

New Innovative Technique that Determines Obstacles Remotely: Tsunami Waves, Thanks to the Processing, Analysis and Reconstruction of Two-Dimensional 2D Digital Images, Using Benzzoubzir-Laguerre or Legendree Orthogonal Moments, in Real Time via Server Connection

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Abstract

This article will add to the more developed field of ICT (Information, Communication and Technology) and applied mathematics, new creative approaches applied for the first time to the field of 2D digital image processing. The first invention relates to the creation of a new technique for processing, analyzing, and reconstructing two-dimensional (2D) digital images. Experimental results conclusively demonstrate the effectiveness of this new contribution, characterized by the presence of a large number of orders ($n=p+q$) that yields all reconstructed stages of the digital images, along with a remarkable convergence of reconstruction errors, regardless of the order values used. This contrasts with older methods, where reconstruction or analysis was either impossible or possible only with a limited number of orders ($n=p+q$) and error divergence. The second invention relates to the creation of a new automated process running on a server accessible to multiple computers. Each computer is connected to a camera to signal the presence of tsunami waves, thanks to fast and efficient computer procedures. The field of invention relates more particularly to the processing, analysis, and reconstruction of two-dimensional (2D) digital images using Benzzoubeir-Laguerre or Legendree orthogonal moments in real time, to determine obstacles remotely: tsunami waves, a connection from a server accessible to several computers linked to cameras.

Keywords:

New Innovative Technique, Tsunami Waves, Digital Images, Benzzoubzir-Laguerre, Legendree Orthogonal Moments, Server Connection

1. Introduction

While the orthogonal moments of Legendre [3], Zernike [4] [17], Chebyshev [3], Krawchuk [5], Hahn [6], and Racah [7] have been widely used in 2D image analysis and processing, Laguerre moments, whether discrete or continuous, have never been applied or used with digital images. Therefore, the main objective of this article [1] [2] is to eliminate all the difficulties of image reconstruction using conventional Laguerre moments and to offer, for the first time in the field of information, communication, and

technology (ICT), a powerful and efficient new technique [1] [2] that ensures a completely complete reconstruction of 2D digital images with a high number of orders, accompanied by a remarkable and significant convergence of the reconstruction error rate. Since then, moments and moment functions have been employed as shape descriptors in a variety of applications in image analysis, such as visual pattern identification used by Hu [8], Alt [28], Dudani [9], Wong & Hall [10], Maitra [11] and Hupkens [12]; object classification [13], object recognition using invariant moments [14], object image classification [15], texture classification [16][18]... In parallel, and with regard to the presence of other applications, the reconstruction of digital images has prompted several researchers to improve new mathematical tools and computer algorithms to achieve the desired experimental results. In this regard, Legendre and Zernike moments [25],[19],[24] have been intensively researched in the recent past, and several new techniques have emerged, involving device detectors based on orthogonal moments. Furthermore, the notion of complex moments was introduced by Abu-Mostafa [26] as a simple and straightforward technique for deriving a set of invariant moments. Hu [27] stated that geometric moments are not orthogonal moments, given that the generating basis of monomials is not an orthogonal basis. And to avoid the problem of numerical divergence during reconstruction, Teague [24] proposes a method for reconstructing moments using Zernike invariants for any order. And AbuMustapha [26] introduces another method for extracting invariant moments from complex moments. Also, Khotanzad uses Zernike moments for pattern recognition...

Mukundan [19] provided a fast algorithm for computing the Zernike and Legendre moments. Considerable effort has been devoted to finding efficient algorithms for calculating moments, as demonstrated by L. Yang [20]. Liao's work [21] is based on the reconstruction

