A Novel Complex Feedback Independent Component Analysis Algorithm and its Application to Fingerprints Image Separation

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Summary
Independent Component Analysis (ICA) is now a day become a more stable and sophisticated statistical method to analyze any multivariate data. There are various ICA algorithms already available to find the independent components as well as to separate the sources from the mixtures. Reviews show that researchers have focused more on separation of simulated data rather than the real life data. Most of the available algorithms are not able to completely solve the separation problem of the real life mixtures of images and audios. We have proposed a novel algorithm known as complex feedback ICA algorithm (complex-FEBICA), which is a gradient based algorithm with feedback architecture. It is well known that in the complex domain, rotational invariance can be found; complex-FEBICA is highly applicable to the real life mixtures. We have applied our algorithm to different fingerprint mixtures either created artificially or real life mixtures and have demonstrated the successful separation of fingerprints from mixtures. We are also able to separate m fingerprints out of less than m unknown mixtures.

Keywords:
Independent component analysis (ICA), rotational invariance, complex domain, FEBICA, fingerprints.

1. Introduction

In most of the real world situations, we have access to the signals that are embedded in noise or sometimes even mixed. Practically, we don't know how these signals were mixed and what the mixing matrix was? Such mixtures are called blind mixtures. The method of extracting original sources from these mixtures are known as **"Blind Source Separation (BSS)"** and such problem is termed as **"Blind Source Separation Problem (BSSP)"**. Independent component analysis has been widely used to solve blind source separation problem (BSSP) in a variety of applications.

BSSP can be mathematically defined as follows:
Let X: $\Phi \rightarrow \mathbb{R}^n$ *be an independent random vector, and let* $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$ *be finite and measurable. An ICA of* $Y = g \circ X$ *is called BSS of (X, g). Given a full rank* $n \times m$ *matrix A called mixing matrix, an ICA of Y = AX is called BSS of X.*

Again, in the case of *square BSS*, m = n; which means that the mixing matrix A is invertible. From definition it is clear that **Y** is the observed variable or signals, **A** is the mixing matrix and **X** is the original signals and in real life situations, we only observed **Y**. Let $\mathbf{A} = (a_1|...|a_n)$ with $a_i \in \mathbb{R}^n$, we can write,

$$Y = AX \quad (1)$$
$$= (a_1|...|a_n)X \quad (2)$$
$$= \sum_{i=1}^{n} a_i X_i \quad (3)$$
$$= \sum_{i=1}^{n} \left(\frac{a_i}{\alpha_i}\right)(\alpha_i X_i), \alpha_i \in \mathbb{R}^n; \alpha_i \neq 0. \quad (4)$$

Thus, it is clear that multiplying the sources with non-zero constants does not change their independence, so **A** can only be found up to scaling. Furthermore, permuting the sum in the index **i** above do not change the model, so only the set of columns of **A** can be found, but not their order. Thus, we can see both the scaling indeterminacy and the permutation indeterminacy. So, we can get large number of solutions after applying ICA. Some kind of normalization often be used for reducing the set of solutions: For example, in the model we could assume that var $(X_i) = 1$ i.e. that the sources have unit variances or that $|a_i| = 1$. These conditions would restrict choices for the **i** to only two and we can see the sign indeterminacy.

In real world situations, there are following types of possible cases:

1) Sensors are equal to the number of sources.
2) Sensors are greater than the number of sources.
3) Sensors are less than the number of sources.

If m > n i.e. in the case of more mixtures than sources, the model above is called *overdetermined or undercomplete*. In the case m < n i.e. in the case of fewer

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mixtures than sources, the model is known as *underdetermined or overcomplete*.

If the mixing is convolutive in nature, we should search for an algorithm that works in the complex domain. Most of the researchers have worked on functional resonance imaging (fMRI) image data taking it as a real valued image. But, the actual data is acquired as complex valued images. When performing an analysis of fMRI data using the complex- valued images, results demonstrate an increased ability to isolate the task related functional changes [1,2] illustrating the importance of performing source separation directly on the acquired data that is complex valued. If we work in the frequency domain and use the Fourier transform of the images we will again get the complex-valued data. Algorithms for ICA of complex-valued data have been proposed in [3, 4, 5, 6, 7, 8]. A popular neural-network approach for ICA utilizes the principle of information maximization (infomax) [9]. This approach utilizes an intuitively meaningful contrast function (mutual information), typically, provides a simple learning rule has shown promising performance on a number of synthetically created BSS problems.

