Comparison of Four Genetic Crossover Operators for Solving Distance-constrained Vehicle Routing Problem

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Summary
The vehicle routing problem (VRP) is a very difficult optimization problem. It is an important NP-hard problem that has many real-life applications. The problem is seeking to obtain an optimal tour with minimum distance or cost to serve n customers by m vehicles, such that each vehicle starts from the depot, every customer is visited only once, and all vehicles end tour at the depot. There are many variations of the problem. In this paper, we consider distance-constrained VRP (DVRP) in which entire distance traveled by each vehicle is within a predetermined distance limit. Many exact, heuristic, and metaheuristic methods had been applied to solve the VRP and its variations. We propose to apply genetic algorithm (GA) to solve the problem. In GA, crossover operator plays an important role and hence, selection of good crossover operator leads to efficient GA. We compared four crossover operators on TSPLIB instances to determine the best operator. The experimental study shows that the sequential constructive crossover is superior to the other crossover operators in terms of solution quality for the problem.

Key words:
Vehicle routing problem, Distance-constrained, NP-hard, Genetic algorithm, Sequential constructive crossover.

1. Introduction

Many important problems in our life are very complicated; thus, they represent a challenge for computer algorithms. In this paper, we consider one of these problems, called the vehicle routing problem (VRP). The VRP is an important NP-hard problem, which is one of the most studied combinatorial optimization problems. The VRP is concerned with determining the optimal routes for a set of vehicles to serve a set of customers, which was introduced by Dantzig and Ramser before more than 50 years [1]. Since then many researchers have been considered exact and approximate solutions for the problem as a result of its importance for many real-life applications such as transportation networks, shipments delivery, street cleaning etc.

The VRP has many variants which have been discussed in the literature, such as VRP with time windows where each customer must be served during a time period. In this paper, we consider especially distance-constrained VRP (DVRP) where the total traveled distance by each vehicle in the solution is restricted by a maximum possible traveled distance. The DVRP includes determining a set of vehicles routes such that each customer is served only once by one vehicle, each vehicle starts and ends its tour at the depot, and the total traveling distance by every vehicle in the solution is less than or equal to a maximum allowed distance.

Let us define DVRP as follows. Suppose N = {1, 2, . . ., n} be a set of cities (or nodes or customers), city 1 is the depot and there is a group of m identical vehicles. Also, suppose, D = [dij] be a distance matrix that associates every pair of cities. The matrix D will be symmetric if dij=dji, or asymmetric otherwise. This paper considers asymmetric DVRP. The maximum possible traveled distance (Dmax) for any vehicle is given. The problem is to obtain any optimal tour set having least distance to visit n cities using m vehicles, each vehicle tour is beginning from and ending at the depot, and every city is visited only once, and the entire traveled distance by every vehicle is within the predetermined travelled distance limit, Dmax. The VRP is discussed richly in the literature, but the DVRP especially is not a common variant [2].

Since VRP is one of the NP-hard problems, it is observed that obtaining a solution using exact methods is very difficult, while the heuristic and metaheuristic methods are better and more suitable to obtain near-optimal solutions in a short time. Genetic Algorithm (GA) is one of the common metaheuristic algorithms that is applied to solve many NP-hard combinatorial optimization problems, specially, the VRP. GA could find good solutions for these problems in a reasonable time.

In this paper, the aim is to develop an efficient GA for solving the DVRP. In GA, crossover operator plays an important role. So, we compare four crossover operators, namely, Sequential Constructive Crossover (SCX) [3], Cycle Crossover (CX) [4], Partially Mapped Crossover (PMX) [5], and Alternate Edge Crossover (AEX) [6] for solving the problem on some TSPLIB instances.

This paper is organized as follows: section 2 is a literature review of crossover operators, section 3 describes the GA and provides illustrations of the four crossover operators, section 4 presents experimental results and discussion of the applied four crossover operators, and finally section 5 presents conclusion and future works.

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2. Literature Review

In the literature, many GAs were developed to solve different variants of the VRP. Each researcher aims to improve GA by proposing new approaches or designing new operators. GA operators include parents' selection, crossover, mutation, and survivor selection. Since crossover operators have an important role in GA, many crossover operators were proposed for the VRP.