of images using various moment calculation methods. Mamistvalov [22] edited the correct version of the fundamental theorem on moment invariants in arbitrary dimensions and showed how to use it to derive N-D affine moments. Teague [23] presented moments with orthogonal functions of the bases, with the additional property of minimal information redundancy in a moment set. Mukundan [3] proposed an application of Chiebechef moments for reconstructing images on an orthogonal basis. He compares the orthogonal moments of Zernike and Legendre with moments based on Chiebechef's hypergeometric functions. Similarly, Mukundan reconstructed the images through these moments, which verified the condition of orthogonality and the condition of Cartesian distribution. Advantageously, in our article (S. Benzoubeir) [1][2], we present for the first time a new and innovative technique for reconstructing 2D digital images using a new and innovative series of Benzoubeir-Laguerre orthogonal moments. These new moments can be effectively used as modeling tools in the analysis, reconstruction, and processing of 2D images. Article [1] also provides mathematical and computational justifications for the creation of these new moments, as well as the reasons for the creation of four mathematical formulas: Benzoubeir-Laguerre orthogonal moments on a continuous and discrete basis, the intensity of the 2D digital image, and the expression and new characteristics of the Benzoubeir-Laguerre orthogonality coefficient.

Article [1] also presents justifications and reasoning for the new naming of the mathematical formulas, as well as the technique for reconstructing digital images using an innovative computational algorithm. Our new technique given in the articles (S.Benzoubeir) [1], [2] completes the need of the various existing applications; and the experimental results conclusively prove the effectiveness of this new technique; that it can be considered as a new descriptor of reconstruction. The renewed interest in the use of moments, as described in articles [1] and [2], stems from the parallel development of new techniques for the processing, analysis, and reconstruction of 2D digital images, which meet the following criteria: Improved digital image quality during reconstruction or analysis; ensured computational stability when the analyzed image size is large; increased reconstruction orders ($n=p+q$); minimized execution time; and minimized reconstruction error regardless of the size of the processed image. Based on the understanding of the

importance of moments in image processing and analysis discussed in previous articles, we can deduce that the processing and analysis of two-dimensional (2D) digital images using orthogonal moments is the most interesting because their use as discriminating attributes allows us to highlight relevant structural data. Furthermore, this family of moments is perfectly suited to representing or reconstructing objects with minimal data redundancy. In all these applications, moments and their extensions radial, complex, orthogonal polynomial, or hypergeometric have played important roles in characterizing the image shape and extracting features that remain constant under the overall transformation of the digital image.

Furthermore, thanks to the reconstruction, analysis, and processing of 2D digital images, numerous applications can be implemented across various high-tech fields—that is, the most advanced technology allowing for a wide range of applications in the most in-demand new technologies, such as the cybersecurity sector: Remote obstacle detection for aircraft and other means of transport; detection of tsunami waves for long-distance signaling, thus saving lives; and medical imaging, for determining the degree of infection in affected organs. Here we have the choice between two reconstruction methods: Legendre or Benzoubeir-Laguerre; we provide the reconstruction order and the error variation.

The remainder of this article is organized according to the following steps: The general definition of continuous and discrete Benzoubeir-Laguerre orthogonal moments and the reconstructed image intensity in sections A, B, and C is described in Section 2. Legendre moments are then defined in a continuous and discrete basis in Section 3. In Section 4, the reconstruction phases are defined, including the narrative appearance phase, the reconstruction phases using the Legendre method and the Benzoubeir-Laguerre method in sections a, b, c, and d, and the divergence phase in B for the 20x20 syntitic image L. The rule of three for determining the height of tsunami waves is presented in Section 5. A flowchart of a new automated process running on a server accessible to computers connected to cameras is defined in Section 6. The case of computers independent of the given server is discussed in Section 7. The overall conclusion is given in Section 8.

2. New Definition of Orthogonal Benzzoubeir-Laguerre Moments

A- Definition of Continuous Orthogonal Benzzoubeir-Laguerre Moments

Theorem1:

Continuous Orthogonal Benzzoubeir-Laguerre Moments as defined by:

$$BL_{pq} = \frac{1}{(c_{(BL)})^2} \iint_D \tilde{L}_p(x) \tilde{L}_q(y) f(x, y) dx dy, \tag{15}$$

$$\tilde{L}_p(x) = e^{-x} L_p(x), \quad \tilde{L}_q(y) = e^{-y} L_q(y), \tag{16}$$

$L_p(x), L_q(y)$: Laguerre polynomials of order (p, q) respectively.

f , Presents the intensity of the pixel coordinates (i, j) ,

$D = [0,1]$, The new field of definition the variable (x,y) .

