On the Use of Non-Uniform FFT for Fast and Secure Wireless Communication

Mohamed Ayari [1†]

[1] Faculty of Computing and Information Technology, Northern Border University –Kingdom of Saudi Arabia;
[†] Syscom Laboratory, National Engineering School of Tunis, Tunis El-Manar University, Tunisia

Abstract
The present work sets out to present a new secure and fast image cryptosystem based on two-dimensional non-uniform fast Fourier transform (2D-NUFFT) so as to improve the security level of system data. Taking into consideration the benefits of its image compression properties, the Discrete Wavelet Transform (DWT) in a two-dimensional index is introduced and implemented in the novel cryptosystem to decrease the global time-computation and to Boost Data Security Level. The Singular Value Decomposition (SVD) as well as the randomization process, are performed with 2D-NUFFT to corroborate the efficiency and robustness of the proposed algorithms in contrast to the conventional one investigated in recent works apropos of accuracy, stability and time efficiency.

Key words:
Image encryption; 2D-NUFFT; 2D-DWT; Singular Value Decomposition; Time-efficiency

1. Introduction

The fast evolution of technology has obliged the security field to follow it closely in order to ensure a minimum of confidentiality, availability and integrity of both digital data and communication. Day-by-day, the cyber-warfare is growing due to the potential non-stop hacking tools and techniques. This has urgent the researchers and scientists to develop powerful and robust methods capable to circumvent the fascinated hacking and the subtle craftiness of the hackers. The security in image processing becomes a vital component to ensure a secured submission of digital images from transmitter to specified receiver. Digital images are the subject of many applications such as military [1-2], biometric [3], cloud computing [4-5] and several other domains. Optical encryption techniques have held a special place in image security applications and many approaches have been developed in this perspective. Indeed in [6], an encryption algorithm using two chaotic logistic maps and an external key of 80-bit has been developed and analyzed using statistical, key sensitivity and key space properties so as to prove the security of the image encryption process.

In the same trends, many image encryption algorithms have been implemented using different techniques such as the chaotic tent map [7], DNA and chaotic logistic maps [8] and fractional chaotic time series [9]. These techniques have suffered from several attacks mainly in the applications pertaining to real-time image transmission over the communication channels. By the way, many algorithms are concentrated on the handling of 2D- matrix defining the plain-image by applying another encipherment approaches in particular, wave transmission [10], Hartley transform (HT) [11], Gyrator transforms (GT) [12], 2D-discrete Fourier transform (2D-DFT) [13], 2D- fractional Fourier transform (2D-FrFT) [14], Fresnel transform (FrT) [15-16], Cosine transform (CT)[17] and others [18-20].

To increase the security level, the researchers have combined these aforementioned transforms with random or chaotic masks to obtain a solid image encryption process. However for some of them, the correlation between plain and cipher images can be detected and the algorithm can be broken easily by the attackers [21-22]. Other image encryption algorithms present a good resistance against hacking due to the implementation of the special encryption key as investigated in [23-26]. Nevertheless despite their robustness and solidity, these algorithms remain quasi-slow in terms of computational time. From this perspective, we propose in this work a novel fast and efficient image encryption approach based on singular value decomposition (SVD) algorithm and 2D-NUFFT which is considered as a new special transform used recently in image encryption applications.

This paper is ordered as follows: the different techniques defining our novel approach such as two-dimensional Discrete Wavelet Transform (2D-DWT), 2D-NUFFT and singular value decomposition are presented and evaluated in section II. A detailed description of the novel symmetric secure image cryptosystem is the subject of Section III. Section IV investigates and evaluates the proposed system within the frame of image encryption uses. Section V presents our conclusions and future work.

Manuscript received September 5, 2020
Manuscript revised September 20, 2020

DOI: 10.22937/IJCSNS.2020.20.09.13

2. Theory

In this section, a brief theoretical background of 2D-DWT and 2D-NUFFT are presented and evaluated in terms of computational complexity as well as the singular value decomposition (SVD).

2.1. S Two-Dimensional Discrete Wavelet Transform (2D-DWT)

Let ϕ be a 2D-scaling function derived from unidimensional one taking the following form:

$$\phi(x,y)=\phi(x)\phi(y) \qquad (1)$$

Let ψ be its analogous 2D-wavelet producing an orthonormal basis of $L^2(R)$. Three 2D-wavelets can be defined as follows:

$$\psi^1(x,y)=\phi(x)\psi(y) \qquad (2)$$

$$\psi^2(x,y)=\psi(x)\phi(y) \qquad (3)$$

$$\psi^3(x,y)=\psi(x)\psi(y) \qquad (4)$$

Where ψ^1,ψ^2,ψ^3 define respectively the changes according to rows, columns and diagonals. These three 2D-wavelets extract the details of an image f at different scales and in different directions (see Figure 1).

