Exclusive Conjugate Quasi Orthogonal Space Time Block Code Family with Full Rate Full Diversity Following Feedback Approach

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Abstract
This paper presents a subfamily of quasi-orthogonal space time block codes (QOSTBC), named Exclusive Conjugate (EC-QOSTBC), with full rate, full diversity and simple decoding at the receiver. Our approach estimates feedback constants at the receiver and the transmitter in turn modifies the transmitted symbols based on the estimated constants. This approach is an alternative method to those used in the literature to overcome the challenge of no existing STBC with full diversity and full rate with simple decoding for more than two transmitted antennas. Our simulation results show that the proposed family has better performance in terms of BER-SNR when compared to other codes presented in the literature. Finally, the estimated feedback constants in this work consists of only two real constants.

Keywords:
MIMO, Alamouti code, STBC, QOSTBC, 5G.

1. Introduction

Mobile traffic data is experiencing an exponential increase due to the demand for high quality video, smart phones, IoT, etc. For example, data traffic increased by more than 60% in 2016 compared to the previous year. As the demand increases, new approaches are necessary to improve the quality of service. One of the promising approaches in 5G, the next generation of mobile networks, is massive multiple input multiple output (MIMO) systems. Massive MIMO systems consist of large numbers of antennas that could simultaneously serve hundreds of users [1].

MIMO systems have been studied by many researchers in the literature since they represent an attractive approach to increase the reliability for wireless communications. The main concept of MIMO systems is to take advantage of multipath propagations between the transmitter and receiver which can be modeled as several independent fading links. This concept was originally introduced by Alamouti and was subsequently named for him as Alamouti codes. Alamouti codes are the first and only codes with full rate full diversity in space time block code techniques [2][3].

Quasi orthogonal space time block codes (QOSTBC) is an alternative technique to provide full rate full diversity with pair-wise decoding. However, a feedback approach is introduced in the literature to overcome the pair-wise decoding challenge. Successful implementation removes the need to decode two symbols together, which in turn decreases the complexity at the receiver. This approach is defined by estimating constants at receiver. The transmitter then modifies the transmitted symbols in the next quasi time slots based on the estimated feedback constants from receiver [4-6].

In [5] and [6], the authors follow the feedback method and estimate different feedback constants for the Jafarkhani codes presented in [4]. Therefore, the self-interference from adjacent symbols that drives the need for pair-wise decoding, is eliminated. The result is that the feedback scheme achieves full rate and full diversity with simple and single receiver-based decoding. However, the performance of the two schemes is poor in terms of bit error rate vs. signal to noise ratio (BER-SNR).

In this paper, we will present a new family of Quasi Orthogonal STBC for four transmitted antennas and N received antennas. The new family of codes needs only two real constants to achieve full diversity and full transmission rate. The proposed code improves the (BER-SNR) performance compared to the two feedback schemes in [5],[6]. In addition, the size of estimated feedback is less than the feedback size present in [6].

1.1 Alamouti Code Analysis

Alamouti codes, as mentioned before, are the only complex codes that achieve full diversity and full rate with linear decoding. It has been shown that it is impossible to design a full rate full diversity orthogonal complex code for systems with more than two transmitter antennas [3]. Table 1 shows an Alamouti code where the rows represent time slot and the columns represent specific transmitter antenna. The * sign denotes the conjugate operation and T_s is one slot time.

Table 1: Alamouti code

Transmit time	Antenna 1	Antenna 2
t	s_1	s_2
$t+T_s$	$-s_2^*$	s_1^*

Manuscript received October 5, 2024
Manuscript revised October 20, 2024
https://doi.org/10.22937/IJCSNS.2024.24.10.23

Therefore, research results focused on extending Alamouti code-based systems with more than two transmitter antennas are of interest. It has previously been proven in the literature that the rate for complex orthogonal STBC with full diversity is less than one when the number of transmitter antennas is greater than two [7]. In this section, we will focus on Alamouti code analysis and the next section will describe QOSTBC.