Davis [7] developed ordered crossover (OX), where the offspring is created by randomly selecting 2 crossover points and copying nodes between these points from one parent into the offspring and complete the remaining nodes in the same order that they appear in the other parent. Goldberg and Lingle [5] developed PMX, where the offspring is created by randomly selecting 2 crossover points and copying nodes between these points from one parent into the offspring and complete the remaining nodes from the other parent in mapped process. Oliver et al. [4] developed CX, where offspring is created by taking values and positions from one of the parents, so nodes are copied from each parent in alternated cycles. Blanton and Wainwright [8] proposed two crossover operators, namely, merge#1 and merge#2 that use global knowledge to examine solution space.

Ahmed [3] proposed SCX that produces an offspring with better edges from each parent. Also, it introduces new, better edges that are even not included in any one of the parents. It works by selecting sequentially the nodes with the least distance from both parents, the experimental results on traveling salesman problem (TSP) showed that it is superior to other compared crossover operators. Also, there are two edge-based crossover operators, namely, AEX [6] and edge recombination crossover (ERX) [9]. In AEX, the offspring is created by selecting edges alternatively from each parent, or randomly selecting a feasible edge if there is an infeasible edge. In ERX, the offspring inherits edges from the parents as much as possible and the common edges in parents will have a priority.

Krunoslav et al. [10] made a comparison of 8 crossover operators for solving the VRP and showed through experimental result that AEX is best among them. Rachid et al. [11] studied the performance of many crossover operators based on experiments for solving the VRP; and found that PMX is better than OX and that OX is better than merge #2. Also, a comparative analysis of three crossover operators PMX, CX and OX to solve the TSP was reported in [12] and showed through experimental results that PMX is better than CX, and CX is better than OX. Based on these researches some of the best crossover operators are selected for comparison in this paper, namely, SCX, PMX, CX and AEX to assess their goodness for solving the DVRP.

3. Genetic Algorithm

Genetic Algorithm (GA) was introduced by John Holland in 1960 which is based on reproduction, selection, and evolution that happens naturally [13]. The main principle of GA is that only the fittest individuals can survive. GA works with individuals, which are encoded as chromosomes, each represents a possible solution to a given problem. GA creates an initial population of possible solutions, and evaluates them based on a fitness function, then selects best chromosomes for reproduction or creating new generations repeatedly. GA creates new offspring by applying genetic operators (crossover and mutation) on selected chromosomes (parents). GA repeats this process of generating, evaluating, and selecting individuals iteratively until reaching a stopping criterion that terminates the algorithm such as reaching a specific number of generations. There are several parameters that affect GA performance. Population size is one of these parameters, it specifies how many chromosomes will be created initially, a large population size will increase searching time, while a small population size will affect the search results and it may be not enough to obtain good results. Also, there are probabilities for both crossover and mutation, which determine the possibility of applying the operator on the parents. Moreover, the stopping criterion is one of the GA parameters.

3.1 Genetic Encoding

To solve any problem by GA we must look for encoding, or the way to represent solutions as chromosomes. We consider path representation with m-1 extra nodes (where m is the number of vehicles), these extra nodes represent replications of the depot to indicate the starting of new vehicles [14]. Thus, the chromosome length will be n+m-1 (where n is the number of nodes). So, the distance matrix needs to be modified, to include the new replication of depot node. For this, we add (m-1) copy of the depot (city 1) row and column (i.e., 1st row and 1st column) to the given matrix.

3.2 Fitness Function and Selection Operator

The aim of the DVRP is to find routes with minimum traveling distance. Thus, the objective function is to minimize the total distance of routes in each chromosome and the fitness function is the inverse of the objective function. The selection operator selects, based on their fitness values, the chromosomes that will be used for crossover to create offspring in the next generation. One of the common selection operators is fitness proportionate selection, and many such selection operators are available in the literature. We are going to use stochastic remainder selection method [15] for our GAs.