Evidently, there are many functions ϕ that can be selected to produce a mother wavelet as the two-dimensional Gaussian function given below:

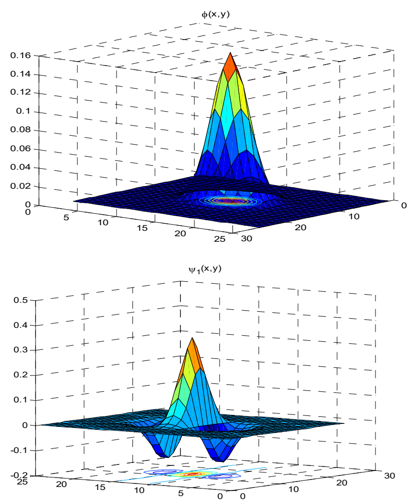
$$\phi\left(\frac{x}{\sigma_x},\frac{y}{\sigma_y}\right)=\frac{1}{2\pi\sigma_x\sigma_y}exp\left(-\frac{x^2}{2\sigma_x{}^2}-\frac{y^2}{2\sigma_y{}^2}\right) \qquad (5)$$

Where σ_x and σ_y are the dilation parameters.

The Marr wavelet function also known as Ricker wavelet or Mexican Hat wavelet function is derived from the negative normalized second derivative of the above equation (5). It can be expressed as follows:

$$\psi\left(\frac{x}{\sigma_x},\frac{y}{\sigma_y}\right)=\frac{1}{2\pi\sigma_x\sigma_y}\left(2-\frac{x^2}{\sigma_x{}^2}-\frac{y^2}{\sigma_y{}^2}\right)exp\left(-\frac{x^2}{2\sigma_x{}^2}-\frac{y^2}{2\sigma_y{}^2}\right)$$

This wavelet is often used to model seismic data [42] and as efficient tool in automatic Image Registration [43].



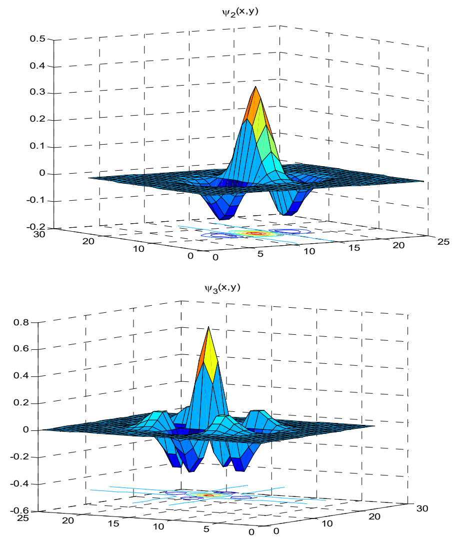


Fig. 1 Different details associated with 2D-wavelets $\psi^l(x,y); 1\le l\le 3$.

We note that the wavelet family $\left\{\psi^1_{j,n,m}(x,y),\psi^2_{j,n,m}(x,y),\psi^3_{j,n,m}(x,y)\right\}_{(j,n,m)\in\mathbb{Z}^3}$ and its dual are bi-orthogonal Riesz basis [27] of $L^2(R^2)$ referring to separable wavelet bases theorem where:

$$\psi^l_{j,n,m}(x,y)=\frac{1}{2^j}\psi^l\left(\frac{x-2^jn}{2^j},\frac{y-2^jm}{2^j}\right);1\le l\le 3 \qquad (5)$$

Let consider a_j the approximation at scale j defined by:

$$a_j[n,m]=\langle f|\phi_{j,nm}\rangle \qquad (6)$$

and d_j^l the detail at scale j given for $1\le l\le 3$ by:

$$d_j^l[n,m]=\langle f|\psi^l_{j,n,m}(x,y)\rangle \qquad (7)$$

The wavelet can be represented by the following expression:

$$\left[a_J,\{d_j^1,d_j^2,d_j^3\}_{1\le j\le J}\right] \qquad (8)$$

The analysis phase (decomposition) can be described by the following equations:

$$a_{j+1}[n,m]=a_j*\bar{h}\bar{h}[2n,2m] \qquad (9)$$

$$d^1_{j+1}[n,m]=a_j*\bar{h}\bar{g}[2n,2m] \qquad (10)$$

$$d^2_{j+1}[n,m]=a_j*\bar{g}\bar{h}[2n,2m] \qquad (11)$$

$$d^3_{j+1}[n,m]=a_j*\bar{g}\bar{g}[2n,2m] \qquad (12)$$

where the sign * represents the convolution product; the notation *XY[k,l]* designating the product *X[k]Y[l]*; *X* and *Y* are dummy variables and can be any filters such that $\bar{X}[k]=X[-k]$.