In this paper we have proposed a new feedback based ICA algorithm that work with the complex weights and in the complex domain. The feedback architecture is very similar to the Hopfield network; i.e. the self-feedback of the neuron is zero. The non-linearity used in our case is hyperbolic tangent of tan-sigmoid. Introducing this non-linearity we create a narrow boundary, which is helpful in distinguishing the different independent components. As, probability of finding different independent vectors reduces with the limited or reduced space and consequently the probability of finding a single independent vector in that reduced space increases.

The feed-back architecture is helpful in decreasing the mutual information. We successfully applied our algorithm to the artificially created fingerprints mixtures as well as real-life fingerprints mixture data obtained from some security agency. Also, complex valued neural networks enable us to automatically capture good rotational behavior of complex numbers and hence very useful in the case of rotated fingerprints mixture.

2. Methodology

In the complex-valued neural networks, all parameters are complex numbers. We have applied the complex methodology individually to the real and imaginary part of the existing gradient based method. The second order structure of a complex random vector can be found in [11,12,13,14] and higher- order statistical structure can be found in [15]. Any complex number can be notated as follows:

$$z = z_R + jz_I \in C, j = \sqrt{-1} \tag{5}$$

With modulus defined by,

$$|z| = \sqrt{z^* z} = \sqrt{z_R^2 + z_I^2} \tag{6}$$

The Euclidean norm of vector z can be denoted as:

$$||z||^2 = z^{ct} z, \tag{7}$$

ct = conjugate transpose or the Hermitian adjoint. Any random vector (r. v.) can be defined as follows:

$$\vec{x} = \vec{x}_R + j\vec{x}_I \tag{8}$$

The expectation value of the complex r. v. can be given by,

$$E_{\vec{x}} = E_{\vec{x}_R} + E_{\vec{x}_I} \tag{9}$$

And the complex covariance matrix can be given by,

$$cov[\vec{x_1}, \vec{x_2}] = E_{\vec{x_1}, \vec{x_2}}[(\vec{x_1} - E_{\vec{x_1}}[\vec{x_1}]).(\vec{x_2} - E_{\vec{x_2}}[\vec{x_2}])] \tag{10}$$

If the two complex vectors are uncorrelated then the covariance of $\vec{x1}$ and $\vec{x2}$ must be zero. The covariance matrix must be a square matrix. Based on the covariance matrix, the eigen analysis or SVD analysis can be done to find out the direction of the principal components. This can be simplified by separate analysis of both the real and imaginary part of the covariance matrix.

Using the PCA, the *second - order dependency* will be removed. At the same moment, if the case is of greater number of sensors than sources, we can reduce the dimension of the data to get less number of sources. The complex weights has been given by,

$$W = W_R + jW_I \tag{11}$$

and the feedback weights has been given by,

$$W_{fb} = W_{fbR} + jW_{fbI} \tag{12}$$

with diagonal zero i.e.

$$W_{fb}(k,k) = 0 \tag{13}$$

The non-linear function has been defined as,

$$nl = tanh(logsig(Y_R + Y_I)) \tag{14}$$

Where,

$$Y = fft(Y) \tag{15}$$

And

$$Y_z = W * y_z - W_{fb} * nl_z \tag{16}$$

The function is bounded over the entire complex plane. The final weight update equation for infomax using natural gradient has been given with the reference of [9](Entropy maximization) as,

$$\Delta W = \eta[1 + \frac{nl''}{nl'} y^{ct}]W \tag{17}$$

$$W = W + \Delta W \tag{18}$$

and the final independent vectors can by found using,

$$I.V. = W * Y + mean \quad (19)$$

The convergence of complex FEBICA can be shown as same derived in [6].