3.3 Four Crossover Operators and their Illustrations

Crossover is the most significant operator in GAs which is applied on a pair of selected parent chromosomes to produce one (or two) offspring chromosome(s). One can use one-point or multi-point crossover operator. Among various crossover operators found in the literature for the TSP and related combinatorial optimization problems, four crossover operators, namely, SCX, CX, PMX, and AEX, are proposed to implement for our problem, and then their results will be compared for finding best operator among them. For illustration of these crossover operators, we consider n=7 and m=2. We construct modified matrix by adding one copy of the depot (city 1) row and column (i.e., 1st row and 1st column) to the original distance matrix, which is reported in Table 1. We use same parent chromosomes P_1: 1 2 4 8 3 6 5 7 and P_2: 1 3 8 5 2 7 4 6 for the illustration. The objective value of each offspring is calculated by adding the tour distance of each vehicle in the produced offspring. The total distance of 1st parent is 75 with the maximum vehicle distance is 54 and the other vehicle distance is 21. The total distance of 2nd parent is 72 with the maximum vehicle distance is 56 and the other vehicle distance is 16.

3.3.1 Sequential Constructive Crossover (SCX)

SCX [3] was developed for the TSP and then is applied successfully to many other combinatorial optimization problems [16-22]. It produces an offspring with better edges from each parent. Also, it introduces new better edges that are not included in any of the parents. Thus, the possibility of producing better offspring is high. It adds the first city to the offspring, then it looks sequentially for the following city in both parents and chooses the one with less distance value. If all the following cities are infeasible, then it considers all cities sequentially 1,2, …, n, and selects the first valid city instead of the infeasible following city. As an illustration, first 'city 1' will be added to the offspring, and hence the partially resulting offspring is "1 - - - - - - -". Then, the successors of 'city 1' in both parents are 'city 2' and 'city 3' respectively. Now, distance from 'city 1' to both 'city 2' and 'city 3' are 2 and 11 respectively. Hence, the city with the least distance, that is, 'city 2' is added to the offspring, and the partially resulting offspring is "1 2 - - - - - -". The successors of 'city 2' are 'city 4' and 'city 7' with distances are 8 and 6 respectively in the parents, and thus 'city 7' will be added. The partially resulting offspring becomes "1 2 7 - - - - -". Then, there is no successor of 'city 7' in P_1, and thus P_1 is searched sequentially from the beginning looking for a valid city and found 'city 4' as the first valid city. Successor of 'city 7' in P_2 is 'city 4' with lower distance 10, and thus city '4' is added into the offspring, and the partially resulting offspring is "1 2 7 4 - - - -". Then, continue the same process to get a complete chromosome, and the final offspring is "1 2 7 4 6 3 8 5", where the maximum vehicle distance is 37, the other vehicle distance is 19, and the total distance is 56. The offspring distance is better than both parents.

3.3.2 Cycle Crossover (CX)

CX [4] produces an offspring by copying cities from each parent in alternating cycles. It selects cycle from parent 1 and another from parent 2 and it continues the same process until getting a complete offspring chromosome. Thus, each city takes its position from one of the parents. As an illustration, the first city from P_1 'city 1' is added to the offspring, and the partially resulting offspring is "1 - - - - - - -", the corresponding city in P_2 is also 'city 1', and thus a cycle occurs and parents' role is exchanged. The second city is added from P_2, and the partially resulting offspring is "1 3 - - - - - -", the corresponding city in P_1 is 'city 2', which is added to the offspring at its position in P_2, and thus the partially resulting offspring is "1 3 - - 2 - - -". The corresponding city of 'city 2' in P_1 is 'city 3', which is already visited, and thus a cycle occurs and parents' role is exchanged. After that, the first valid city in P_1 'city 4' is added to the offspring and the partially resulting offspring is "1 3 4 - 2 - - -". Then, continue the same process to get a complete chromosome, and the final resulting offspring is "1 3 4 8 2 7 5 6", where the maximum vehicle distance is 33, the other vehicle distance is 31, and the total distance is 64. The offspring distance is better than both parents.