$c_{(BL)}$: Orthogonal Benzzoubeir-Laguerre coefficient.

Since, the continuous orthogonal moments of Benzzoubeir-Laguerre are defined in finite interval $[0,1]$, then the discretization is realizable.

B- Definition of Discreet Orthogonal Benzzoubeir-Laguerre Moments

Theorem2:

On an image coordinate space $(i, j) \in \{0,1,2,\dots, N-1\}$; the above moment integral has new following discrete approximation in interval $[0,1]$:

$$BL_{pq} = \frac{1}{(N-1)^2 (c_{(BL)_n})^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \tilde{L}_p(x_i) \tilde{L}_q(y_j) f(i, j), \tag{17}$$

$p, q = 0.1.2\dots,$

C- Image reconstructed by orthogonal Benzzoubeir-Laguerre moments

Theorm3: Image reconstructed by the orthogonal Benzzoubeir-Laguerre polynomials in the discrete base can be similarly expressed by:

$$f(i, j) = \sum_{m=0} \sum_{n=0} BL_{mn} \tilde{L}_m\left(\frac{i}{N-1}\right) \tilde{L}_n\left(\frac{j}{N-1}\right), \tag{18}$$

$m, n = 0.1.2\dots,$

3. The Continuous and Discrete Orthogonal Legendre Moments

A-Continuous Orthogonal Legendre Moments

The two most important orthogonal moments that have found several applications in the field of image shape representation are the Legendre moments and the Zernike moments. The Legendre moments of order $(p+q)$ are defined as:

$$\lambda_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^1 \int_{-1}^1 P_p(x) P_q(y) f(x, y) dx dy, \tag{19}$$

$p, q = 0,1,2,3,\dots$

Where $P_n(x)$ is the Legendre polynomial of order n .

B- Discreet Orthogonal Legendre Moments

On an image coordinate space $(i, j) \in \{0.1.2\dots N-1\}$ the above moment integral has the following discrete approximation:

$$\lambda_{pq} = \frac{(2p+1)(2q+1)}{(N-1)^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} P_p\left(\frac{2i-N+1}{N-1}\right) \times P_q\left(\frac{2j-N+1}{N-1}\right) f(i, j) \tag{20}$$

The inverse moment transform which follows from the orthogonal of Legendre polynomials in the continuous domain, can be similarly expressed with:

$$f(i, j) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \lambda_{mn} P_m\left(\frac{2i-N+1}{N-1}\right) P_n\left(\frac{2j-N+1}{N-1}\right), \tag{21}$$

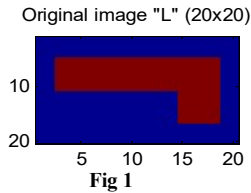
$i, j = 0,1,2,3,\dots, N-1.$

4. The Reconstruction Phases

Overall, there are 8 phases of reconstruction: Initiation phase, beginning of reconstruction phase, centered reduced phase, dilation phase, contour emergence phase, exact reconstruction phase, exact reconstruction phase, divergence phase; of interest to us are the contour emergence phase, the exact reconstruction phase, and the divergence phase.

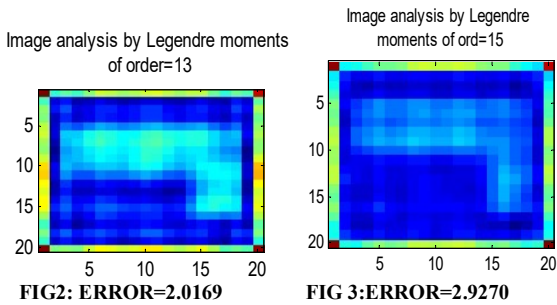
A. Definition of the contour appearance phase, the exact reconstruction phase, and the divergence phase

The contour emergence phase and the exact reconstruction phase are two phases that provide all the image information, with a significant number of reconstruction orders, a convergent error variation, and minimal execution time. The divergence phase results in the complete absence of all image information, accompanied by error divergence, except for reconstruction using Benzoubeir-Laguerre moments, where the error is convergent with a higher order. We performed experiments on the image L with dimensions 20x20.