An Alamouti code-based system implementation consists of two transmitter antennas and one or more receiver antennas. Figure 1 below shows a depiction of an Alamouti system. The transmitter sends symbols s_1 and s_2 over transmitters 1 and 2 at the first time slot. At the next slot, the transmitter sends $-s_2^*$ and s_1^* over transmitters 1 and 2, respectively.

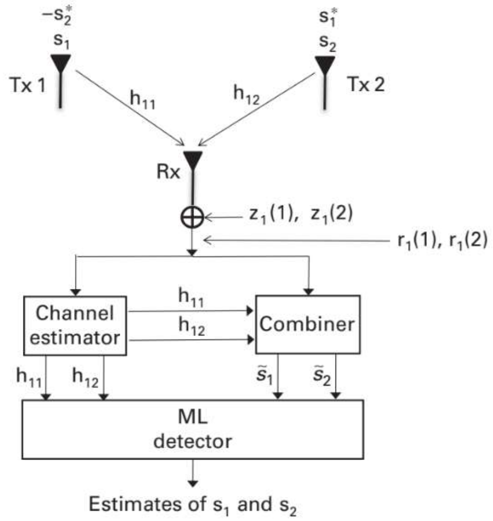

Fig. 1 Alamouti system model [3]

The two symbols are transmitted over two different channels, h_{11} and h_{12}. At the receiver antenna, the two symbols are received as noise corrupted signals given by

$$R_1(1) = h_{11}s_1 + h_{12}s_2 + z_1(1) \quad (1)$$

$$R_2(2) = -h_{11}s_2^* + h_{12}s_1^* + z_1(2) \quad (2)$$

where $R_i(j)$ is the received symbols with noise from receiver antenna i a time slot j. $z_i(j)$ is the noise over channel between receiver i and transmitter antennas at time slot j.

The combiner's function is to estimate the value of transmitted symbols. $\tilde{s}_1$ and $\tilde{s}_2$ denote the corresponding symbols at receiver for the transmitted symbols s_1 and s_2. They can be expressed as

$$\tilde{s}_1 = h_{11}^*R_1(1) + h_{12}R_1^*(2) \quad (3)$$

$$\tilde{s}_2 = h_{12}^*R_1(1) + h_{11}R_1^*(2) \quad (4)$$

After simple simplification $\tilde{s}_1$ and $\tilde{s}_2$ can be expressed as

$$\tilde{s}_1 = \left(\sum_{i=1}^{2}|h_{1,i}|^2\right)s_1 + h_{11}^*z_1(1) + h_{12}z_1^*(2) \quad (5)$$

$$\tilde{s}_2 = \left(\sum_{i=1}^{2}|h_{1,i}|^2\right)s_2 + h_{12}^*z_1(1) + h_{11}z_1^*(2) \quad (6)$$

The last step is to use a maximum likelihood estimator to estimate which symbols were originally transmitted. The decoding process is linear, and this is because the code matrix as shown in Table 1 is orthogonal. The orthogonality means any two columns in code matrix are orthogonal and thus carry non-overlapping information content. This property ensures that the decoding process is linear.

The rest of this paper is organized as follows: Section 2 presents the system model that is considered. Section 3 details the new proposed code. Section 4 presents the derivative of the Exclusive Conjugate QOSTBC family. Section 5 summarizes the simulation results for the new code compared to two schemes presented in the literature. Sections 6 contains the conclusions.

2. System Model

Consider a wireless communications system with N receiving antennas and four transmission antennas as shown in Figure 1. The receiving antennas are denoted by Rx_j where $j \geq 1$. The transmission antennae are denoted by Tx_i where $i \in \{1,2 \ldots 4\}$. The channel attenuation between Tx_i and Rx_j is represented by h_{ji}.

