On a modification of the GL-models constructing method

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Abstract
The work is devoted to fault-tolerant multiprocessor systems and graphological models that reflect the response of the system to the occurrence of various multiplicity processors failures. Systems consisting of several subsystems that are not identical in terms of reconfiguration capabilities are considered. In particular, a modification of forming edge functions method of the model is proposed, which allows constructing a single model of a cyclic form without dividing into submodels.

Key words:
fault-tolerant multiprocessor systems, graphical-logical models, reliability calculation.

1. Introduction

Fault-tolerant multiprocessor systems (FMPS) are widely used in the field of complex objects management, where high demands are placed on reliability (in aviation, rocket science, in medical and banking systems, etc.). The calculation of the reliability characteristics of the FMPS is carried out by various methods [1-17] including statistical, which involve statistical tests with models of the FMPS behavior in the failure flow (in particular, GL-models [5,6]). All known methods are quite complicated. This is especially true for systems that behave differently when faults of the same multiplicity appear (in some cases there is a failure of the FMPS, in others - not). Modern FMPS include dozens, and sometimes hundreds of processors, often consist of several subsystems whose components differ in physical parameters. Even worse, when the system includes a subsystem or group of processors that are not equivalent in terms of reconfiguration capabilities. Below we consider just such a case, namely, when some processors of the system can perform the functions of a failed processor, while others do not. In such cases, sometimes it is necessary to divide the system into 2 or more subsystems, use a sliding reserve and other methods, and all this is reflected in the GL-models - they become more complex. At the same time, simpler solutions are sometimes possible.

Formulation of the problem

Below we propose a simpler solution to the problem of constructing a GL-model for FMPS, which has several subsystems of the form X and Y with the following properties

- any subsystem is capable of reconfiguring until the number of processors that fail has not exceeded the maximum possible. (basic FMPS);
- processors of subsystem Y are capable of taking on the functions of X type subsystems processors, but the opposite is impossible.

2. The edge functions formation of the new model

The GL-model is an undirected graph, each edge of which corresponds to a Boolean edge function. The arguments of such functions are the values of the system state vector components, i.e. a vector whose components correspond to the states of the system processors and take the value 1 if the corresponding processor is operational, and 0 if it fails. If the edge function takes a zero value, the corresponding edge is excluded from the graph. The graph connectivity corresponds to the health of the system as a whole.

In [5,6], methods were proposed for constructing GL-models with various properties. In particular, the MEL-model (minimum edges losing) described in [19] is convenient in that it loses one edge when an acceptable number of failures appears and two when an extra one failure appears. The proposed methods allow you to build models of the so-called basic systems, i.e. those that remain operational until the number of failures does not exceed the set value (k-out-of-n systems) [1, 10-16]. The basic GL-models will be denoted by K(m, n) where n is the number of processors in the system, and m is the maximum allowable number of failures. For other types of systems, which are called nonbasic, methods are proposed for modifying the basic GL-models [6], the purpose of which is to maintain the adequacy of the FMPS model behavior in a failure flow. GL-models are used to perform experiments in statistical calculations of reliability parameters, as well as safety [8] of fault-tolerant multiprocessor control systems.

When constructing a GL-model for FMPS, consisting of several subsystems, submodels are usually formed for each of the subsystems, which are then combined into a common model. Often such a model is hierarchical and quite complex. The task of simplifying GL-models is not only of theoretical importance. It is clear that the simpler the model

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is, the less time it takes to perform one experiment with it. This means that at the same time with a simpler model more statistical experiments can be performed, and therefore, the accuracy of calculating the reliability parameters of the system increases.

Let's return to the statement of the problem. We first consider a simple case. Let the system S consist of several subsystems. Each subsystem X_1, X_2 … X_r includes processors capable of reconfiguration only in its subsystem. There is also a subsystem Y, consisting of universal processors that can take on the functions of any processor in system S. All subsystems are 1-fault-tolerant, however, if all processors in the Y subsystem are operational and if any one processor fails in any X_i subsystem, its functions can be satisfied by the subsystem Y. Then the following is true.

Statement. The edge functions of the cyclic type GL-model for system S can be represented as follows: for the submodel Y as well as for the MEL model of a 1-fault-tolerant FMPS, for all subsystems X_i as well as for the MEL model of a 2-fault-tolerant system.