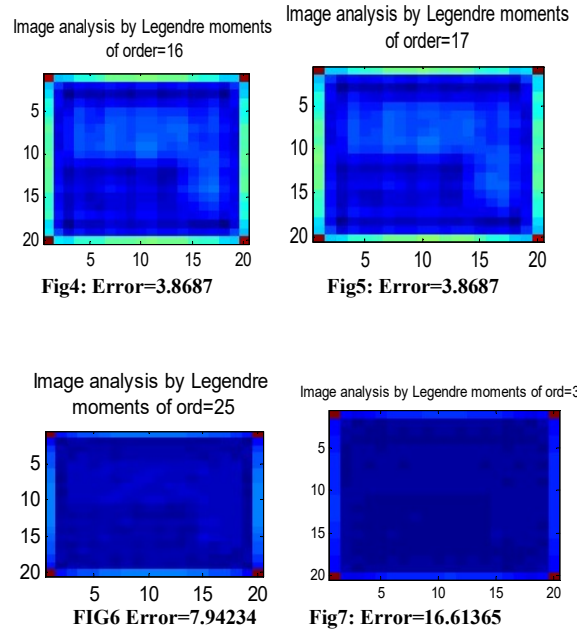
a. the contour appearance phase by Legendre

When the order varies in the set $\{12...15\}$, the error varies exclusively between 2.01et 2.93, and the images processed according to the set of orders are:



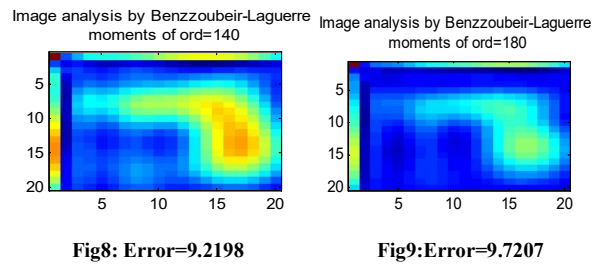
b .Exact reconstruction phase by Legendre

$$order_{Leg} \in \{16....30\} \Rightarrow Error_{Leg} \in [3.8687, 16.6136]$$



c.The contour appearance phase by Benzoubeir-Laguerre

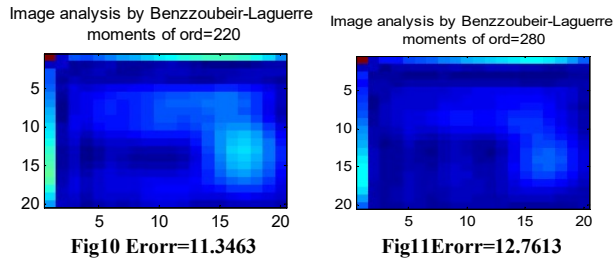
$$order_{Ben-Lag} \in \{140...180\} \Rightarrow Error_{Ben-Lag} \in [9.2198, 9.7207]$$



The order varies between 140 and 180; the error varies strictly in the vicinity of 9. And the dark color begins to decrease, to give the overall structure of the outline of the letter "L".

d. Exact reconstruction phase by Benzoubeir-Laguerre

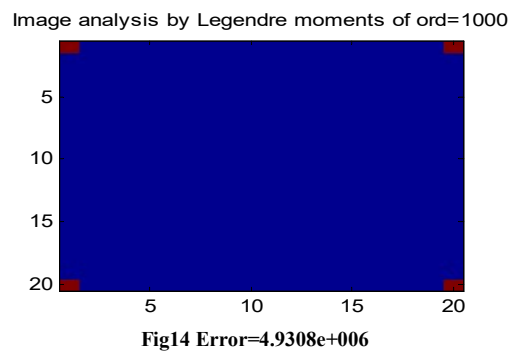
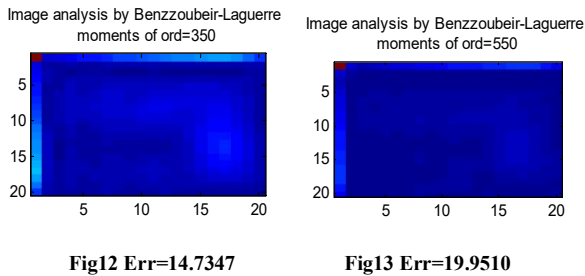
$$order_{Ben-Lag} \in \{220 \dots 550\} \Rightarrow Error_{Ben-Lag} \in [11.3463 ; 19.9510]$$



$$order_{Leg} \in \{60..100\} \Rightarrow Error_{Leg} \in [129.0905, 58.705]$$

$$order_{Leg} \geq 100 \Rightarrow (Error_{Leg} \rightarrow \text{diverge})$$

All values of the order strictly greater than 100; the error diverges sharply with a transmutation of $10^3, 10^6$.