During each quasi-time slot, the four signals $s = [s_1, s_2, s_3, s_4]$ are encoded and transmitted over four transmission antennas according to the below codeword matrix

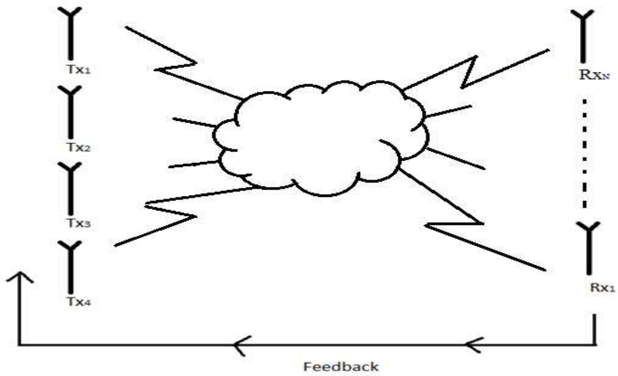

Fig. 2 4xN MIMO system

$$S = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix} \tag{7}$$

Each row represents a time slot. Each column represents an individual transmitter antenna. For example, the intersection of second row with third column represents the symbol that is transmitted over Tx_3 at the second time slot.

At the receiver, the received signal can be expressed as

$$\tilde{R} = H_{4x4}S_{4x1} + z_{4x1} \tag{8}$$

where H is the channel matrix given by

$$H_{4x4} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{12} & -h_{11} & h_{14} & -h_{13} \\ h_{13}^* & -h_{14}^* & -h_{11}^* & h_{12}^* \\ h_{14}^* & h_{13}^* & -h_{12}^* & -h_{11}^* \end{bmatrix}, S_{4x1} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \tag{9}$$

and z_{4x1} represents the additive white gaussian noise (AWGN) at each time slot. The rows in H_{4x4} represent time slots and the columns represent symbol number. For example, $-h_{11}^*$ in the third row and third column represent symbols, s_3, transmitted at slot number three over the channel, h_{11}, i.e. transmitter number one, Tx_1. The original symbols can be recovered by multiplying $\tilde{R}$ by H_{4x4}^H as shown below where $(.)^H$ denotes the Hermitian operator.

$$Y = H_{4x4}^H * H_{4\times 4} * S_{4\times 1} + H_{4x4}^H z_{4x1} \tag{10}$$

$$= \begin{bmatrix} \gamma & \alpha & 0 & 0 \\ -\alpha & \gamma & 0 & 0 \\ 0 & 0 & \gamma & -\alpha \\ 0 & 0 & \alpha & \gamma \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + H_{4x4}^H z_{4x1}$$

Where,

$$\gamma = \sum_{i=1}^{4} |h_{1i}|^2 \tag{11}$$

And α is

$$h_{12}h_{11}^* - h_{12}^* h_{11} + h_{14}h_{13}^* - h_{14}^* h_{13} \tag{12}$$

We can note from Y that the decoding process is not simple but pairwise because of the self-interference from adjacent symbols. To overcome this challenge, we will follow a feedback approach to estimate constants. These constants are sent to the transmitter to modify the transmitted symbols which in turn reduce the magnitude of the off- diagonal values in matrix Y to zero. More details of this method are provided in the next section.

3. Proposed Method

The feedback approach has been followed by many researchers in the literature to improve the performance of QOSTBC. The approach is used to estimate one or more constants at the receiver. These constants are sent back to transmitter to modify transmitted symbols. Note that the symbol extraction procedure results in a system of equations describing the interdependence of each symbol on another due to the non-zero α terms present in the off-diagonal elements. Therefore, we seek ways to simplify the resultant system by minimizing these terms. If we examine α, we will find that α is a purely imaginary number and given by the following equation:

$$\alpha = j2(Re(h_{11})Im(h_{12}) - Re(h_{12})Im(h_{11}) + Re(h_{13})Im(h_{14}) - Re(h_{14})Im(h_{13})) \tag{13}$$

We can rewrite $\alpha = j(a + b)$ where a is

$$2\big(Re(h_{11})Im(h_{12}) - Re(h_{12})Im(h_{11})\big) \tag{14}$$

and b is

$$2\big(\mathrm{Re}(h_{13})\mathrm{Im}(h_{14})\text{-}\mathrm{Re}(h_{14})\mathrm{Im}(h_{13})\big) \tag{15}$$