3.3.3 Partially Mapped Crossover (PMX)

PMX [5] produces an offspring by selecting 2 randomly crossover points, then it copies a sequence of cities between these points from the first parent to the offspring. After that, it adds the remaining cities from the second parent, and it uses a mapping process when the city is already added from the first parent. As an illustration, suppose that random crossover points r_1 and r_2 are 3 and 5. Subsequence between these two points in P_1 "4 8 3" is copied to the offspring at the same position, and thus partially resulting offspring is "- - 4 8 3 - - -". After that, the remaining cities are copied from P_2, starting after point r_2. First city to add from P_2 is 'city 7', which is a valid city, and thus the partially resulting offspring is "- - 4 8 3 7 - -". The next city is 'city 4', and it is invalid since it is already visited, and thus the mapping to its corresponding city in the subsequence between the crossover points in P_1 is needed, the partially resulting offspring is "- - 4 8 3 7 5 -". Then, continue the same process to get a complete chromosome, and the final resulting offspring is "1 2 4 8 3 7 5 6", where the maximum vehicle distance is 37, the other vehicle distance is 21, and the total distance is 58. The offspring distance is better than both parents.

3.3.4 Alternate Edge Crossover (AEX)

AEX [6] produces an offspring by choosing edges from each parent alternatively, and in case of infeasibility it chooses randomly a feasible edge. As an illustration, first edge “1 2” is added from P_1, and the partially resulting offspring is “1 2 - - - - - -”. Then, the edge “2 7” from P_2 is added and the partially resulting offspring is “1 2 7 - - - - -”. After that, there is no following edge for ‘city 7’ in P_1, and thus a random valid city is added, for example, ‘city 3’ and the partially resulting offspring is “1 2 7 3 - - - -”. Then, continue the same process to get a complete chromosome, and the final resulting offspring is “1 2 7 3 8 4 6 5”, where the maximum vehicle distance is 41, the other vehicle distance is 25, and the total distance is 66. The offspring distance is better than both parents.
For the above example, it is found that SCX is the best among the four crossover operators.

3.4 Survivor Selection

After applying the crossover operator survivor selection operation [3] is applied, that selects chromosomes for the next generation, which is also called chromosomes replacement. It considers parents and offspring chromosomes with high fitness value only which compete. Survivor selection includes two approaches: generational where all or large group of chromosomes are replaced, or steady-state where only one chromosome or few chromosomes are replaced. We use a steady-state survivor selection that considers fitter chromosomes from both parents and offspring.

3.5 Mutation Operator

Mutation operator ensures the diversity of the population. The mutation is applied with a probability, which is usually very low. We apply exchange mutation; where two positions are randomly chosen, and their values are exchanged.

4. Experimental Results and Discussion

The aim of the experiments is to compare the performance of four crossover operators: SCX, CX, PMX, and AEX. GAs using these crossover operators are encoded in C++ and run on a personal computer with intel core i5-1.6 processor and 8 GB RAM under MS Windows 10. Computational tests are performed on fifteen asymmetric TSPLIB instances, with instance size ranges from 17 to 171. The common parameters are selected for the algorithms as follows: population size is 70, probability of crossover is 1.0, probability of mutation is 0.09, and maximum of 20,000 generations as the stopping condition. The tests were performed 20 times for each instance.