Here we observed that the luminous color was diminished until its total disappearance, subsequently giving an exact and complete structure to the geometric shape of the letter "L". Even the appearance of the blue color in the background of the overall image is identical to that of the initial image. Always with an error that varies precisely in the vicinity of the value 11, for any order varying between 220 and 240. The error varies in the vicinity of the value 12 for any order, ranging between 250 and 280. Finally, the error varies in the vicinity of the value 13, for all orders varying between 290... 300. In the overall range $\{350...550\}$, the error varies successively within the interval $[14.7347, 19.9510]$.

b. Benzoubeir-Laguerre's method

In this phase, there is a total absence of all information from the letter "L", without any error discrepancy.

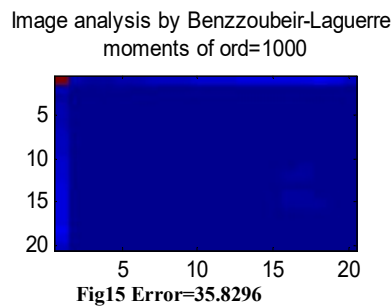
$$order_{Ben-Lag} \leq 1000 \Rightarrow \begin{cases} Error_{Ben-Lag} \leq 35.8296 \\ Error_{Ben-Lag} \mapsto \text{dons not diverge} \end{cases}$$

A. Divergence phase

a. Legendre's method

In this case, there is a complete absence of all the information associated with the letter "L". And the error variation is composed of 3 notable classes which are:

$$order_{Leg} \in \{42...50\} \Rightarrow Error_{Leg} \in [40.2534, 65.7048]$$



The invention relates to the creation of an innovative method for processing and analyzing two-dimensional (2D) digital images using novel orthogonal Benzoubeir-Laguerre or Legendre moments. These new techniques address the computational needs of various

existing applications. Experimental results remarkably demonstrate the effectiveness of this technique, which can be considered a novel device descriptor. This property makes Benzoubeir-Laguerre and Legendre orthogonal moments superior to conventional moments such as geometric moments and Zernike moments. Experimental results also demonstrate their representational and reconstruction capabilities. Overall, the goal of the invention is to obtain experimental results that verify the following properties:

- Minimize execution time during computation.
- Ensure computational stability when dealing with large image sizes.
- Minimize the reconstructed error rate.

Remarque: The purpose of the invention is to ensure stability when the image size is large, regardless of its nature: synthetic or real, knowing that the real image contains normal waves, will be processed and reconstructed according to the reconstruction phases: like the image "L"; until the image containing tsunami waves is captured, in the actual application domain.

5. The Rule of Three for Determining the Height of Tsunami Waves

To determine the placement of computer-linked cameras to capture images containing both normal waves and the higher-height waves of the tsunami, the rule of three is used: Given the height (h) and normal extension (d) of a normal wave, and if the extension (D) of the tsunami waves is known, considering the impact on people and other activities, then, according to the rule, we deduce:

$$H = \left(\frac{h \times D}{d} \right)$$

6. Definition of A Flowchart for A New Automated Process Running on A Server Accessible to Computers Connected to Cameras

The process of processing, analyzing and reconstructing a two-dimensional 2D digital image, using the source image, follows these steps:

