## A Generic Multi-Level Algorithm for Prioritized Multi-Criteria Decision Making

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#### Summary

Decision-making refers to identifying the best alternative among a set of alternatives. When a set of criteria are involved, the decision-making is called multi-criteria decision-making (MCDM). In some cases, the involved criteria may be prioritized by the human decision-maker, which determines the importance degree for each criterion; hence, the decision-making becomes prioritized multi-criteria decision-making. The essence of prioritized MCDM is raking the different alternatives concerning the criteria and selecting best one(s) from the ranked list. This paper introduces a generic multi-level algorithm for ranking multiple alternatives in prioritized MCDM problems. The proposed algorithm is implemented by a decision support system for selecting the most critical short-road requests presented to the transportation ministry in the Kingdom of Saudi Arabia. The ranking results show that the proposed ranking algorithm achieves a good balance between the importance degrees determined by the human decision maker and the score value of the alternatives concerning the different criteria.

#### Keywords:

Decision Support System, Multi-Attribute Decision Making, Ranking, Heuristic

## 1. Introduction

Decision-making is a daily action that is a part of our lives. The decisions may be simple, such as what we should eat or wear, or serious such as investment or strategic decisions of a country. Regardless of the various complexity levels of different decision problems, they have several alternatives and criteria. However, the large number of alternatives and criteria complicates the decision-making process. Consequently, developing decision-making techniques gained the attention of many researchers [1, 2]. Decision-making involves identifying the best alternative from a set of alternatives [3, 4]. Decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems [5]. MCDM is an important branch of decision-making theory [6]. MCDM is a significant part of decision theory, systems engineering,

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management science, applied statistics, etc. Further, MCDM methods have been adopted in widespread applications, including talent selection, medical diagnosis, investment questions, and other fields [7-10].

Based on the solution space, MCDM problems are classified into two classes: continuous and discrete. Hence, two categories of MCDM methods have been developed, namely multi-objective decision-making (MODM) and multi-attribute decision-making (MADM). MODM methods address the persistent problems where there are an unlimited number of alternatives. MADM methods address discrete problems where there exists a predetermined number of alternatives [11]. The primary role of the Decision Maker (DM) in MADM methods is to select the best alternative or rank the different alternatives concerning the different criteria.

On the other hand, the DM of MODM methods is concerned with designing the 'most' promising alternative concerning limited resources [1]. The MADM method is the focus of this paper. In recent literature, MCDM usually refers to discrete MCDM [6]. Henceforth, the MCDM is used in this paper instead of the MADM.

MCDM problems may be choice, ranking, or sorting problem. In choice MCDM problems, the best alternative concerning the different criteria is selected. In ranking MCDM problems, the different alternatives are ordered from the best to the worst ones. In sorting MCDM problems, the best k alternatives are selected [12]. In addition, the used criteria may be prioritized based on a certain importance degree. The importance degrees of the different criteria can be directly defined by the decision maker or by conducting pairwise comparisons in which one of several methods can be employed. These methods include the eigenvector method, weighted least square method, entropy method, Analytic Hierarchy Process (AHP), linear programming technique for multidimensional analysis of preference (LINMAP), etc. [13]. Selecting the most appropriate method depends on the nature of the problem. However, the

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same problem on the same criteria may have different environments, circumstances, and concerns.

This paper proposes a generic, iterative, multi-level algorithm for addressing the MCDM problems. The proposed algorithm assumes that the human decision-maker prioritizes the different criteria to be flexible in considering the different environments, circumstances, and concerns. The objective of the proposed algorithm is to rank the given alternatives from the best to the worst ones. The proposed algorithm is implemented in a real-world problem in which many villages' short road requests have to be ranked based on several criteria, including the expected traffic, security, and geographical, internal, and external factors. The number of villages' short road requests is too large and cannot be executed simultaneously due to the limited budget. Hence, the objective of the proposed algorithm is to rank the villages' short road requests from the most to the least critical ones.