We report Max, Best solution, Average solution and Average time of convergence (in second) of 20 runs by the algorithms for different number of vehicles (m) and different values of D_{max}. In Table 2 (a & b), we used $D_{max}=\infty$ to find Max(1), whereas in Table 3 (a & b), we used $D_{max}=0.9*Max(1)$ to find Max(2) and new solution.
In Table 2 (a & b), when m=1 (only one vehicle is used) the unrestricted DVRP (with $D_{max}=\infty$), could be considered as the usual TSP. Thus, the DVRP results could be compared to the optimal/best known solution of the TSP. To the best of our knowledge, there are no optimal known solutions for the DVRP, thus it is difficult to assess the efficiency of the proposed GAs with more than one vehicle precisely. For the smallest instance (br17) all crossover operators found good solutions which are equal to the optimal/best known solution. When m=1, SCX found solutions near to the optimal solutions in most of the cases. Also, compared to other crossover operators, SCX found the best solutions in 73.33% of the test cases, noting that the best solutions that are equal to the results of other crossover operators are not included in this percentage. Moreover, SCX found the best maximum distance for a single vehicle in the solution for 44.44 % of the test cases. AEX found the best solutions for three cases and it found a solution equals to SCX solution in one case only. For other cases, when m>1, SCX obtains better solutions compared to other three operators. In general, SCX obtains better results than other crossover operators, especially for large-sized instances. However, the average computational time to find solutions is better with CX then AEX.
In Table 3 (a & b), the results for the restricted DVRP (with $D_{max}=0.9*Max(1)$) are shown for m=2 and 3. In some cases, there was no result found with this maximum distance, thus another close value for maximum distance was applied. In the comparison of the four crossover operators, SCX outperforms other compared crossover operators, especially for large-sized instances and could be considered as the best one. For this test also, the average computational time is better with CX in most of cases. It is to be noted that SCX showed very good results and was superior to other crossover operators for solving the TSP [7]. Moreover, SCX showed very good results on the DVRP in this experiment but for some of the instances still best solution is not achieved.

5. Conclusion and Future Works

This paper presented a comparison of four GA crossover operators, SCX, PMX, CX, and AEX for solving the DVRP. Two sets of experimental tests were performed on some TSPLIB instances. The first test was unrestricted, and the maximum distance allowed for each vehicle was set to a very large value, and in the second test maximum distance was restricted by multiplying 0.9 to the maximum distance found in the first test or restricted to another appropriate value. Among the four crossover operators, SCX finds the best quality solutions in both unrestricted and restricted cases DVRP. However, for some instances still best solution is not achieved by SCX. So, one can improve the simple GA to obtain better solutions by hybridization with some local search methods.

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Table 1: The modified distance matrix.

City	1	2	3	4	5	6	7	*8*
1	99999	2	11	10	8	7	6	99999
2	6	99999	1	8	8	4	6	6
3	5	12	99999	11	8	12	3	5
4	11	9	10	99999	1	9	8	11
5	11	11	9	4	99999	2	10	11
6	12	8	5	2	11	99999	11	12
7	10	11	12	10	9	12	99999	10
8	99999	2	11	10	8	7	6	99999

Table 2 (a): Unrestricted DVRP test results (SCX and CX results).

			SCX				CX			
Inst.	**n**	**m**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**
br17	**17**	1	39	39	39.00	2.91	39	39	41.05	1.51
		2	39	39	39.00	4.64	39	39	41.00	2.81
		3	31	42	42.00	5.06	31	42	43.90	1.62
ftv33	**34**	1	1329	1329	1352.95	11.56	1475	1475	1649.00	2.80
		2	1316	1342	1374.00	11.13	822	1470	1605.95	3.04
		3	1195	1328	1380.95	12.49	1188	1420	1582.50	3.13
ftv35	**36**	1	1490	1490	1499.65	13.57	1689	1689	1820.55	3.03
		2	1463	1489	1510.65	12.80	1641	1667	1829.10	3.17
		3	1393	1511	1521.65	14.67	933	1584	1789.70	3.25
ftv38	**39**	1	1549	1549	1582.20	14.35	1652	1652	1875.95	3.25
		2	1533	1559	1598.50	7.90	1309	1666	1900.85	3.56
		3	886	1574	1592.70	8.25	1576	1771	1914.75	3.52
p43	**43**	1	5639	5639	5646.40	9.63	5641	5641	5655.75	3.83
		2	5617	5637	5642.50	19.69	5624	5644	5664.45	3.87
		3	5631	5703	5712.30	20.39	5642	5714	5739.60	4.12
ftv44	**45**	1	1663	1663	1726.05	9.38	1828	1828	2063.20	3.91
		2	1609	1635	1698.80	17.44	1376	1935	2126.85	3.93
		3	1547	1665	1739.00	20.70	1714	1987	2141.10	4.07
ftv47	**48**	1	1815	1815	1852.40	20.99	2134	2134	2353.60	4.05
		2	981	1879	1945.80	21.67	1400	2011	2394.15	4.04
		3	1672	2000	2065.10	20.67	1155	2252	2404.70	4.25
ry48p	**48**	1	15309	15309	15532.10	19.63	16270	16270	17847.20	4.06
		2	15339	15790	16041.40	10.42	15640	16091	18628.80	4.27
		3	15074	16016	16336.90	21.77	14434	16975	18718.00	4.36
ft53	**53**	1	7468	7468	7916.90	24.59	8189	8189	9807.35	7.21
		2	7396	7679	8115.45	26.45	4515	8989	9986.10	8.56
		3	7447	7900	8297.25	12.73	8457	9008	9860.85	4.68
ftv55	**56**	1	1635	1635	1727.70	28.00	1960	1960	2307.70	9.88
		2	1051	1732	1817.30	18.67	1089	2057	2317.25	9.75
		3	1117	1795	1868.40	28.25	1000	2223	2411.90	9.26
ftv64	**65**	1	1996	1996	2100.40	35.00	2394	2394	2691.60	10.60
		2	1986	2012	2090.20	36.44	2368	2481	2696.95	10.70
		3	1879	1999	2113.05	37.66	1172	2577	2751.20	9.00
ft70	**70**	1	40548	40548	41082.60	18.76	42243	42243	43599.30	11.40