3. Applications and Results

The complex FEBICA has been successfully applied to the following mixtures:

1) The complex FEBICA has been applied firstly on the two synthetic fingerprints mixtures created in the lab using the NITZEN fingerprint scanner. In figure 1, the original fingerprints and their mixtures have been shown. The fingerprints mixtures was created using the same Fingerprints but with a rotation of 90^0. In figure 2, the separation results have been shown using standard infomax algorithm, fastICA algorithm and complex-FEBICA algorithm respectively.

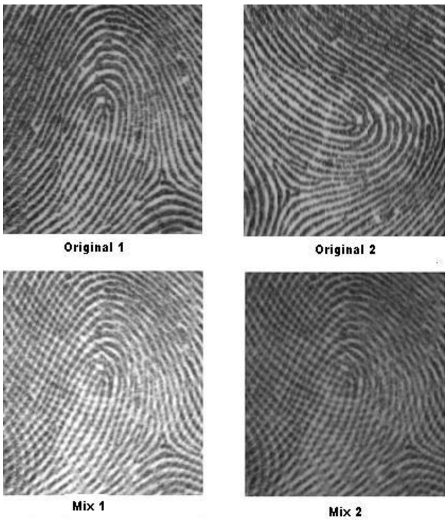

Fig.1. Original Fingerprints and their Mixtures

2) The second data as shown in figure 3, has been obtained using the **Iboss** fingerprint scanner (Indian product), which has a facility to scan overlapped fingerprints. The separated fingerprints using *FastICA algorithm, Natural Gradient - Flexible ICA algorithm, Robust Joint Approximate Diagonalization of Eigen matrices (JADE) algorithm, Self Adaptive Natural Gradient algorithm with nonholonomic constraints, Robust Second Order Blind*

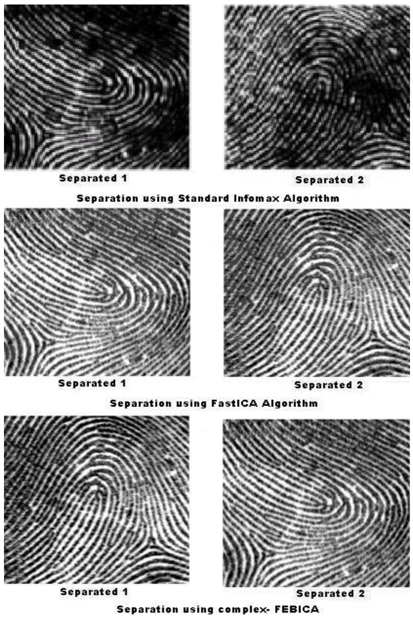

Fig. 2. Separation Results (mixed images are correlated by 98% while separated images are having 0% correlation)

Identification with Robust Orthogonalization algorithm and Complex - FEBICA algorithm has been shown in Figure 4.

Fig. 3. Overlapped fingerprints obtained from iboss fingerprint scanner

3) The image mixture (single image) shown in figure 5 has been obtained from some security agency. In this case, we don't know the number of the original fingerprints. Now, objective was to separate out the fingerprints as well as to find out the original sources. Complex FEBICA has been applied to this image and five separated fingerprints have been found from a single image. The same has been crosschecked and justified again with the security agency. None of the existing algorithm was found successful for separation of the present fingerprint mixture. The complex

FEBICA results as well as the result obtained from other existing algorithms have been comparatively shown in figure 6.