The remaining sections of the paper are organized as follows: Section 2 overviews the different MCDM methods. Section 3 presents the problem formulation and proposed ranking algorithm. Section 4 includes the implementation of the proposed algorithms in the villages' short road requests ranking problem. Finally, the paper is concluded, and future work directions are presented in section 5.

## 2. Related Works

The first attempts at developing MCDM models were presented in the early 1970s, followed by many DM models and related analyses in various applications [14]. Hwang and Yoon introduced an MCDM model named Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), which assumes the chosen alternative should have the shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS) [15, 16]. Duckstein and Opricovic presented another MCDM model to address with decision problems conflicting and noncommensurable criteria named VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) [17, 18]. Julong developed the grey system theory to solve DM problems with poor, incomplete, or uncertain information [19]. Yager introduced a weighted averaging aggregation operator called ordered weighted aggregation (OWA) to deal with discrete MCDM problems [20]. Yakowitz et al. studied attribute domination concerning the ordinal ranking and whether it can be detected if one assumed an additive value function [21]. Saaty proposed a structured technique called the analytic hierarchy process (AHP) for organizing and analyzing complex decisions based on mathematics and psychology [22, 23]. Yingming proposed a DCDM model called maximizing deviation method for multi-indices decisions to automatically identify the objective weight coefficients [24]. Zanakis et al. compared the performance of eight MADM models under different scenarios, including TOPSIS, ELECTRE, multiplicative exponential weighting (MEW), simple additive weighting (SAW), and four versions of AHP [25]. Keršulienė et al. introduced an MCDM model called step-wise weight assessment ratio analysis (SWARA) to address the MCDM problems in which the weights of the attributes significantly vary [26].

However, due to the increasing data uncertainty, information complexity in real-world MCDM problems, and the Fuzziness of human thinking, the fuzzy and uncertainty theory has been gradually combined in recent MCDM models. Pei and Zheng proposed a fuzzy MCDM model based on a revised score function and an accuracy function of intuitionistic fuzzy sets [14]. Kahraman and Çebi extended the Fuzziness in the axiomatic design (FAD) method to address the problems of Fuzzy Multiple Attribute Decision Making (FMADM). The proposed method has a hierarchical structure, supports both crisp and fuzzy inputs, and can be used for ranking problems [1]. Liao and Xu developed a fuzzified version of the preference ranking for enrichment organization method evaluation (PROMETHEE) method that works in the intuitionistic fuzzy environment [27]. Tosuna and Akyzb attempted to handle the decision maker's bias in the supplier selection problem by presenting an iterative, fuzzy MCDM model [28]. Xie et al. used the dual probabilistic linguistic evaluation to reflect the certainty and uncertainty in the assessment of a decision maker [29]. Gupta et al. proposed a multi-attribute group decision-making (MAGDM) model in an interval-valued intuitionistic fuzzy environment by combining extended TOPSIS and linear programming methods [30]. Wang et al. studied the fuzzy-rough-setbased MCDM model, which can be successfully applied for attribute reduction in categorical data [31]. Deng et al. employed the three-way decision (3WD) to solve the MCDM problem in a multi-scale decision information system (MS-DIS) fuzzy environment [2].

#### 3. Proposed Methodology

There are many applications for MCDM, such as: prioritizing central government spending, ranking students for research scholarships, choosing projects for funding, etc... All these applications involve alternatives that are ranked based on multiple criteria. The proposed methodology provides a generic approach that aims at helping the human DM for selecting or rank the possible alternatives. Fig. 1 shows the steps of the proposed Multi-Level Algorithm for Prioritized Multi-Criteria Decision Making.



Fig. 1 The proposed Prioritized Multicriteria Decision Making.

The following subsections present the problem formulation and the pseudocode of the proposed prioritized multi-criteria decision-making algorithm in detail.