For proof, we consider the following possible situations of failure of the system S as a whole.

1. Failure of 2 or more processors of the subsystem Y.

The constructed model loses 2 edges (or more) in the submodel Y, which leads to a loss of connectivity in the model graph.

2. Failure of 2 processors in the subsystem X_i and 2 processors in the subsystem X_j.

The model loses 2 edges: one edge in each corresponding submodel.

3. Failure of more than 2 processors in the X_i subsystem.

In the corresponding submodel, 2 or more edges are lost according to the properties of the MEL model [19].

4. Failure of 2 processors in the X_i subsystem and one processor in the Y subsystem.

Model loses one edge in submodels X_i and Y

All other combinations of processor failures do not lead to the failure of the S system as a whole, and the constructed model loses no more than one edge, which means that model graph does not lose its connectivity.

Based on the above properties of the basic MEL model, we believe that the validity of the statement is proved.

Consequence. If subsystems of the X_i form have each their own fault tolerance degree m_{Xi}, then when constructing a ring-type model of the system S, edge functions are formed for each such submodel in the same way as for the (m_{Xi} + 1) fault-tolerant MEL model.

The proof of this proposition can be carried out similarly to the proof of the statement.

Example

As an example, consider a simple version of the fault-tolerant multiprocessor system S: one subsystem X, consisting of specialized processors x_1, x_2, …x_8, resistant to 2 failures, and one 1-fault tolerant subsystem Y of universal processors y_1, y_2, y_3, y_4. In addition, if 3 processors in the X subsystem fail and all processors of the Y subsystem are working properly, the S system as a whole remains operational (but not contrariwise). The GL-model of the system S constructed in accordance with the statement and consequence is shown in Fig. 1.

$f_1 = x_1 \vee x2 \vee x_3x_4$

$f_2 = x_1x_2 \vee x_3 \vee x_4$

$f_3 = (x_1 \vee x_2)(x_1x_2 \vee x_3x_4)(x_3 \vee x_4) \vee x_5x_6x_7x_8$

$f_4 = x_1x_2x_3x_4 \vee (x_5 \vee x_6)(x_5x_6 \vee x_7x_8)(x_7 \vee x_8)$

$f_5 = x_5 \vee x_6 \vee x_7x_8$

$f_6 = x_5x_6 \vee x_7 \vee x_8$

$f_7 = y_1$

$f_8 = y_2$

$f_9 = y_3$

$f_{10} = y_4$

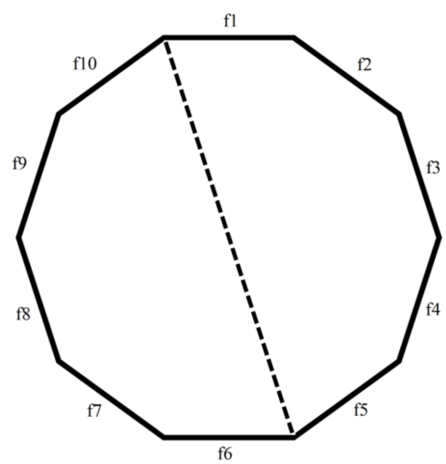

Fig. 1 Example of a GL-model of FMPS, consisting of two unequal subsystems

From the above reasoning, it becomes clear that the behavior in the failure flow of processors of both the S system and the constructed model will be adequate. For example, if the processors x_1, x_2, x_5 fail, only one edge function f_3 becomes equal to 0, and the model loses one edge. If, at the same time, processor y_2 failure appears, then one more edge disappears, the graph of the model loses its connectivity, which indicates that the FMPS S is out of order.

The edge indicated by a dotted line is shown on the figure only in order to illustrate one of possible the ways of transformation of the considered GL-model from basic to nonbasic [6].

3. Conclusion

The case is considered when a fault-tolerant multiprocessor control system of some object has several subsystems, of which only one consists of universal processors capable of performing the functions of any failed processor in the system if the system has been reconfigured. Other processors are capable to be reconfigured only within the limits of their subsystem. Each of the subsystems has its own degree of fault tolerance; a system of universal processors is 1-fault tolerant. A modification of the well-known method for constructing basic GL-models is proposed, which allows us to solve the problem in the framework of cyclic GL-models without dividing the model into separate submodels.

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