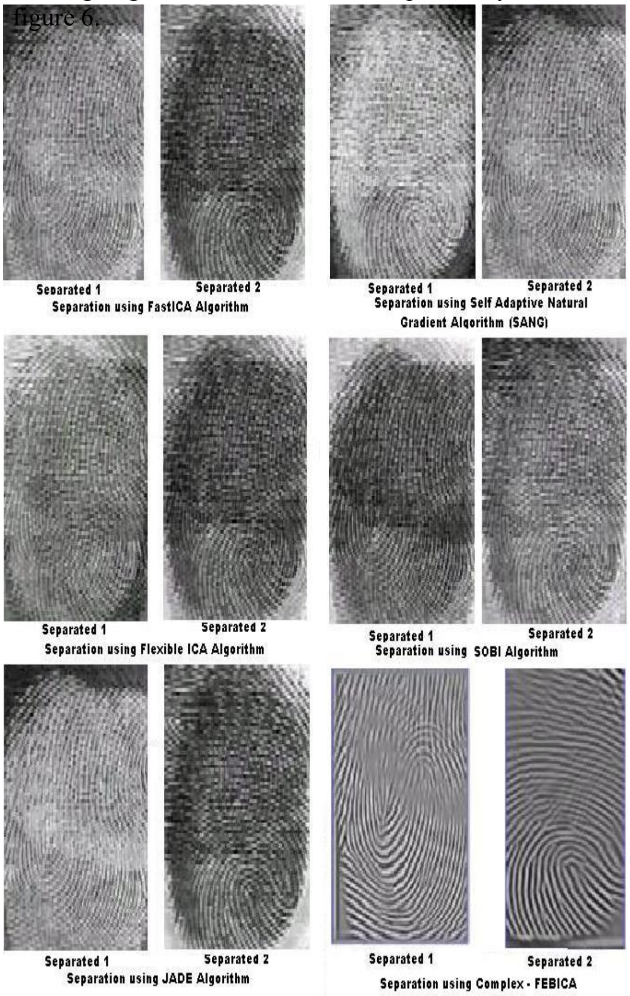

Fig. 5. Real Spot Fingerprints mixture

Fig. 4. Separated Fingerprints using different existing algorithms

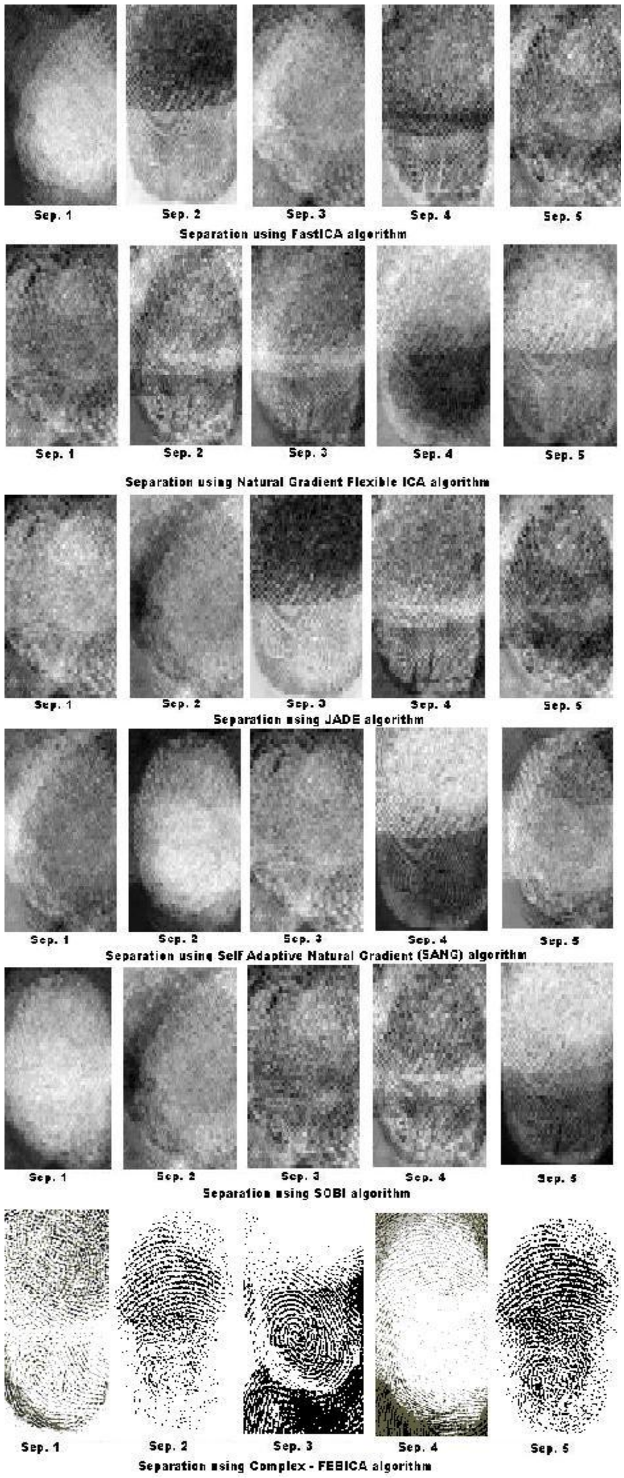

Fig. 6. Five Separated fingerprints from a single image

3. Conclusion

In this paper, the new complex domain based feedback independent component analysis (complex FEBICA) algorithm has been proposed and exciting separation results have been demonstrated. The new area of real - life fingerprints separation has been identified as a target application of ICA for providing analysis in the security areas. Separation of the fingerprints and then matching with the database is of great use. The results reported are just the algorithmic output. Use of some of the filtering techniques can improve the quality of the obtained fingerprints images. The same algorithm is applicable to any image mixture. The reported result demonstrates that this algorithm is having potential to extract the features even when there are less number of sensors than sources. Complex FEBICA can be further enhanced to get better results.

References

[1] V. D. Calhoun, T. Adali, G. D. Pearlson, P. C. M. van Zijl, and, J. J. Pekar. “Independent Component Analysis of FMRI Data in the Complex Domain.” Magn. Res. Med., 2002.

[2] V. D. Calhoun, T. Adali, G. D. Pearlson, and J. J. Pekar. ”On Complex Infomax Applied to Complex FMRI Data.” Proc. ICASSP, Orlando, FL, 2002.

[3] G. Desodt and D. Muller, “Complex ICA Applied to the Separation of Radar Signals,” in Proc. EUSIPCO, Barcelona, Spain, vol. I, pp. 665 - 668, 1990.

[4] E. Bingham and A. Hyvarinen, “A Fast Fixed-Point Algorithm for Independent Component Analysis of Complex- Valued Signals,” Int. J. Neural Syst., vol. 10, pp. 1-8, 2000.