#### 2.1 Problem Formulation

A multi-criteria decision-making problem consists of a finite number of alternatives  $A=\{a_1, a_2, ..., a_n\}$ , a finite number of criteria  $C=\{c_1, c_2, ..., c_m\}$ , and the score for each alternative concerning each criterion, as shown by the given score matrix where  $w_{ij}$  represents the score of  $a_i$  concerning  $c_j$ :

$$S = \begin{pmatrix} w_{11} & \cdots & w_{1m} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nm} \end{pmatrix}$$
(1)

The problem is to evaluate the alternatives and rank them regarding the different criteria in descending order. The DM can then select the best (one/ many) among them. Each criterion has an important degree,  $\theta$ , where  $\theta_j > 0$ . It indicates the priority for the given criterion concerning the given problem. The importance degrees of the different criteria can be determined by the human decision maker, which satisfies the following equation:

$$\sum_{i=1}^{m} \theta_i = 1 \tag{2}$$

The input criteria are ordered based on the importance degree, as follows:

$$\theta_1 \ge \theta_2 \ge, \dots, \ge \theta_m \tag{3}$$

The proposed algorithm assumes that each criterion has a *satisfaction range* that denotes the upper and lower bounds of the human decision-maker's satisfaction concerning this criterion's score. The satisfaction range is evaluated as shown below.

$$R_{j} = \begin{bmatrix} c_{j}^{max} - \left(c_{j}^{max} * (1 - \theta_{j})\right) & , c_{j}^{max} \end{bmatrix}$$
(4)

where  $c_j^{max}$  and  $\theta_j$  are the maximum reported score and importance degree of the criterion  $c_j$ , respectively. As defined in Eq. 4, the satisfaction degree is designed to narrow the range when the importance degree increases to limit the power of less important criteria in ranking the different alternatives. The alternative with a score value within the satisfaction range survived to the next round of the proposed algorithm. The surviving alternatives are ranked based on the next criterion. This process continues until all alternatives are ranked concerning all criteria. The notations used by the proposed algorithm are summarized in Table 1.

Table 1: Notations used by the proposed ranking algorithm

| Notation        | The meaning of the notation                  |  |  |  |
|-----------------|--|--|--|--|
| n               | The number of alternatives                   |  |  |  |
| m               | The number of criteria                       |  |  |  |
| Α               | The list of alternatives                     |  |  |  |
| С               | The list of criteria                         |  |  |  |
| S               | The score of alternatives for each criterion |  |  |  |
| $A_r$           | The list of ranked alternatives              |  |  |  |
| a <sub>i</sub>  | The ith alternative                          |  |  |  |
| c <sub>j</sub>  | The jth criteria                             |  |  |  |
| w <sub>ij</sub> | The score of $a_i$ with respect to $c_j$     |  |  |  |
| $\theta_{j}$    | The importance degree for the jth criteria   |  |  |  |
| $c_j^{max}$     | The maximum score for the jth criteria       |  |  |  |
| R <sub>i</sub>  | The satisfaction range for the jth criteria  |  |  |  |

#### 2.2 Prioritized Multicriteria Decision Making

In this subsection, an iterative Multi-Criteria Decision-Making algorithm is presented. The proposed algorithm aims to rank a set of alternatives based on multiple criteria to help human decision-makers. Given the importance degree of each criterion, the proposed algorithm applies a set of phases for ranking the given alternatives. Each phase consists of several rounds that may reach m, the number of criteria. The pseudocode of the Prioritized Multicriteria Decision Making is given in Algorithm 1.

Algorithm 1 starts with sorting the criteria list in descending order based on their importance degree. The score matrix is then evaluated according to the applied problem to evaluate each alternative concerning each criterion. Then, the algorithm iterates in phases to accumulate the ranked alternatives list,  $A_r$ , which is eventually returned. In each phase, the alternative list is sorted based on the score values for each criterion. An alternative is excluded from the next round in the current phase if its score is below the lower bound of the satisfaction range of the current criterion.