			SCX				CX			
Inst.	**n**	**m**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**
		2	39980	40964	41551.80	36.36	41963	42947	44567.10	10.90
		3	22295	41947	42351.20	42.87	28358	44032	45231.80	6.06
ftv70	**71**	1	2087	2087	2232.95	38.88	2769	2769	2957.30	11.80
		2	2138	2164	2182.55	37.38	1900	2640	2892.60	11.80
		3	1988	2108	2205.25	40.85	2420	2704	3010.50	11.50
kro124p	**100**	1	42067	42067	42820.20	65.63	57000	57000	60546.30	16.80
		2	35422	42421	43380.90	34.46	43881	58543	60945.80	15.50
		3	35612	43471	44203.80	74.58	48262	57209	62683.30	16.70
ftv170	**171**	1	3440	3440	3539.20	184.70	13071	13071	13327.30	30.50
		2	3400	3430	3561.25	88.20	10927	12638	13473.70	28.60
		3	2397	3401	3577.05	87.95	11092	13398	13632.40	27.80

Table 2 (b): Unrestricted DVRP test results (PMX and AEX results).

			PMX				AEX			
Inst.	**n**	**m**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**
br17	**17**	1	39	39	40.85	23.100	39	39	39.00	2.89
		2	39	39	40.25	24.98	39	39	39.25	3.08
		3	31	42	44.90	29.04	31	42	42.00	2.73
ftv33	**34**	1	1447	1447	1609.30	15.36	1286	1286	1346.90	3.24
		2	1212	1379	1563.90	31.09	1195	1302	1336.85	6.37
		3	1347	1498	1626.50	15.60	1195	1328	1359.90	6.82
ftv35	**36**	1	1667	1667	1807.00	21.41	1479	1479	1522.90	6.31
		2	1621	1647	1828.75	15.70	1382	1489	1508.80	3.36
		3	1086	1657	1861.85	15.75	1393	1511	1535.90	3.45
ftv38	**39**	1	1718	1718	1858.95	15.72	1536	1536	1604.65	3.46
		2	1226	1757	1914.10	16.43	1480	1587	1643.15	3.77
		3	874	1789	1931.70	16.24	1543	1629	1679.05	3.88
p43	**43**	1	5635	5635	5654.85	33.98	5736	5736	5755.65	5.18
		2	5619	5639	5669.25	31.95	5715	5735	5770.45	4.64
		3	5621	5693	5723.25	30.23	5634	5809	5848.70	4.92
ftv44	**45**	1	1863	1863	2055.75	16.55	1932	1932	2061.55	4.26
		2	1097	1798	2131.25	16.56	1652	1870	2055.30	4.35
		3	1356	1936	2110.15	16.54	1850	1968	2116.35	4.47
ftv47	**48**	1	2080	2080	2349.45	16.54	2209	2209	2353.00	4.49
		2	2069	2142	2375.45	16.58	1134	2257	2400.25	4.50
		3	1712	2271	2463.50	16.78	1230	2521	2728.60	4.95
ry48p	**48**	1	16424	16424	18359.80	32.50	18144	18144	18648.00	4.59
		2	9699	15895	18059.20	32.82	16942	17845	18733.10	9.06
		3	14612	16204	18215.00	16.98	13969	20247	20695.20	9.55
ft53	**53**	1	8389	8389	9629.40	16.77	8524	8524	8975.10	7.80
		2	8324	8507	9816.20	16.76	6149	8519	9014.40	5.16
		3	9049	9049	9993.10	16.99	8561	9082	9677.75	9.28
ftv55	**56**	1	1975	1975	2300.00	17.33	2731	2731	2819.00	5.45
		2	1995	2068	2301.30	17.33	1487	2695	2919.65	5.56
		3	1154	2134	2375.15	17.26	2013	2801	2969.65	5.75
ftv64	**65**	1	2363	2363	2644.90	17.82	3805	3805	4024.60	6.62