The synthesis phase (reconstruction) can be obtained from the following expression:

$$a_j[n,m]=\tilde{a}_{j+1}*hh[n,m]+\tilde{d}^1_{j+1}*hg[n,m]+\tilde{d}^2_{j+1}*gh[n,m]+\tilde{d}^3_{j+1}*gg[n,m] \qquad (13)$$

The symbol ~ denotes the effect of the over-sampling operation obtained by the insertion of a zero value between two consecutive samples.

The two dimensional fast wavelet transform which is an accelerated form of the 2D-DWT can be implemented by applying numerical filters (decomposition and reconstruction) as described above where the detailed process is presented in Figure 2

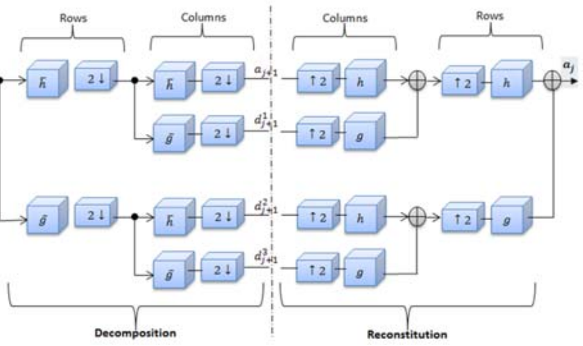


Fig.2 Implementation of 2D-DWT process in accelerated version

The time efficiency and computational storage of this technique are obtained in O(NT log NT) [26] where NT is the total number of pixels for the image I to be analyzed.

2.2. 2D-NUFFT

During the last decade, the 2D-NUFFT and its inverse 2D-INUFFT have confirmed their efficiency in many domains like numerical methods in electromagnetics [28-29][41], tomography [30], magnetic resonance imaging (MRI) [31] astronomy [32] and so on. However, these powerful mathematical transformations have not been yet used in image encryption applications except the work presented in [26]. The 2D-NUFFT and 2D-INUFFT can offer the possibility to improve the security level of data by benefiting from their features such as decomposition technique, speediness and the non-uniformity property.

Indeed, Fourier transform (FT) in non-equispaced form can be calculated straightforwardly by its original and conventional formula; nonetheless, their eminent characterizations principally the symmetry and the exponential basis orthogonality cannot be ensured by the FFT algorithm for non-uniform input data. This restriction has been resolved in [33-34] to deliver great precisions and rapid computation for non-uniform FFT (NUFFT) in 1D-problem.

Moreover, referring to Dutt-Rokhlin interpolation method [33] and different mathematical foundation, 2D non-equispaced fast Fourier transform (2D-NUFFT) and its inverse (2D-NUFFT) as depicted in Figure3, have been successfully developed and implemented in our research work in [28-29].

The 2D-NUFFT algorithm keep the same time computation as the conventional one which is O(NT log N_T) where N_T is the total number of non-equispaced pixels. N_T should be written as 2^n number and considered as a part of the encryption key that will be used in the image processing applications.

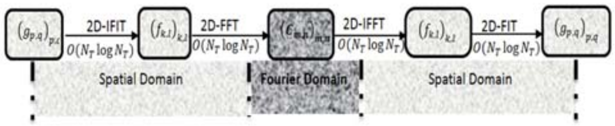


Fig.3 2D-NUFFT and 2D-INUFFT schemes based on Fast Interpolation Transforms (FIT)

2.3. Singular Value Decomposition

The singular value decomposition (SVD) [35]–an efficient mathematical tool–is considered as the workhorse of various approaches both in mathematics and applied fields. During last decade, an eminent application is the implementation of the SVD in image encryption so as to enhance the security degree of confidential data.

Indeed, SVD presents a forward and backward process due to the multiplication order associated with the components resulting from the decomposition phase of the image to be investigated. SVD is a consistent matrix decomposition method which decomposes the image into three independent linear components.

Let **A** be an ***n* x *m*** real matrix (*i.e., n* samples and *m* variables); this matrix can be decomposed into three different matrices:

- a matrix of new orthogonal component (U=($u_1,u_2,\ldots,u_m$)) to represent linear combinations of the original samples.
- a square matrix **S** containing in diagonals the singular values.
- a matrix of new orthogonal vectors (V=($v_1,v_2,\ldots,v_n$)) to represent linear combinations of the original variables.

It can be expressed mathematically as follows:

$$A = U \times S \times VT \quad (14)$$

Where U and V are respectively the m x m left and singular orthogonal matrix and n x n right singular orthogonal matrix. The transcript T denotes a transpose.