On the other hand, the alternatives that satisfy the satisfaction range of the current criteria are important, and further moved to the next round to be ranked based on the next criterion. This process continues until all alternatives reserve their rank in the ranked alternatives list. By the end of each round in each phase, the number of remaining unranked alternatives is checked, and if this number is 1, this alternative is appended to the ranked list. The next section presents a case study for the proposed algorithm applied to a real-world application.

Algorithm 1: The proposed Multi-Criteria Decision-Making algorithm

**Input:** Number of alternatives (n), number of criteria (m), importance degree for criteria ( $\theta$ ). **Output:** The list of ranked alternatives,  $A_r$ .

- 1 Determine the alternatives list, A.
- 2 Determine the criteria list, C.
- 3 Sort the criteria list, C, based on importance degree,  $\theta$ , as shown by Eq. (3)
- 4 Evaluate each alternative concerning each criterion, S.

5  $A_r \leftarrow \varphi$ . 6 While  $(|A_r| < n)$  $T \leftarrow \varphi.$ 7 8 For j = 1 to m9 Sort alternatives list based on  $c_i$  using the score matrix S. 10 Compute satisfaction range,  $R_i$  using importance degree for criterion  $c_j$ , as shown by Eq.4. 11  $T \leftarrow T \cup \{a_k : w_{k,i} \in R_i \}.$ If (|T| = 1) Break. 12 End 13 14  $A_r \leftarrow A_r \cup T.$ 

#### 15 $A \leftarrow A - T$ . If (|A| = 1)16 $A_r \leftarrow A_r \cup A.$ 17 18 Break. 19 End 20 End 21 Return the list of ranked alternatives, Ar.

## 4. A real-world application

The proposed ranking algorithm has been applied in a knowledge-based decision-support system for prioritizing the requests for short-road links presented to the Transportation Ministry, Kingdom of Saudi Arabia. Remote villages and cities submit requests to build roads linking them to the main roads, but the budget only allows the implementation of some requests submitted at a time. Hence, a ranking algorithm was required to prioritize the submitted requests based on many criteria, which are combined into four major criteria: (1) Expected Traffic (Trips/day), (2) Security and Geographical Factors Weight, (3) Internal Factors Weight, and (4) External Factors Weight. The different criteria and sub-criteria used in the ranking process are described below.

#### 4.1 Criteria Description

In the short-road requests ranking problem, four major criteria are used in the ranking process, as shown in Fig. 2. The used criteria are the expected traffic, the internal factors weight, the external factors weight, and security and geographical factors weight. The first criterion is the expected traffic on the short road if constructed. The expected traffic is computed by calculating the expected trips per day (trips/day), which in turn depends on the number of establishments in the village, the existence of basic services, and the village's population.



Fig. 2 The short road request criteria.

Fig. 3 shows the taxonomy of the establishments, which are classified into two major classes: regular and irregular. Regular establishments refer to governmental and semigovernmental establishments such as schools, police offices, prince's offices, fire stations, hospitals, clinics, gas stations, water sources, etc.



Fig. 3 The expected traffic criterion and its dependent sub-criteria.

On the other hand, irregular establishments refer to private establishments such as farms, factories, markets, etc. The basic services include electricity, land telephone lines, etc. The population data is processed, and several subcriteria have been derived, such as the number of children who are less than six years, number of elementary boys' schools, number of elementary girls' schools, number of boys' middle schools, number of girls' middle schools, number of boys' high schools, number of girls' high schools, number of employed women, number of employed men, the number of families living in the village, and the elderly.

