Numerical Study of Turbulence Kinetic Energy Budget in Accelerated Channel Flow

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Abstract
Turbulence kinetic energy consisted of three terms: Production, Diffusion and Dissipation which are always in interaction with each other. In this article the budgets of different terms are considered in turbulent accelerated flows in channels. A specific code is used, in which the turbulence model is employed. The channel has 2 heights respectively. The Reynolds accelerating flow varies from 7000 to 45200. The study occurs under various number of accelerations at various distances from channel wall. It was found that the budget of production and dissipation are higher compared with the other term near wall region and the budget of production and dissipation have similar trends near wall region. Further it was found that accelerated flow makes a lag time in the increase of different terms in the way which it strongly depends on the amount of exerted acceleration and distance from channel wall.
Keywords
Turbulence Kinetic energy; Accelerated Flow; Energy Budget; Channel Flow

1. Introduction

Internal flows of fluids have possessed main part of Fluid Mechanics Science. Today, use of these flows in industry is clear to everyone. It could be claimed that major part of flows in practical works are turbulent flows. In many practical uses, it is necessary for the fluid to pass through a path by pipe or channel and to move from a point to another. As it was mentioned, except for a few cases, the flow inside these sections has turbulent nature. One of the main properties of turbulent flow to interprets and describe turbulence phenomenon is turbulence kinetic energy. Turbulence kinetic energy has some components called as turbulence kinetic energy terms. These terms include production, diffusion and dissipation terms, which are introduced in rest of this paper. In a turbulent flow, these three terms of turbulence kinetic energy (TKE) are constantly in interaction with each other. In another division, fluid flows can be divided to steady and unsteady fluid flows. In steady flows, fluid properties and parameters are fixed in time scope and are not changed; although despite this is true in unsteady flows. One of the most important parameters in fluid is the momentum. If momentum of fluid flow is changed in time scope, the flow is called accelerated flow. Accelerated flows can have accelerating or slowing or pulse momentum. Although analyzing steady and unsteady internal turbulent flows has been topic of many articles, analysis of portion of different terms of TKE in accelerated flow with accelerating momentum in channel has been studied by no research. Here, the aim by accelerated flow is such flow, in which the momentum of fluid from a section is increased over the time. Airflow inside a channel while passing through a convergent section or while starting a fan is an example of this type of turbulent and unsteady flow.

Empirical study of turbulent and accelerated flow has been presented by a few scholars. He and Jackson [1] have performed some experiments for turbulent and accelerated flow inside pipe and published the results related to different radiuses of pipe. They announced relatively complete results with almost all required properties of fluid to achieve other fluid properties. An interesting and unique feature in response to applied momentum of flow is related to different time lags: a lag in turbulence production response, a lag in redistribution of TKE among its three components and a lag associated with turbulence radial diffusion. As a result of these 3 lags, intensity of turbulence is decreased in accelerated flow [1]. Kurokawa and Morikawa [2] conducted experimental investigations in field of in-pipe accelerated flow. In this experiment, 5 different amounts are investigated for increasing and decreasing momentum. The momentums were controlled through ball valve opening and closing by electric motor. As a conclusion, Kurokawa and Morikawa found that firstly, not only the pattern and behavior of flow are different between increasing and decreasing momentum, but also behavior of flow with high increasing momentum and flow with low increasing momentum are different. Under conditions that there is large increasing momentum compared to viscosity effects, the thickness of boundary layer (BL) is low and the viscosity effects are less distributed in the flow and this means high critical Reynolds. Secondly, under condition of different decreasing momentums, the profile of average velocity is lower than increasing momentum. During entire time of applying momentum, regime and pattern of flow is not changed and if it is turbulent, it can't be laminar. Thirdly, in flows with increasing momentum, critical Reynolds with the square is suitable momentum; it means that a little increase in momentum can increase critical Reynolds significantly [3]. In another experimental study, Nishihara, Nakahata,

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Kinsley and Sasaki [3] have studied effect of accelerated flow on history of passage of laminar to turbulent flow. In this study, they have presented effect of increasing momentum on history of transfer from laminar to turbulent flow for 3 different profiles of velocity. In this project, lag is called and analyzed as passage from laminar to turbulent flow. The results obtained from the study showed that time lag in transfer of laminar to turbulent flow are related to history of applied momentum. [3]