3. Novel Secure Image Cryptosystem

Before exposing our proposal encryption/decryption schemes, a dual channel secure communication based on chaotic system is considered as a transmission data model between sender and receiver ensuring a good security level of data in the context of public communication networks. The dual communication channels are introduced to distinguish between the synchronization phase and the encryption process. The synchronization between both master and slave hyper-chaotic systems, is guaranteed through channel B; by the way, the encryption process of plain image is performed via channel A as depicted in Fig. 4. Besides, this model can be applied to send any form of digital data.

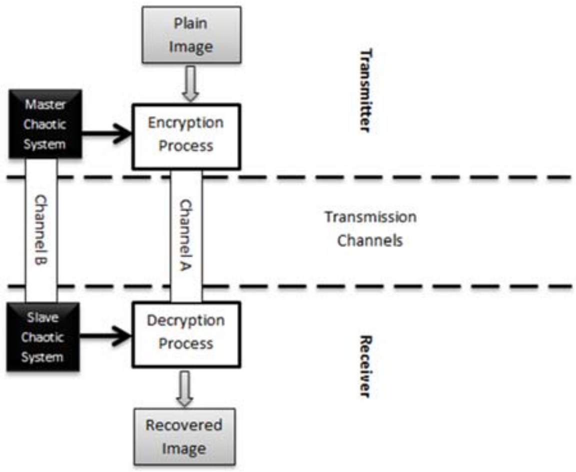


Fig.4 Chaos-based secure communication scheme with dual-channels

In what follows, we expose a novel symmetric and secure image cryptosystem with its forward and backward schemes built on the combination between the aforementioned techniques 2D-DWT, 2D-NUFFT and SVD.

Let us take a plain-image $I_P = F(x,y)$ (x and y are the coordinates in spatial domain) where the dimension is $M*N$.

Firstly, so as to guarantee a good protection level, we blind the plain-image by adding it with another secret image I_s and we obtain:

$$L(x,y) = \left[I_p(x,y) + I_S(x,y)\right] \quad (15)$$

Next, the 2D-DWT is applied on the obtained image L to produce:

$$2D\text{-}DWT\{L(x,y)\} \quad (16)$$

To randomize and improve the security degree of the input data, the Chaotic Random Phase Mask (CRPM) is called where its expression can be written trigonometrically in the following manner:

$$CRPM = e^{j\frac{\pi}{2}S(x,y)} \quad (17)$$

Where $S(x,y)$ denotes the randomized sequence of numbers generated by the chaos function based on linear congruence.

Then, the multiplication between the mask (16) by the result obtained in equation (15) yields:

$$\left\{2D\text{-}DWT\{L(x,y)\} * e^{j\frac{\pi}{2}S(x,y)}\right\} \quad (18)$$

The notation * designate entry-wise product [36] of matrices with same dimensions.

The *2D-NUFFT* transform is then applied in (18) to generate the following distribution:

$$\left\{2D\text{-}NUFFT\left\{2D\text{-}DWT\{L(x,y)\} * e^{j\frac{\pi}{2}S(x,y)}\right\}\right\} \quad (19)$$

It is noticed that the application of this non-uniform transform needs an identification of the selected anisotropic mesh for implementation [37].

To increase the security degree, the result depicted in (19) is decomposed into three components by applying SVD technique:

$$[U,S,V] = SVD\left\{2D\text{-}NUFFT\left\{2D\text{-}DWT\{L(x,y)\} * e^{j\frac{\pi}{2}S(x,y)}\right\}\right\} \quad (20)$$

As the last step of the proposed encryption process, these three components are separately randomized to deliver three ciphered images I_{c1}, I_{c2} and I_{c3}.

The choice of randomized process here is very advantageous in particular in presence of a malicious "adversary" or attacker who intentionally attempts to feed a bad input to the algorithm. It is ubiquitous in the field of cryptography.

It is noticed that the diffusion concept is present at (20) in the context of the information security.

$$I_{C1}(x,y) = R\big(U(x,y)\big) \quad (21)$$

$$I_{C2}(x,y) = R\big(S(x,y)\big) \quad (22)$$

$$I_{C3}(x,y) = R\big(V(x,y)\big) \quad (23)$$

These ciphered images can be transmitted via unsecured channels since the secure process of image is considered as too strong.