Now, we can send $k_1 = \frac{1}{a}$ and $k_2 = -\frac{1}{b}$ as feedback to the transmitter. These constants provide estimates of channel transmission effects to the transmitter. The transmitter in turn will multiply these constants by the symbols transmitted through antennas 1 and 4, respectively. The new resulting matrix H_{4x4} will now be

$$\begin{bmatrix} k_1h_{11} & h_{12} & h_{13} & k_2h_{14} \\ h_{12} & -k_1h_{11} & k_2h_{14} & -h_{13} \\ h_{13}^* & -k_2h_{14}^* & -k_1h_{11}^* & h_{12}^* \\ k_2h_{14}^* & h_{13}^* & -h_{12}^* & -k_1h_{11}^* \end{bmatrix} \tag{16}$$

Correspondingly, the value of Y will be

$$Y = H_{4x4}^H * R + H_{4x4}^H z_{4x1} \tag{17}$$

$$= \begin{bmatrix} \gamma & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + H_{4x4}^H z_{4x1}$$

We note that the value of α is zero since

$$\begin{aligned} \alpha &= j(a+b) \\ &= j2[Re(k_1h_{11})Im(h_{12}) - Re(h_{12})Im(k_1h_{11}) \\ &\quad + Re(h_{13})Im(k_2h_{14}) - Re(k_2h_{14})Im(h_{13}))] \\ &= j2[Re\left(\frac{h_{11}}{a}\right)Im(h_{12}) - Re(h_{12})Im\left(\frac{h_{11}}{a}\right) \end{aligned} \tag{18}$$

$$+Re(h_{13})Im\left(\frac{h_{14}}{b}\right) - Re\left(\frac{h_{14}}{b}\right)Im(h_{13}))]$$
$$= j2[(Re(h_{11})Im(h_{12}) - Re(h_{12})Im(h_{11}))/a$$
$$-(Re(h_{13})Im(h_{14}) - Re(h_{14})Im(h_{13}))/b]$$
$$= j2[1-1]=0$$

The resultant system of equations is now free from inter-symbol dependence and therefore the receiver can estimate the transmitted symbols at each time slot without the need to obtain the information contained in another time slot. For the general case where we have N receiver antennas, the value of a is

$$\sum_{j=1}^{N}\left(Re(h_{j1})Im(h_{j2}) - Re(h_{j2})Im(h_{j1})\right) \quad (19)$$

And b is given by

$$\sum_{j=1}^{N}\left(Re(h_{j3})Im(h_{j4}) - Re(h_{j4})Im(h_{j3})\right) \quad (20)$$

We can note that the feedback size is two real constants. In comparison to the literature, the feedback size of the proposed codeword matrix is half the size of the work presented in [6] and it is the same size as those presented in [5]. Simulation results summarizing the performance of our code as functions of SNR and BER will be compared to those in [5] and [6], in section 5.

4. Derivation of Exclusive Conjugate QOSTBC Family

The self-interference from adjacent symbols is encapsulated by α value in the following equation:

$$Y = \begin{bmatrix} \gamma & \alpha & 0 & 0 \\ -\alpha & \gamma & 0 & 0 \\ 0 & 0 & \gamma & -\alpha \\ 0 & 0 & \alpha & \gamma \end{bmatrix} + H_{4x4}^{H} z_{4x1} \quad (21)$$

where α, as explained above, is given by

$$h_{12}h_{11}^{*} - h_{12}^{*}h_{11} + h_{14}h_{13}^{*} - h_{14}^{*}h_{13} \quad (22)$$

The α value is in an exclusive conjugate format assigned to the family name that the code belongs to. Moreover, this value can be simplified into two real constants a and b. In fact, the proposed code belongs to a family which we named as Exclusive Conjugate QOSTBC (EC-QOSTBC). All codewords in this family have the same performance in terms of bit error rate verse signal to noise ratio. A simulation result for a few codes belonging to this family will be presented in the result section.