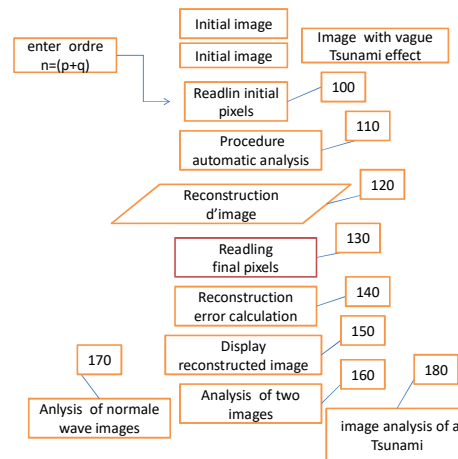


Fig 16: Flowchart of a new, fast, automated process running on a server accessible to computers connected to cameras

The said process (110) uses a new, powerful, and fast technique to reconstruct the said source image given in step (120). According to mode (150), the process displays said analyzed image, specifying the reconstruction rate according to step (140). According to mode (160), the process analyzes the two reconstructed images (source image and final image); and if the reconstructed image is identical to the initial reconstructed source image, then there is no tsunami phenomenon, according to mode (170). Conversely, according to mode (180), the process signals the presence of the tsunami. The signal is equivalent to the display of the address and phone number; It is perceived by monitoring personnel who, through their actions, signal the existence of the tsunami, using alarm bells or other means, in order to save lives; illustrate this in the flowchart of the fig16. The server continues its execution until the tsunami waves are captured.

Creation of a new automated procedure based on fast algorithms that meet new quality standards for the analysis and reconstruction of 2D images. This procedure analyzes and reconstructs 2D images in less time, with high reconstruction quality and error of the reconstruction order values.

The second invention describes the process for processing, analyzing, and reconstructing two-dimensional 2D digital images through the creation of a new automated process running on a server accessible to multiple computers connected to cameras, in order to coordinate access to the process, as shown in the fig17.



Fig17: Computers connected to the camera depend the server

The purpose of the invention is to provide a connection between different computers, to meet the needs of any application required: the absence or presence of the Tsunami phenomenon. The invention relates to the creation of a new and innovative technique for determining remote obstacles: computer security, through the processing, analysis, and reconstruction of 2D digital images using Benzoubeir-Laguerre or Legendre orthogonal moments, to determine remote obstacles: tsunami waves, at the level of coastal cities, i.e., the presence of hotels, housing, and other activities.

Another invention concerns the creation of a new automated process, running on a server accessible to multiple computers, each connected to a camera, to signal the presence of tsunami waves using fast and efficient computer procedures. The invention aims to remotely identify obstacles to tsunami waves and allows any application to run with minimal time and high reconstruction quality for various types of 2D digital images.

7. Case of Computers Independent of the Server

Independent of the server, a camera connected to a computer uses fast computer procedures to reconstruct 2D digital images. and compare with images containing normal waves, until the image containing the tsunami waves is captured. And it signals the stoppage of compilation and the display of the address and telephone number, as seen by the surveillance personnel. who declare the existence of the Tsunami phenomenon by means of doorbells or other means, in order to save people's lives; fig18



Fig18: computer linked to the camera independent of the server

8. Conclusion

The purpose of the invention is to utilize a powerful technique to facilitate and meet any application required, in real time and much shorter; i.e., fast and efficient new computer programming procedures. And the technique of orthogonal moment analysis plays a very important role in various applications, in the field of artificial intelligence and computer science. The fundamental idea of this theory is the projection of the data space onto a completely orthogonal basis, in order to extract useful information from the image. the latter is extracted in the form of projection coefficients called moments; the orthogonal moments of Benzoubeir-Laguerre or Legendre remain, which are important. Furthermore, the invention proposes a new technique for automatic and real-time processing, analysis and reconstruction, using a connection associated with a server, to analyze, process and reconstruct 2D two-dimensional digital images in real time, to deduce the information requested: Tsunami effect. The process relates more specifically to the processing, analysis, and reconstruction of two-dimensional (2D) digital images using Benzoubeir-Laguerre or Legendre orthogonal moments in real time, to determine tsunami waves. This involves connecting a server, accessible to multiple computers linked to the cameras.

The invention allows for several applications across various high-tech fields, i.e., the most developed technology. In the field of computer security: Determining remote obstacles for aircraft and also some means of transport... another sector is the medical imaging sector for determining the degrees of infection for infected organs, with a higher order, and error convergence; and other applications in applied mathematics, and image processing 2D and 3D.

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