The symmetric encryption process can be summarized in the following flowchart (Fig. 5):

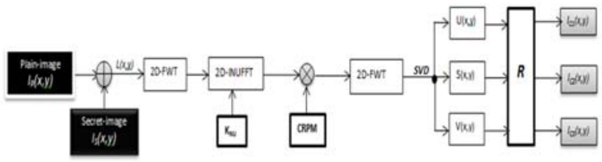


Fig.5 Proposed encryption process

In the decryption phase, the received cipher-images I_{c1}, I_{c2} and I_{c3} are respectively derandomized to obtain three segments U, S and V as follows:

$$U(x,y) = R^{-1}\big(I_{C1}(x,y)\big) \quad (24)$$

$$S(x,y) = R^{-1}\big(I_{C2}(x,y)\big) \quad (25)$$

$$V(x,y) = R^{-1}\big(I_{C3}(x,y)\big) \quad (26)$$

where R^{-1} represents the derandomization function which is applied here to remove the randomness process already performed in the encryption phase.

By multiplying with the same order U, S and V, the transitional decrypted image is obtained as follows:

$$T(x,y) = [U \times S \times V^T] \quad (27)$$

The inverse transform 2D-INUFFT should be directly applied to T(x,y) taking into account the CRPM conjugate already given in (17), we find:

$$2D\text{-}INUFFT\{T(x,y)\} * Conj\left\{e^{j\frac{\pi}{2}S(x,y)}\right\} \quad (28)$$

Then, the inverse 2D-IDWT is directly applied on (28) to obtain the following distribution:

$$2D\text{-}IDWT\left\{2D\text{-}INUFFT\{T(x,y)\} * Conj\left\{e^{j\frac{\pi}{2}S(x,y)}\right\}\right\} \quad (29)$$

To recover the plain-image I_p, the result (29) is multiplied by the conjugate of the secret image I_S as follows:

$$I_p(x,y) = \left[\left\{2D\text{-}IDWT\left\{2D\text{-}INUFFT\{T(x,y)\} * Conj\left\{e^{j\frac{\pi}{2}S(x,y)}\right\}\right\}\right\} \times I_S{}^*(x,y)\right] \quad (30)$$

The symmetric decryption process can be summarized in the following flowchart (Fig.6):

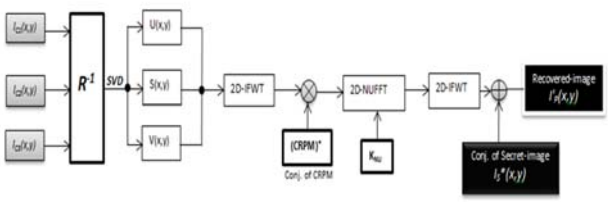


Fig.6 Proposed decryption process

4. Analysis of the Novel Secure Image Cryptosystem

The investigation of the proposed novel encryption/ decryption process is guaranteed by computing the time complexity and accuracy and comparing to algorithms already investigated in recent scientific publications.

The proposed approach gathers different powerful tools such as 2D-DWT and its inverse 2D-IDWT, 2D-NUFFT and its inverse 2D-INFFT, the chaotic random phase function and its conjugate, the singular value decomposition, the randomization and derandomization process as well as the selected secret image which is applied with the plain-image as a first encryption key. These techniques are implemented not only to reinforce the robustness of the encryption/decryption algorithm but also to increase the security level of the input data. .

In what follows, we identify the computational time of the proposed algorithms in both forward and backward ways so as to measure the efficiency and stability of this novel secure image cryptosystem. To simplify the calculus, we assume that the plain and secret images have the same dimension N*N.

Table 1: Time efficiency of the encryption process

Encryption process		
Step	Description	Complexity
1	Application of *the secret image I_S* on the plain-image I_P.	$O(N^2)$
2	Application of *2D-DWT*	$O(N log_2 N)$
3	Encrypting by chaotic random phase function	$O(\frac{N^2}{2})$
4	Performing of *2D-NUFFT*	$O(N^2\ log_2 N)$
5	Application of *SVD technique*	$O(N^2)$
6	Application of *the selected randomization* process on each components resulting from *step5* to obtain encrypted image	$O(1)$
Total time complexity of encryption		$O(2N^2\ log_2 N)$

Table 1: Time efficiency of the decryption process

Decryption process		
Step	Description	Complexity
1	Application of the selected derandomization process	$O(N^2)$
2	Application of *SVD technique*	$O(N^2)$
3	Application of *2D-INUFFT*	$O(2N^2\ log_2 N)$
4	Decrypting by conjugate random phase function	$O(\frac{N^2}{2})$
5	Performing of *2D-IDWT*	$O(N log_2 N)$
6	Application of *the conjugate of the secret image I_S** to obtain decrypted image *I'_P*.	$O(N^2)$
Total time complexity of decryption		$O(2N^2\ log_2 N)$

Based on the results depicted in Table 1 able 2, the total time efficiency of the proposed algorithm is the sum of both encryption and decryption total time efficiencies let $O(4N^2 \log_2 N)$ which is belonging to quadrarithmic-time class. Therefore, the proposed novel symmetric algorithm is considered as an efficient and fast approach due to its computational time that set apart from other techniques based on 2D-FrFT developed in recent works [26][38-40] in the secure image cryptosystems sphere.