The second criterion is the weight of the internal factors, which reflects the extent to which a village is attractive to the population. It is calculated based on several sub-criteria, including whether the village has regular establishments, a housing plan, a water source, and basic services. The third criterion is the external factors' weight which reflects the importance of the village from the government's perspective. It is calculated based on several sub-criteria, including whether the village is a part of the government development plan, is a tourism place, is grazing land, or is a part of the managerial join plan. The final criterion is the security and geographical factors' weight. The weight of this criterion is calculated based on whether the village is a border village, the geographic nature of the village (valley, mountain, valley-mountain, or sandy), is the village becomes isolated due to mountain avalanches, and the road length.

## 4.2 Applying the Proposed Algorithm

This section demonstrates the application of the proposed ranking algorithm to rank several short-road requests. The used case study consists of eight alternatives (short road requests). The computed score values of all alternatives are presented in Table 2. The proposed algorithm assumes that the human decision-maker prioritizes the criteria, and each criterion has an Importance Degree. The importance degrees of the different criteria are shown in Table 3. In this case study, the human decision-maker decided that the descending order of the criteria concerning the importance degree is the expected traffic, the security and geographic factors weight, the internal factors, with importance degrees 40%, 30%, 20%, and 10%, respectively.

Table 2: The score matrix

| Alternative           | Expected<br>Traffic | Security and<br>Geographic<br>Weight | Internal<br>Weight | External<br>Weight |
|-----------------------|---------------------|--------------------------------------|--------------------|--------------------|
| $A_{I}$               | 817                 | 8                                    | 1                  | 4                  |
| $A_2$                 | 530                 | 85                                   | 16                 | 6                  |
| $A_3$                 | 866                 | 56                                   | 31                 | 0                  |
| $A_4$                 | 999                 | 182                                  | 14                 | 0                  |
| $A_5$                 | 1532                | 1                                    | 33                 | 5                  |
| $A_6$                 | 627                 | 8                                    | 15                 | 6                  |
| <i>A</i> <sub>7</sub> | 2067                | 3                                    | 39                 | 12                 |
| $A_8$                 | 1024                | 168                                  | 37                 | 5                  |

Table 3: The importance degree of different criteria

| Criteria             | Expected<br>Traffic | Security and<br>Geographic<br>Weight | Internal<br>Weight | External<br>Weight |
|----------------------|---------------------|--------------------------------------|--------------------|--------------------|
| Importance<br>Degree | 40%                 | 30%                                  | 20%                | 10%                |

Fig. 4 shows the initial state of the case study in which eight alternatives need to be ranked and an empty ranking table. In addition, the human decision-maker determines the importance of degrees for different criteria.

Fig. 5 shows the first phase of the proposed ranking algorithm. This phase starts with eight unranked alternatives and ends with ranking A8 as the most important alternative. *Initial State:* 

In each round during the phase, the available alternatives are sorted concerning the current criterion, and the satisfaction range of this criterion is computed. The alternatives that belong to the satisfaction range are moved to the next round, while the alternatives whose values are outside the range are excluded from the current phase and introduced again in the next phase.

Fig. 6 shows the second phase of the proposed ranking algorithm. This phase starts with seven unranked alternatives and ends with ranking A4 as the most important among the seven alternatives. In the second round, during the second phase, the number of alternatives that belong to the satisfaction range is one; hence, this alternative is appended to the ranked alternatives, and the phase is terminated. By the end of the second phase, the total number of ranked alternatives was 2. This process continues during phases 3, 4, and 5. The number of alternatives ranked during phases 3, 4, and 5 is 2, 1, and 1, respectively.

Fig. 7 shows the sixth phase of the proposed ranking algorithm. This phase starts with two unranked alternatives and ends with ranking both. In the first round, only A1 is selected and appended to the ranked list, and no more rounds are executed. By the end of the phase, the number of remaining unranked alternatives is 1; hence, it is directly appended to the ranked list.