			PMX				AEX			
Inst.	**n**	**m**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(1)**	**Best sol**	**Avg. sol**	**Avg. time**
		2	1813	2379	2669.75	17.89	2886	3875	4040.05	6.74
		3	1566	2376	2680.10	18.00	2696	3973	4143.55	6.90
ft70	**70**	1	42801	42801	43609.30	18.06	44560	44560	45280.40	12.488
		2	42118	43102	44723.80	18.70	44662	45646	46027.10	7.35
		3	35448	43704	44913.10	18.86	28536	46791	47301.90	7.51
ftv70	**71**	1	2647	2647	2909.15	18.22	4002	4002	4305.00	7.26
		2	1416	2725	2919.85	18.22	2238	4098	4332.20	7.35
		3	1812	2658	2933.85	18.36	2506	4161	4424.45	7.53
kro124p	**100**	1	66025	66025	68620.40	42.41	104210	104210	106554.00	11.17
		2	57104	65880	70121.40	33.16	86423	103786	107459.00	11.32
		3	38118	69026	71622.90	35.06	90865	105161	109035.00	11.60
ftv170	**171**	1	15059	15059	15400.50	25.55	10752	10752	11180.50	23.96
		2	10596	15247	15477.40	25.70	7931	10940	11280.00	24.19
		3	7622	15385	15667.10	49.79	8830	11038	11381.00	24.39

Table 3 (a): Restricted DVRP test results (SCX and CX results).

			SCX				CX			
Inst.	**n**	**m**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**
br17	**17**	2	31	42	42.00	2.61	31	42	43.95	2.18
		3	31	42	41.45	4.30	31	42	43.90	1.66
ftv33	**34**	2	749	1387	1442.45	12.66	1129	1490	1617.85	3.06
		3	957	1382	1395.05	13.91	683	1509	1652.30	5.82
ftv35	**36**	2	1036	1508	1579.80	13.84	970	1665	1806.60	3.17
		3	886	1517	1538.10	14.33	1019	1668	1842.75	3.21
ftv38	**39**	2	827	1617	1703.95	15.46	974	1744	1908.35	3.50
		3	708	1727	1795.25	16.82	1055	1854	1964.90	3.51
p43	**43**	2	5449	5703	5720.80	21.66	5445	5701	5727.05	3.93
		3	5445	5721	5727.00	10.36	5445	5748	5795.50	4.24
ftv44	**45**	2	1017	1752	1844.30	19.35	1047	1967	2128.15	7.04
		3	1112	1706	1800.30	9.69	1051	1922	2114.75	5.55
ftv47	**48**	2	1771	1886	1987.05	22.15	1184	2111	2366.15	8.29
		3	990	1945	2067.90	19.95	914	2175	2468.95	7.01
ry48p	**48**	2	9456	16295	16987.30	10.40	13117	16589	18739.80	6.91
		3	8963	16572	16868.00	22.34	11443	16979	19221.30	8.84
ft53	**53**	2	5047	7832	8392.75	25.31	5328	8484	9744.40	4.53
		3	5663	7955	8405.15	26.71	6237	8881	9700.65	8.10
ftv55	**56**	2	928	1741	1830.80	27.90	1253	2142	2319.55	10.10
		3	707	1839	1904.20	13.53	742	2117	2420.60	4.88