Moreover, the accuracy and performance of this approach can be identified by computing the Mean Square Error (MSE) between plain and decrypted images (i.e. Ip and I'p). Indeed, the MSE indicator benchmarks the performance degree and assesses the average squared difference between pixel values of both authentic and recovered images. It can be defined from the following equation:

$$MSE = \frac{1}{M*N}\sum_{1\leq x\leq M}\sum_{1\leq y\leq N}\left|I'_p(x,y) - I_p(x,y)\right|^2 \quad (31)$$

This algorithm has been developed in MATLAB environment and the cameraman image of size 256×256 is used as the plain-image and peppers image as the secret image with the same size (See Fig. 7). The simulation results prove a high similarity between recovered and plain image since the obtained MSE is very low (i.e. less than 10-27) showing a good quality of the decrypted image that cannot be detectable from human eye.

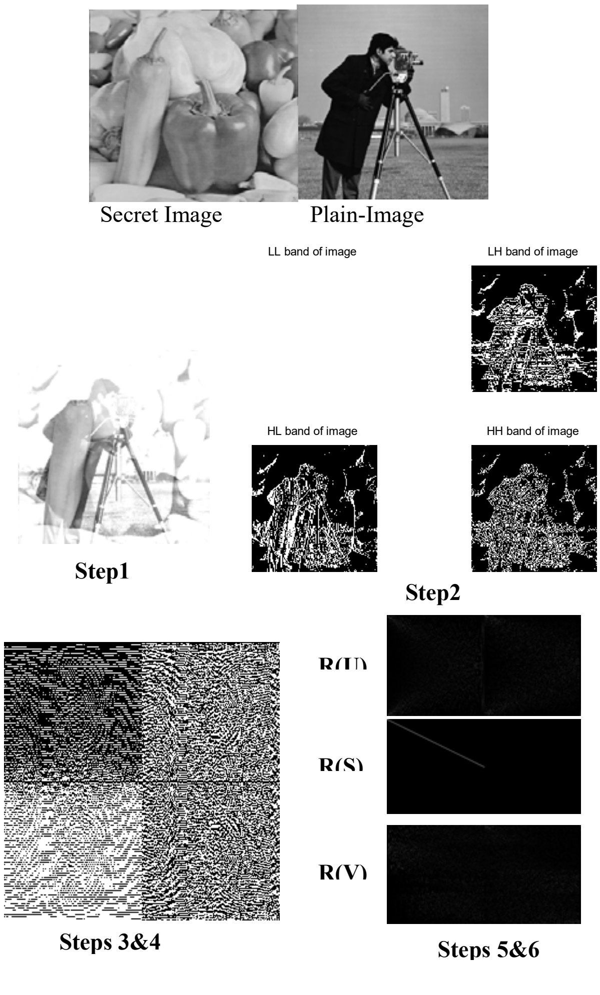


Fig.7 Different steps of encryption scheme

5. Conclusions

A novel fast symmetric approach based on 2D-NUFFT and singular value decomposition technique has been successfully presented and developed in the context of security analysis and enhancements of image cryptosystems. The presence of the chaotic random phase function as well as the randomization process makes the system too strong against any attack from unauthorized users. The analysis of both encryption and decryption algorithms proves the validity, stability and fastness of the proposed cryptosystem in terms of time complexity and accuracy that set apart from recent efficient approaches based on 2D-fractional Fourier transform. The Equal Modulus Decomposition as well as double chaotic random phase mask can be a good additional solution for our proposed symmetric cryptosystem to enhance the security level of input sensitive data.