#### 4.3 Results Analysis

Table 4 shows the ranked alternatives, which represent the short-road requests. Based on Table 4, the proposed algorithm attempts to balance the importance degrees determined by the human decision maker and the score values of each alternative concerning the different criteria. For example, although A7 has a high score concerning the expected traffic criterion, it comes third in the ranked list because of the relatively low score regarding the other criteria. Hence, the proposed algorithm does not suffer from bias to the most important criterion and considers the score values of the other criteria.

Table 2: The score matrix

| Alternative           | Expected<br>Traffic | Security and<br>Geographic<br>Weight | Internal<br>Weight | External<br>Weight |
|-----------------------|---------------------|--------------------------------------|--------------------|--------------------|
| $A_{I}$               | 817                 | 8                                    | 1                  | 4                  |
| $A_2$                 | 530                 | 85                                   | 16                 | 6                  |
| $A_3$                 | 866                 | 56                                   | 31                 | 0                  |
| $A_4$                 | 999                 | 182                                  | 14                 | 0                  |
| $A_5$                 | 1532                | 1                                    | 33                 | 5                  |
| $A_6$                 | 627                 | 8                                    | 15                 | 6                  |
| <i>A</i> <sub>7</sub> | 2067                | 3                                    | 39                 | 12                 |
| $A_8$                 | 1024                | 168                                  | 37                 | 5                  |

| Alternative | E-T  | S-G-W | I-W | E-W |
|-------------|------|-------|-----|-----|
| A1          | 817  | 8     | 1   | 4   |
| A2          | 531  | 85    | 16  | 6   |
| A3          | 866  | 56    | 31  | 0   |
| A4          | 999  | 182   | 14  | 0   |
| A5          | 1532 | 1     | 33  | 5   |
| A6          | 628  | 8     | 15  | 6   |
| A7          | 2068 | 3     | 39  | 12  |
| A8          | 1025 | 168   | 37  | 5   |

| Criteria   | E-T | S-G-W | I-W | E-W |
|------------|-----|-------|-----|-----|
| Importance | 40% | 30%   | 20% | 10% |
| importance | 40% | 30%   | 20% | 10  |

Fig.4 The initial state of the used case study.

| Ranked Alternatives           |  |  |  |  |  |
|-------------------------------|--|--|--|--|--|
| Alternative E-T S-G-W I-W E-W |  |  |  |  |  |
|                               |  |  |  |  |  |
|                               |  |  |  |  |  |
|                               |  |  |  |  |  |
|                               |  |  |  |  |  |
|                               |  |  |  |  |  |
|                               |  |  |  |  |  |
|                               |  |  |  |  |  |
|                               |  |  |  |  |  |

#### Phase I: Total number of ranked alternatives < 8

| Round-1: Descending sort |      |       |     |     |  |
|--------------------------|------|-------|-----|-----|--|
| Alternative              | E-T  | S-G-W | I-W | E-W |  |
| A7                       | 2068 | 3     | 39  | 12  |  |
| A5                       | 1532 | 1     | 33  | 5   |  |
| A8                       | 1025 | 168   | 37  | 5   |  |
| A4                       | 999  | 182   | 14  | 0   |  |
| A3                       | 866  | 56    | 31  | 0   |  |
| A1                       | 817  | 8     | 1   | 4   |  |
| A6                       | 628  | 8     | 15  | 6   |  |
| A2                       | 531  | 85    | 16  | 6   |  |

| Round-2: Descending sort |      |       |     |     |  |
|--------------------------|------|-------|-----|-----|--|
| Alternative              | E-T  | S-G-W | I-W | E-W |  |
| A4                       | 999  | 182   | 14  | 0   |  |
| A8                       | 1025 | 168   | 37  | 5   |  |
| A3                       | 866  | 56    | 31  | 0   |  |
| A7                       | 2068 | 3     | 39  | 12  |  |
| A5                       | 1532 | 1     | 33  | 5   |  |
| A1                       | -    | -     | -   | -   |  |
| A6                       | -    | -     | -   | -   |  |
| A 2                      |      |       |     |     |  |