			SCX				CX			
Inst.	**n**	**m**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**
ftv64	**65**	2	1692	2122	2197.50	16.98	1614	2470	2651.60	11.80
		3	1005	2006	2093.45	17.43	1289	2436	2667.40	5.64
ft70	**70**	2	34865	42217	42722.70	26.17	24524	42794	44393.80	12.80
		3	17623	43206	43962.70	19.70	22076	43930	45230.50	11.00
ftv70	**71**	2	1475	2257	2284.10	19.57	1495	2580	2846.80	11.70
		3	1522	2129	2238.45	19.97	1089	2707	2966.80	12.20
kro124p	**100**	2	25671	43357	44548.10	68.56	33017	59348	67955.60	9.17
		3	29048	42866	44965.60	34.49	33994	60338	63765.30	18.10
ftv170	**171**	2	2182	3516	3788.25	86.99	7494	13001	13501.50	30.60
		3	1682	3382	3585.45	87.54	6070	12917	13526.10	31.40

Table 3 (b): Restricted DVRP test results (PMX and AEX results).

			PMX				AEX			
Inst.	**n**	**m**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**
br17	**17**	2	31	42	44.20	29.19	31	42	42.00	2.60
		3	31	42	45.45	14.47	31	42	42.20	3.53
ftv33	**34**	2	971	1470	1667.55	15.62	877	1336	1373.55	7.09
		3	1012	1525	1648.95	15.77	877	1362	1397.00	6.58
ftv35	**36**	2	1018	1616	1850.75	16.19	886	1491	1527.70	6.86
		3	807	1758	1895.05	16.11	886	1517	1558.55	6.93
ftv38	**39**	2	963	1746	1952.15	16.05	1068	1626	1701.45	7.55
		3	869	1822	1968.50	16.19	1013	1665	1706.65	6.33
p43	**43**	2	5553	5689	5714.40	33.66	5488	5800	5827.50	9.05
		3	5445	5738	5910.05	34.83	5444	5876	5927.35	10.26
ftv44	**45**	2	947	1875	2155.05	33.45	1258	2154	2501.05	9.55
		3	942	1993	2206.35	35.20	1192	1922	2132.35	9.04
ftv47	**48**	2	1305	2169	2363.15	34.38	1368	2546	2951.30	4.93
		3	1216	2263	2463.25	36.12	1222	3216	3534.50	5.24
ry48p	**48**	2	10619	16387	18364.80	28.71	10919	18242	19412.80	9.75
		3	9977	16870	18671.90	16.77	9479	19974	21665.80	10.38
ft53	**53**	2	5460	8906	9887.90	16.91	5139	9601	10124.50	11.45
		3	4058	8284	9702.10	17.12	5118	9479	9831.05	11.20
ftv55	**56**	2	1189	1981	2258.55	33.15	1764	3108	3502.00	5.99
		3	819	2111	2464.20	35.45	1202	3008	3256.70	12.20
ftv64	**65**	2	1525	2358	2580.70	36.78	2334	4497	4708.20	14.03
		3	1004	2422	2678.25	37.47	2118	3963	4476.00	8.77

			PMX				**AEX**			
Inst.	**n**	**m**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**	**Max(2)**	**Best sol**	**Avg. sol**	**Avg. time**
ft70	**70**	2	33731	43555	44646.10	18.34	31218	45940	46648.70	15.79
		3	29222	43617	45225.30	18.40	23840	47136	48618.30	13.07
ftv70	**71**	2	1725	2590	2897.50	34.86	2419	4614	4856.70	16.34
		3	1294	2625	2870.35	18.46	1971	4601	4911.95	16.28
kro124p	**100**	2	33642	67106	71329.00	21.12	56528	104953	109467.00	23.62
		3	26561	69515	73574.10	41.71	73624	104368	109649.00	24.94
ftv170	**171**	2	7736	15277	15775.50	26.50	6386	11775	12296.90	53.54
		3	5985	15463	16093.30	25.89	7624	11523	11913.10	51.87