[5] H. Sawada, et al., “A Polar-Coordinate Based Activation Function For Frequency Domain Blind Source Separation,” in Proc. Int. Conf. on Independent Component Analysis and Blind Signal Separation, San Diego, CA. pp. 663-668,2001.

[6] V. Calhoun and T. Adali, “Complex Infomax: convergence and approximation of infomax with complex nonlinearities,” in Proc. NNSP 2002, Martigny, Switzerland, Sept. 2002, pp. 307-316.”

[7] S. Fiori, “Extended hebbian learning for blind separation of complex-valued sources,” IEEE Trans. Circuits Syst. II, vol. 50, NO. 4, PP. 195-202, Apr. 2003.

[8] J. Anemuller, T. Sejnowski, and S. Makeig, “Complex independent component analysis of frequency-domain electroencephalographic data, Neural Networks, vol. 16, no. 9, pp. 13111323, Nov. 2003.

[9] A.J. Bell and T.J.Sejnowski, “An Information Maximization Approach to Blind Separation and Blind deconvolution,” Neural Comput., vol. 7, pp. 1129-1159, 1995.

[10] T. W. Lee, et. al., “A Unifying Information-Theoretic Framework for Independent Component Analysis,”' Int. J. Comp. and Math. with Appl., vol. 31, pp. 1-21, 2000.

[11] F. Neeser and J. Massey, Proper complex random processes with applications to information theory, IEEE Trans. Inform. Theory, vol. 39, no. 4, pp. 12931302, July 1993.

[12] B. Picinbono, Second-order complex random vectors and normal distributions, IEEE Trans. Signal Processing, vol. 44, no. 10, pp. 26372640, Oct. 1996.

[13] P. Schreier and L. Scharf, “Second-order analysis of improper complex random vectors and processes”, IEEE Trans. Signal Processing, vol. 51, no. 3, pp. 714725, Mar. 2003.

[14] B. Picinbono and P. Bondon, “Second-order statistics of complex signals”, IEEE Trans. Signal Processing, vol. 45, no. 2, pp. 411420, Feb. 1997.

[15] P. Amblard, M. Gaeta, and J. Lacoume, Statistics for complex variables and signals Part I: Variables, Signal Processing, vol. 53, pp. 113, 1996.