| Round-3: Descending sort |      |       |     |     |  |
|--------------------------|------|-------|-----|-----|--|
| Alternative              | E-T  | S-G-W | I-W | E-W |  |
| A8                       | 1025 | 168   | 37  | 5   |  |
| A3                       | 866  | 56    | 31  | 0   |  |
| A4                       | 999  | 182   | 14  | 0   |  |
| A7                       | -    | -     | -   | -   |  |
| A5                       | -    | -     | -   | -   |  |
| A1                       | -    |       | -   | -   |  |
| A6                       | -    |       | -   | -   |  |
| A2                       | -    | -     | -   | -   |  |

| Round 4: Descending sort |      |       |     |     |  |
|--------------------------|------|-------|-----|-----|--|
| Alternative              | E-T  | S-G-W | I-W | E-W |  |
| A8                       | 1025 | 168   | 37  | 5   |  |
| A3                       | 866  | 56    | 31  | 0   |  |
| A4                       | 999  | 182   | 14  | 0   |  |
| A7                       | -    | -     | -   | -   |  |
| A5                       | -    | -     | -   | -   |  |
| A1                       | -    |       | -   | -   |  |
| A6                       | -    | -     | -   | -   |  |
| A2                       | -    | -     | -   | -   |  |

 Sort Criteria
 E-T

 Satisfaction Range
 # of Selected-Alternatives
 Decision

 827.2
 2068
 6
 Continue



| Sort Crit      | eria I | -W                         |          |
|----------------|--------|----------------------------|----------|
| Satisfaction R | ange   | # of Selected-Alternatives | Decision |
| 7.4            | 37     | 3                          | Continue |



| Round       | Round-1: Select alternatives within range |       |     |     |  |
|-------------|---|-------|-----|-----|--|
| Alternative | E-T                                       | S-G-W | I-W | E-W |  |
| A7          | 2068                                      | 3     | 39  | 12  |  |
| A5          | 1532                                      | 1     | 33  | 5   |  |
| A8          | 1025                                      | 168   | 37  | 5   |  |
| A4          | 999                                       | 182   | 14  | 0   |  |
| A3          | 866                                       | 56    | 31  | 0   |  |
| A1          | 817                                       | 8     | 1   | 4   |  |

| Round-2: Select Alternatives Within Rang | Round-2: Select A | Alternatives | Within | Rang |
|--|-------------------|--------------|--------|------|
|--|-------------------|--------------|--------|------|

| Alternative | E-T  | S-G-W | I-W | E-W |
|-------------|------|-------|-----|-----|
| A4          | 999  | 182   | 14  | 0   |
| A8          | 1025 | 168   | 37  | 5   |
| A3          | 866  | 56    | 31  | 0   |
| A7          | -    | -     | -   | -   |
| A5          | -    | -     | -   | -   |
| A1          | -    | -     | -   | -   |
| A6          | -    | -     | -   | -   |
| A2          | -    | -     | -   | -   |

| Round-3: Select alternatives within Range |      |       |     |     |  |
|---|------|-------|-----|-----|--|
| Alternative                               | E-T  | S-G-W | I-W | E-W |  |
| A8  | 1025 | 168   | 37  | 5   |  |
| A3  | 866  | 56    | 31  | 0   |  |
| A4  | 999  | 182   | 14  | 0   |  |
| A7  | -    | -     | -   | -   |  |
| A5  | -    | -     | -   | -   |  |
| A1  | -    | -     |     | -   |  |
| A6  | -    | -     |     | -   |  |
| A2  | -    | -     |     | -   |  |

| Ranked Alternatives |      |       |     |     |  |
|---------------------|------|-------|-----|-----|--|
| Alternative         | E-T  | S-G-W | I-W | E-W |  |
| A8                  | 1025 | 168   | 37  | 5   |  |
|                     |      |       |     |     |  |
|                     |      |       |     |     |  |
|                     |      |       |     |     |  |
|                     |      |       |     |     |  |
|                     |      |       |     |     |  |
|                     |      |       |     |     |  |
|                     |      |       |     |     |  |

Number of Ranked Alternatives = 1 Total Number of Ranked Alternatives = 1

Fig.5 The first phase of the proposed ranking algorithm.

