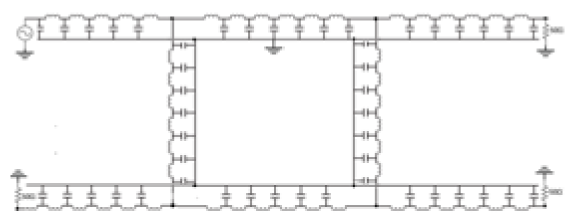

Fig.9. L-C extended equivalent model of the directional coupler

To achieve a quasi-static variation of the current on each cell, we use the length of a unit cell with an order of λg/50. A wavelength of 48 cells corresponds to a quarter of a 12-cell wavelength which is equal to the length of each transmission line section of the directional coupler. Then the length dx of a unit cell is calculated as follows (21).

$$dx = \frac{\lambda g}{48} = \frac{c}{48.f_0.\sqrt{\varepsilon_r}} \qquad ; c = 3.0 * 10^8\, m/s$$

(21)

The characteristic impedance of a lossless transmission line can be written as a function of per- unit- length inductance L’ and capacity C’, as follows (22).

$$Z_0 = \sqrt{\frac{L'}{C'}} \qquad (22)$$

The expressions of the inductance L and the capacity C are independent of the permittivity, as given by (23) and (24)

$$L = L'.dx = \frac{Z_0 \cdot \sqrt{\varepsilon_r}}{c.} \cdot \frac{c}{48.f_0 \cdot \sqrt{\varepsilon_r}} = \frac{Z_0}{48.f_0}$$

(23)

$$C = C'.dx = \frac{\cdot \sqrt{\varepsilon_r}}{c.Z_0} \cdot \frac{c}{48.f_0 \cdot \sqrt{\varepsilon_r}} = \frac{1}{48.Z_0 \cdot f_0} \quad (24)$$

To calculate the values of L', C' and the length dx of a unit cell, we take the permittivity equal to 1.

$$dx = \frac{\lambda g}{48} = \frac{c}{48.f_0 \cdot} = 1.3\,mm \quad ; f_0 = 5GHz$$

(25)

The values of the per unit length inductance L' and capacitance C' are summarized in the table 1

Table 1: The values of the parameters L' and C' of a unit cell.

Z0 = 50Ω	L (nH)	C (pF)	C1+C2 (pF)
Z0	1.5915	0.636	1.536
Z0/sqrt(2)	1.1252	0.9	

4. Results of simulation

The simulation result of the extented equivalent model of the directional coupler is shown in Fig.10.

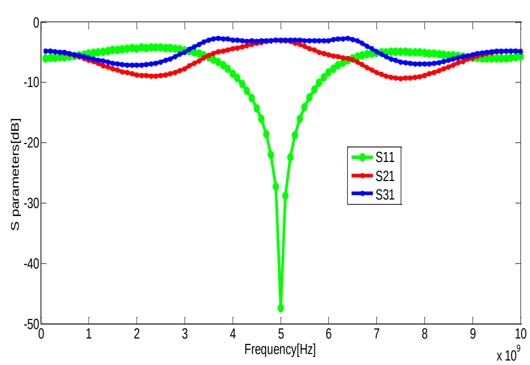

Fig. 10 The S parameters of the EEM of the directional coupler using the WCIP method (the new approach)

It is noticed that a -48dB attenuation of the return loss (S11) at 5GHz (the frequency used in the theoretical calculation) corresponds to a maximum of power transmission from the source to ports 1 and 2 where it is equally divided to generate the insertion losses S21 and S31 of 3dB at 5GHz.

4.1 Design of the directional coupler under the ADS environment using microstrip lines

This design is used for the ideal sections of micro strip transmission lines. The substrate is 1.27 mm in thickness and its permittivity is equal to 10.2.