The first step to derive the new family is to group two Alamouti codes for real symbols as A and B. For example,

$$A = \begin{bmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{bmatrix}, B = \begin{bmatrix} s_3 & s_4 \\ -s_4 & s_3 \end{bmatrix} \quad (23)$$

The second step is to build the Quasi Orthogonal STBC matrix as follow:

$$\begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{B^H} & \mathrm{-A^H} \end{bmatrix} \ \& \ \begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{-B^H} & \mathrm{A^H} \end{bmatrix} \quad (24)$$

This results in two matrix codes with the same characteristics as the proposed code in the system model. Table (2) shows 6 matrix codes that belong to the same family. Each codeword has its own formula as in (19) and (20) for the value of a and b. Note that these codes are not the only codes in the family. Any change in the shape of the two real Alamouti codes will result in the generation of a new code based on the requirements specified in (24). A simulation result of the performance for the six codeword matrix will be presented in the next section.

5. Simulation Results

In our simulation, we assume there are four transmitter antennas and one or four receiver antennas. Where the number of transmitter and receiver antennas were chosen to facilitate a better comparison with the results presented in the current literature. The channel is assumed to be characterized by quasi-static Rayleigh flat fading (i.e. the channel attenuation characteristics do not change over 4 time slots). The modulation used is QPSK. Figure 3 shows a comparison between our code and the other two studies, presented in [5] and [6], in terms of bit error rate (BER) performance. The x-axis represents the required transmission power and is given in terms of signal to noise ratio (SNR) in dB and the corresponding y-axis value is the resultant BER. We can note that the performance of our code exceeds the performance of the codes in the other two studies. As an example, note that to achieve the same BER of our code at 2 dB the code detailed in [6] must transmit signals at 6 dB. Therefore, implementation of our code results in a power saving of 4 dB compared to [6]. Comparing the results of the EC-QOSTBC with those of the study in [5], our code results in a power saving of 6.5 dB. Additionally, we note that implementation of the methods presented in this study results in feedback consisting of two

Table 2: Exclusive Conjugate QOSTBC family

A and B matrix are Alamouti code for real symbols	$\begin{bmatrix} A & B \\ B^H & -A^H \end{bmatrix}$	$\begin{bmatrix} A & B \\ -B^H & A^H \end{bmatrix}$
$A = \begin{bmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{bmatrix}$, $B = \begin{bmatrix} s_3 & s_4 \\ -s_4 & s_3 \end{bmatrix}$	$\begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ s_3^* & -s_4^* & -s_1^* & s_2^* \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix}^1$	$\begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}^2$
$A = \begin{bmatrix} -s_1 & s_2 \\ -s_2 & -s_1 \end{bmatrix}$, $B = \begin{bmatrix} -s_3 & s_4 \\ -s_4 & -s_3 \end{bmatrix}$	$\begin{bmatrix} -s_1 & s_2 & -s_3 & s_4 \\ -s_2 & -s_1 & -s_4 & -s_3 \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4^* & -s_3^* & -s_2^* & s_1^* \end{bmatrix}^3$	$\begin{bmatrix} -s_1 & s_2 & -s_3 & s_4 \\ -s_2 & -s_1 & -s_4 & -s_3 \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ -s_4^* & s_3^* & s_2^* & -s_1^* \end{bmatrix}^4$
$A = \begin{bmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{bmatrix}$, $B = \begin{bmatrix} -s_3 & s_4 \\ -s_4 & -s_3 \end{bmatrix}$	$\begin{bmatrix} s_1 & s_2 & -s_3 & s_4 \\ -s_2 & s_1 & -s_4 & -s_3 \\ -s_3^* & -s_4^* & -s_1^* & s_2^* \\ s_4^* & -s_3^* & -s_2^* & -s_1^* \end{bmatrix}^5$	$\begin{bmatrix} s_1 & s_2 & -s_3 & s_4 \\ -s_2 & s_1 & -s_4 & -s_3 \\ s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & s_3^* & s_2^* & s_1^* \end{bmatrix}^6$

real constants. This effectively reduces the size of the feedback constants presented in [6] by half.