References

[1] Boyat, A. K., & Joshi, B. K. (2015). A review paper: Noise models in digital image processing. arXiv preprint arXiv:1505.03489.
[2] Altowaijri, S., Ayari, M., & El-Touati, Y. (2017). On the Use of Multi-Sensor Technology for Improving Soldiers' Safety and Saudi Border Security. Border Security And Safety, 269.
[3] Ambarwari, A., Herdiyeni, Y., & Hermadi, I. (2018). Biometric Analysis of Leaf Venation Density Based on Digital Image. Telkomnika, 16(4).
[4] Altowaijri, S., Ayari, M., & El Touati, Y. (2019). Impact of Multi-Sensor Technology for Enhancing Global Security in Closed Environments Using Cloud-Based Resources. Journal of Sensor and Actuator Networks, 8(1), 4.
[5] Xia, Z., Zhu, Y., Sun, X., Qin, Z., & Ren, K. (2018). Towards privacy-preserving content-based image retrieval in cloud computing. IEEE Transactions on Cloud Computing, 6(1), 276-286.
[6] Pareek, N. K., Patidar, V., & Sud, K. K. (2006). Image encryption using chaotic logistic map. Image and vision computing, 24(9), 926-934
[7] Li, C., Luo, G., Qin, K., & Li, C. (2017). An image encryption scheme based on chaotic tent map. Nonlinear Dynamics, 87(1), 127-133.
[8] Jain, A., & Rajpal, N. (2016). A robust image encryption algorithm resistant to attacks using DNA and chaotic logistic maps. Multimedia Tools and Applications, 75(10), 5455-5472.
[9] Wu, G. C., Baleanu, D., & Lin, Z. X. (2016). Image encryption technique based on fractional chaotic time series. Journal of Vibration and Control, 22(8), 2092-2099.
[10] Ye, G. (2014). A block image encryption algorithm based on wave transmission and chaotic systems. Nonlinear Dynamics, 75(3), 417-427.
[11] Singh, N., & Sinha, A. (2009). Optical image encryption using Hartley transform and logistic map. Optics Communications, 282(6), 1104-1109.

[12] Khurana, M., & Singh, H. (2018). Data Computation and Secure Encryption Based on Gyrator Transform using Singular Value Decomposition and Randomization. Procedia computer science, 132, 1636-1645.

[13] Liao, X., Li, K., & Yin, J. (2017). Separable data hiding in encrypted image based on compressive sensing and discrete Fourier transform. Multimedia Tools and Applications, 76(20), 20739-20753..

[14] Agarwal, R., & Patidar, V. (2018, July). Double Image Encryption Based on 2D Discrete Fractional Fourier Transform and Piecewise Nonlinear Chaotic Map. In International Conference on Advanced Informatics for Computing Research (pp. 519-530). Springer, Singapore.

[15] Huang, J. J., Hwang, H. E., Chen, C. Y., & Chen, C. M. (2012). Optical multiple-image encryption based on phase encoding algorithm in the Fresnel transform domain. Optics & Laser Technology, 44(7), 2238-2244.

[16] Hennelly, B. M., & Sheridan, J. T. (2004). Random phase and jigsaw encryption in the Fresnel domain. Optical Engineering, 43(10), 2239-2250.

[17] Lima, J. B., Lima, E. A. O., & Madeiro, F. (2013). Image encryption based on the finite field cosine transform. Signal Processing: Image Communication, 28(10), 1537-1547.

[18] Zhou, N., Wang, Y., & Wu, J. (2011). Image encryption algorithm based on the multi-order discrete fractional Mellin transform. Optics communications, 284(24), 5588-5597.

[19] Zhou, N. R., Hua, T. X., Gong, L. H., Pei, D. J., & Liao, Q. H. (2015). Quantum image encryption based on generalized Arnold transform and double random-phase encoding. Quantum Information Processing, 14(4), 1193-1213.

[20] Liang, H. R., Tao, X. Y., & Zhou, N. R. (2016). Quantum image encryption based on generalized affine transform and logistic map. Quantum Information Processing, 15(7), 2701-2724.

[21] Meihua L, Wenqi H, Dajiang L & Xiang P , (2017) Ciphertext-only attack on optical cryptosystem with spatially incoherent illumination: from the view of imaging through scattering medium, Scientific Reports 7:41789.

[22] Carnicer A et al., (2005), Vulnerability to chosen–ciphertext attacks of optical encryption schemes based on double random phase keys, Opt. Letter 30:1644-1646.

[23] Liu, Z., Chen, H., Blondel, W., Shen, Z., & Liu, S. Image security based on iterative random phase encoding in expanded fractional Fourier transform domains. Optics and Lasers in Engineering, 105, 1-5, 2018.

[24] Gong, L., Deng, C., Pan, S., & Zhou, N. (2018). Image compression-encryption algorithms by combining hyper-chaotic system with discrete fractional random transform. Optics & Laser Technology, 103, 48-58.

[25] Ali Khan, F., Ahmed, J., Ahmad, J., Khan, J. S., Ahmad, F., Stankovic, V., & Larijani, H. (2019, February). A novel chaos-based partial image encryption scheme using Lifting Wavelet Transform. In The First International Nonlinear Dynamics Conference.

[26] Altowaijri, S., Ayari, M., & El Touati, Y. (2018). A Novel Image Encryption Approach for Cloud Computing Applications. International Journal Of Advanced Computer Science And Applications, 9(12), 440-445.