## Phase II: Total number of ranked alternatives < 8

| Round-1: Descending sort |      |       |     |     |  |
|--------------------------|------|-------|-----|-----|--|
| Alternative              | E-T  | S-G-W | I-W | E-W |  |
| A7                       | 2068 | 3     | 39  | 12  |  |
| A5                       | 1532 | 1     | 33  | 5   |  |
| A4                       | 999  | 182   | 14  | 0   |  |
| A3                       | 866  | 56    | 31  | 0   |  |
| A1                       | 817  | 8     | 1   | 4   |  |
| A6                       | 628  | 8     | 15  | 6   |  |
| A2                       | 531  | 85    | 16  | 6   |  |

| Round-2: Descending sort |      |       |     |     |  |
|--------------------------|------|-------|-----|-----|--|
| Alternative              | E-T  | S-G-W | I-W | E-W |  |
| A4                       | 999  | 182   | 14  | 0   |  |
| A7                       | 2068 | 3     | 39  | 12  |  |
| A5                       | 1532 | 1     | 33  | 5   |  |
| A3                       | -    | -     | -   | -   |  |
| A1                       | -    | -     | -   | -   |  |
| A6                       | -    | -     | -   | -   |  |
| A2                       | -    | -     | -   | -   |  |

|     | Sort Crit     | eria | E | -T            |                |          |
|-----|---------------|------|---|---------------|----------------|----------|
| Sat | tisfaction Ra | ange |   | # of Selected | d-Alternatives | Decision |
| 82  | 27.2          | 2068 |   | 3             |                | Continue |

Sort Criteria S-G-W

# Round-1: Select alternatives within range Alternative E-T S-G-W I-W E-W A7 2068 3 39 12 A5 1532 1 33 5 A4 999 182 14 0 A3 A1 A6

| Round-2: | Select | alternatives | within | range |
|----------|--------|--------------|--------|-------|
|          |        |              |        |       |

| Ranked Alternatives |      |       |     |     |
|---------------------|------|-------|-----|-----|
| Alternative         | E-T  | S-G-W | I-W | E-W |
| A8                  | 1025 | 168   | 37  | 5   |
| A4                  | 999  | 182   | 14  | 0   |
|                     |      |       |     |     |
|                     |      |       |     |     |
|                     |      |       |     |     |
|                     |      |       |     |     |
|                     |      |       |     |     |
|                     |      |       |     |     |

| Satisfaction Range |     | # of Selected-Alternatives | Decision |
|--------------------|-----|----------------------------|----------|
| 54.6               | 182 | 1                          | Stop     |

Number of Ranked Alternatives = 1 Total Number of Ranked Alternatives = 2

Fig.6 The second phase of the proposed ranking algorithm.

#### Phase VI: Total number of ranked alternatives < 8



Number of Ranked Alternatives in Sixth Phase = 2

Total Number of Ranked Alternatives = 8

Fig.7 The sixth phase of the proposed ranking algorithm.

#### 5. Conclusion and Future Work

This paper has presented a multi-level ranking algorithm for prioritized multi-criteria decision-making (MCDM) problems. The proposed algorithm considers the importance degree of each criterion determined by the human decision maker. It has been applied in a decision support system for ranking the villages' short road requests presented to the ministry of transportation in the Kingdom of Saudi Arabia. In the used case study, the proposed algorithm revealed a good balance between the importance degrees determined by the human decision-maker and the score values of the different alternatives concerning the different attributes. In the future, the proposed algorithm should evolve to consider the uncertainty and missing values using the fuzzy theory.

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