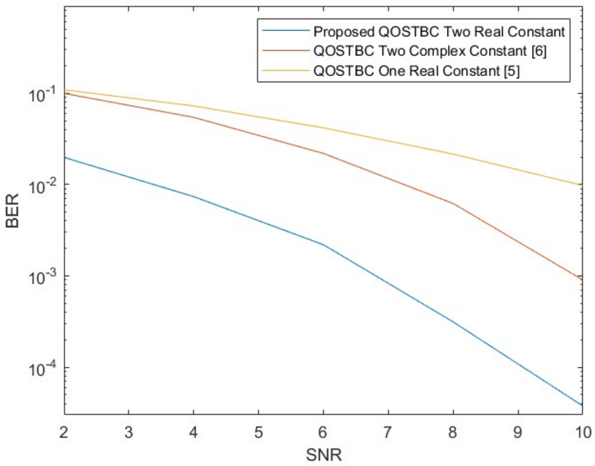

Fig. 3 Performance comparison between our code and other two studies [5] & [6] in terms of bit error rate (BER).

In Figure 4, it can be seen that the performance of the proposed QOSTBC with two real constants feedback exhibits better performance than the QOSTBC with two complex constants feedback presented in [6] when the number of received antenna is four. It can also be noted that the performance of our code at 2 dB is the same as that of the code developed in [6] at 4 dB. Thus, our code results in a required power reduction of 2 dB. It must be mentioned that the work presented in [5] does not support more than one received antenna in their formulation. Therefore, direct comparison is not applicable.

The BER-SNR performance for six codewords from Table (2) was evaluated and presented in Figure 5. The setting for this experiment is like the previous experiment. The channel is assumed to be quasi-static with Rayleigh flat channel fading. The modulation used is QPSK. The result shows that all codewords presented in table (2) result in similar BER performance.

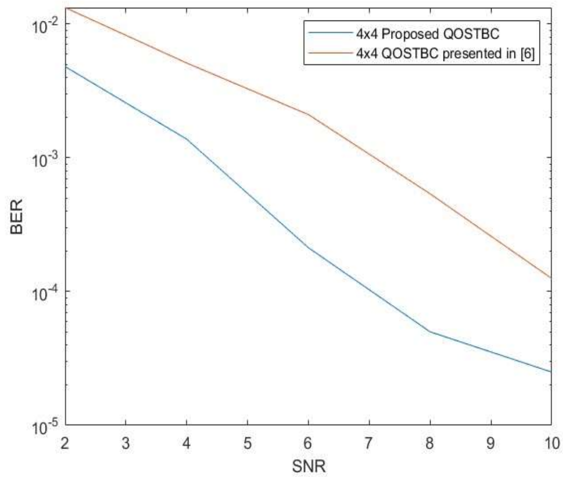

Fig. 4 The BER performance of proposed QOSTBC with two real constant feedback to two complex constant feedback presented in [6].

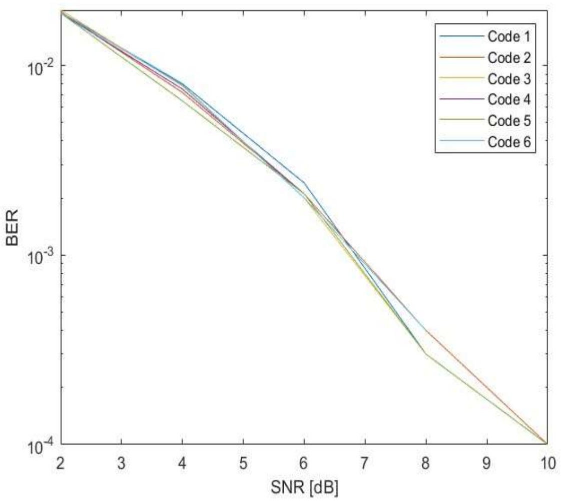

Fig. 5 Performance comparison between different codeword's belongs to the proposed Exclusive Conjugate QOSTBC family

6. Conclusion

This paper presents a new family of QOSTBC with full rate and full diversity that overcomes the challenges of pairwise decoding in 4xN MIMO systems. Our approach is used to estimate the transmission effects at the receiver, send feedback constants to the transmitter, and the modify the transmitted symbols according to these constants to allow the receiver to reconstruct the original symbols using simple single time slot decoding. Our results show that the new family codes exhibited better performance than similar developments completed in the literature in [5],[6]. Moreover, the new family of codes developed in this work needs smaller feedback constant sizes when compared to the work in [6].

Disclosure Statement

The authors report there are no competing interests to declare.

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