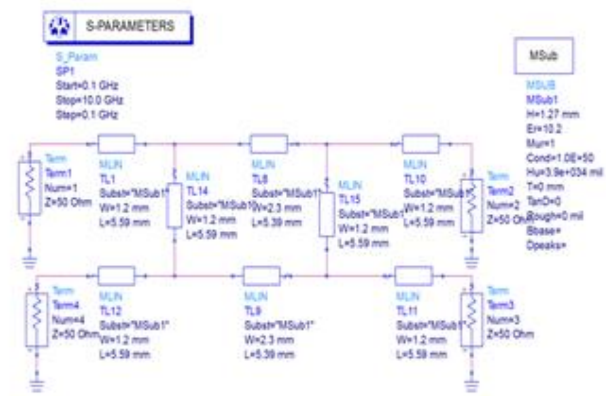

Fig.11 The circuit of the directional coupler under ADS environment

The simulated S parameters with ADS simulator are given by the Fig.12

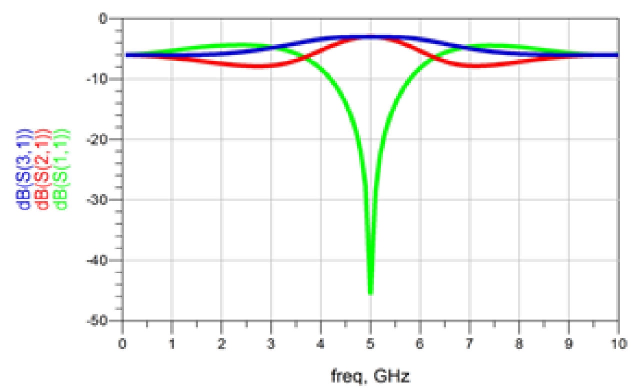

Fig.12 The S parameters of the distributed model of the directional coupler using ADS

A comparison of the simulation resultsgiven by the WCIP and ADS is shown in Fig.13 where a good agreement is noticed.

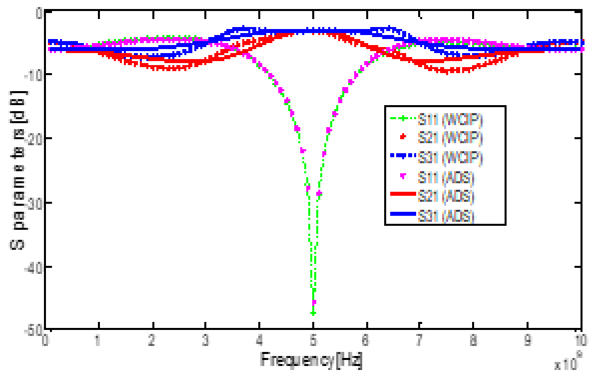

Fig.13. the S parameters of the directional coupler using WCIP and ADS

We can conclude that the extended equivalent model is an accurate model to describe the behavior of the distributed model.

4.2 Study of the simplified equivalent model of the directional coupler

A simplified equivalent circuit of the directional coupler is shown in Fig.14 [15].

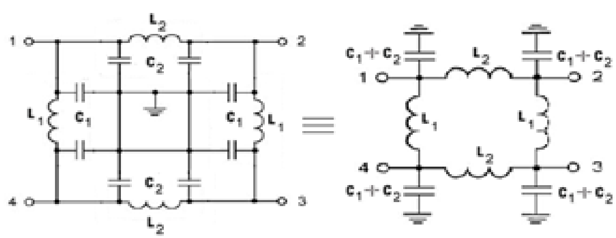

Fig. 14 Equivalent circuits of the directional coupler

The ABCD matrix (Transfer Matrix) of a lossless section of a transmission line is given by (26)

$$\begin{pmatrix} A_T & B_T \\ C_T & D_T \end{pmatrix} = \begin{bmatrix} \cos\theta & jZ_0 \sin\theta \\ j\frac{1}{Z_0}\sin\theta & \cos\theta \end{bmatrix} \quad (26)$$

The ABCD matrix of a pi L-C cell is given by

$$\begin{pmatrix} A_T & B_T \\ C_T & D_T \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}\begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}$$

(27)

The expressions of the inductance L and capacity C can be deduced as

$$L = \frac{Z_0 \sin\theta}{\omega}$$

(28)

$$C = \frac{1}{\omega Z_0}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \quad (29)$$

If θ corresponds to a quarter wavelength (θ =π/2), L and C can simplified to (30) and (31)

$$L = \frac{Z_0}{\omega} \quad (30)$$

$$C = \frac{1}{\omega Z_0} \quad (31)$$

The values of L and C in the simplified equivalent model of the coupler at the frequency 5GHz are then given in table.2