[27] Tang, W. S. (2000). Oblique projections, biorthogonal Riesz bases and multiwavelets in Hilbert spaces. Proceedings of the American Mathematical Society, 128(2), 463-473.

[28] Ayari, Mohamed, Taoufik Aguili, and Henri Baudrand. "New version of TWA using two-dimensional non-uniform fast fourier mode transform (2d-nuffmt) for full-wave investigation of microwave integrated circuits." Progress In Electromagnetics Research 15 (2009): 375-400.

[29] Ayari, M., Altowaijri, S., & El Touati, Y. (2019). Mutual Coupling Between Antennas In Periodic Network Using The Advanced Transverse Wave Approach For Wireless Applications. International Journal Of Advanced And Applied Sciences, 6(1), 99-105.

[30] Tang, J. (2016). The Non-uniform Fast Fourier Transform in Computed Tomography. arXiv preprint arXiv:1605.05231.

[31] Lin, J. M. (2017). Python Non-Uniform Fast Fourier Transform (PyNUFFT): multi-dimensional non-Cartesian image reconstruction package for heterogeneous platforms and applications to MRI. arXiv preprint arXiv:1710.03197.

[32] Ou, T. (2017). gNUFFTW: Auto-Tuning for High-Performance GPU-Accelerated Non-Uniform Fast Fourier Transforms.

[33] A. Dutt, and V. Rokhlin, “Fast Fourier transforms for nonequispaced data II,” Appl. Comput. Harmon. Anal. Vol. 2, pp. 85–100, 1995.

[34] Plonka, G., Potts, D., Steidl, G., & Tasche, M. (2018). Fast Fourier Transforms for Nonequispaced Data. In Numerical Fourier Analysis (pp. 377-419). Birkhäuser, Cham.

[35] Wall, M. E., Rechtsteiner, A., & Rocha, L. M. (2003). Singular value decomposition and principal component analysis. In A practical approach to microarray data analysis (pp. 91-109). Springer, Boston, MA.

[36] Craigen, R., Seberry, J., & Zhang, X. M. (1992). Product of four Hadamard matrices. Journal of Combinatorial Theory, Series A, 59(2), 318-320.

[37] Ayari, M., Aguili, T., & Baudrand, H. (2009). More efficiency of Transverse Wave Approach (TWA) by applying Anisotropic Mesh Technique (AMT) for full-wave analysis of microwave planar structures. Progress In Electromagnetics Research, 14, 383-405.

[38] Wu, J., Liao, X., & Yang, B. (2018). Image encryption using 2D Hénon-Sine map and DNA approach. Signal Processing, 153, 11-23.

[39] Zhang, D., Liao, X., Yang, B., & Zhang, Y. (2018). A fast and efficient approach to color-image encryption based on compressive sensing and fractional Fourier transform. Multimedia Tools and Applications, 77(2), 2191-2208.

[40] Kandar, S., Chaudhuri, D., Bhattacharjee, A., & Dhara, B. C. (2019). Image encryption using sequence generated by cyclic group. Journal of information security and applications, 44, 117-129.

[41] Ayari, M., El Touati, Y., & Altowaijri, S. (2020). Method of Moments versus Advanced Transverse Wave Approach for EM Validation of Complex Microwave and RF Applications. *Journal of Electromagnetic Engineering and Science*, *20*(1), 31-38.

[42] Sinha, S., Routh, P. S., Anno, P. D., & Castagna, J. P. (2005). Spectral decomposition of seismic data with continuous-wavelet transform. *Geophysics*, *70*(6), P19-P25.

[43] Murphy, J. M., Le Moigne, J., & Harding, D. J. (2015). Automatic image registration of multimodal remotely sensed data with global shearlet features. IEEE Transactions on Geoscience and Remote Sensing, 54(3), 1685-1704.

Mohamed Ayari received the Dipl.-Ing., M.S., and Ph.D. degrees in Telecommunications in 2003, 2004, and 2009, respectively, from the National Engineering School of Tunis (ENIT), Tunisia, in collaboration with the National Polytechnic Institute of Toulouse, France, and Virginia Tech, USA. He has been a teacher at several universities since 2003. He has been a permanent research member at the 6'COM laboratory at ENIT from 2003. In 2005 he joined RCEM-Inc. in Toulouse, France. Since 2010, he has been a tenure-track associate professor at the National Engineering School of Carthage (ENICAR), Carthage University, Tunisia. In 2015, he is joined the IT Department of the Faculty of Computing and Information Technology, Northern Border University (NBU), Kingdom of Saudi Arabia, as an assistant professor. His current research interests are electromagnetic (EM) fields, numerical EM methods, and computer-aided design of microwave circuits and antennas. His research interests also include information security, image processing and wireless applications.