Table 2: Values of L-C parameters of the simplified equivalent model

Z0 = 50Ω	L (nH)	C (pF)	C1+C2 (pF)
Z0	1.5915	0.636	1.536
Z0/sqrt(2)	1.1252	0.9	

The result of simulation using ADS is given by the Fig.15

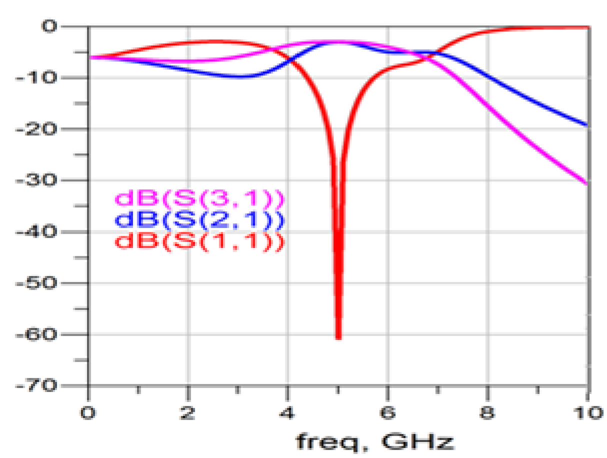

Fig.15 The S parameters of the simplified equivalent model of the directional coupler using ADS

The degradation of the frequency response in the upper band [6-10GHz] proves that the simplified equivalent model is limited in frequencies.

A. Electric Field Distribution on the directional coupler (EEM) using the new approach

As shown in Fig.16 and Fig.17, we can visualize the distribution of the electric field using the new approach to the WCIP method, which is not the case for all simulators of the lumped circuits.

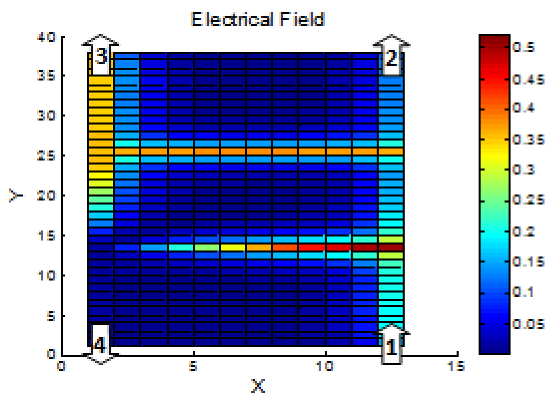

Fig.16. The electric field distribution on the directional coupler (2D view)

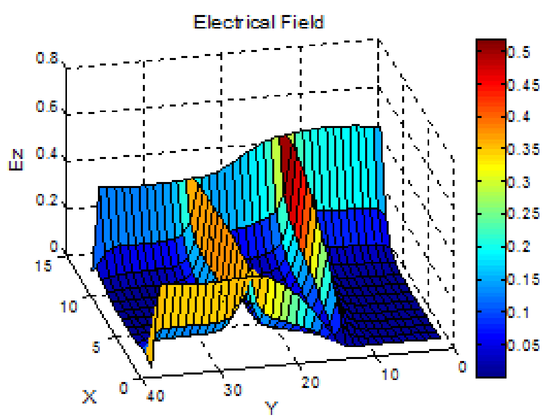

Fig.17 The electric field distribution on the directional coupler (3D view)

It is clear from the electric field distribution that the forth port is decoupled.

5. Conclusion

The theory of a new approach to the WCIP method to analyze periodic lumped circuits has been described in details. To validate our proposed approach, a study of a directional coupler by its extented equivalent model such as a two-dimensional quasi-periodic circuit with many ports has been made. The comparison between the simulation results with the ADS simulator shows a good agreement. The frequency response of an equivalent simplified model shows that this model can be used only for a narrow band of frequency. But, with the extended equivalent model (EEM), the behavior of the distributed circuits using the microstrip technology can be described with accuracy on frequency broadband. This model can be easily studied with less parameter, traced and simulated under our proposed new approach. Moreover, it offers the possibility of the visualization of the electric field and electric current density distributions. As a future work, several applications in metamaterials and electromagnetic bandgaps can be studied with accuracy, more flexibility and within a reduced time.

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