In addition to different empirical studies, some scholars have used computer simulation models. The computer models are generally divided to 3 groups including zero-equation models, single-equation models and two-equation models. In field of investigating pulse flow that is a special type of accelerated flow, Adamovsky and Levandowski [4] presented numerical models for unsteady friction coefficient in describing water hammer phenomenon using their laboratory model. Adamovsky and Levandowski obtained computer outputs for several different Reynolds and compared the results with empirical data and found that water hummer phenomenon could be predicted using unsteady friction models significantly better than models based on steady flows. Then on 2008, new ideas were presented to be used in unsteady friction model by a researcher called Ayoptov [5]. In field of defining the ideas, Ayoptov has tried to present new model in such manner that take less space of computer memory in addition to improve existing model. He has mentioned that one of the advantages of this promoted model is considering difference between solution and simulation of flows with increasing and decreasing momentum [5]. Anus, Koppel and Verdi [6] have also used an experimental method to investigate flow transfer from laminar to turbulent in accelerated in-pipe flow. Se Yun Jang and Chung [7] have simulated in-pipe accelerated flow with rounded section numerically. Green Blot and Mows [8] and Scotti and Paiomli [9] conducted experimental and numerical studies in this field using Laser Doppler anemometers. On the other hand, scholars like Shamar and Vygansy [10] used zero-equation turbulent numerical model; Tow and Rmparyan [11] used single-equation turbulent model and Blodiwex and Colombini [12] used two-equation turbulence model. Lam and Berm Horst [13] also used k-ε model to predict turbulence near the wall. Then, Khaleghi, Pasandidefard, Malek Jafarian and Chung [14] tested different numerical turbulence models and used findings of He and Jackson to evaluate accuracy of the data.

Petasinsky et al [15] investigated effect of presence of polymers in fluid turbulent flow inside channel using direct numerical solution (DNS). According to their results, presence of polymer in turbulent flow inside channel can decrease drag coefficient intensely. In this regard, they presented an analysis of changes in portion of different terms of TKE (especially production term) and found that a part of production of TKE could be dissipated by the property of polymers in fluid flow [15]. In another study, Mansur, Kim and Moyen [16] used DNS and tried to obtain dissipation amount of TKE inside channel using low Reynolds values. According to their analysis of portion of different terms of TKE in channel, it was found that changes in portions of these terms of TKE is more tangible near the wall compared to other parts of channel and portion of different terms of TKE near the wall are mostly sensitive to changes in Reynolds values [16]. Moreover, according to their results, portion of production and dissipation terms of TKE in turbulent flows in channel is dominant and portion of production term is significant near the wall [16]. In another interesting study conducted by Matic Suavely [17] using Reynolds method on 2012, behavior of turbulent flows in atmosphere is investigated. According to the results obtained from the study, this time, similar to other times, TKE was significant near ground surface. In the study, it was found that in field of measurement of turbulent atmosphere flows, the first important portion of TKE is production term and after that, dissipation term was in next position, since TKE dissipation helps atmosphere vortices get smaller [17]. On the other hand, Khaleghi [18], in PhD thesis on 2011, has investigated the length of extension in turbulent accelerated flow based on 4 main specifications of flow, TKE, turbulence stress and turbulence viscosity. In the study, the important point was that in unsteady turbulent flows, relevant equations of extension length in steady flow and the length is increased to some extent and is a function of applied momentum on the flow. [18]

At the present study, portion of different terms of TKE in turbulent and accelerated flow in channel is investigated. This study is conducted using a computer program. As the main nature of studied topic is related to turbulent flows, the first issue competent to be explained here is applied turbulent model. In the computer program, applied turbulent model is enhanced model of k-ε-v^2. The second important point competent to be referred here is applied geometry. In figure 1, studied geometry is illustrated.

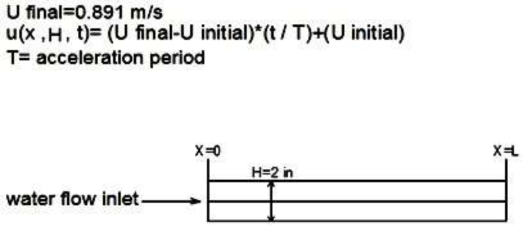

Figure 1: geometry of solution scope

The studied geometry is a channel with height of 2.54cm (2 inches). Length of channel is larger than section dimensions. Velocity of fluid input to channel in certain duration (5, 20, 40, 50 and 100sec) in a section with 3m from inlet span reaches to 0.891m/sec from 0.138m/sec. The certain duration is called acceleration period after this. According to the definition, the lower the acceleration period is, it means that the fluid velocity has reached its final velocity in shorter period and hence, applied momentum has been in higher level. It should be noted that in this study, the focus is just on this section in 3-m distance from inlet span and completely extended flow.

2. The Governing Equations

In this part, the governing equations studied on geometry, turbulence model and boundary conditions are considered. First, the basis and logic of solution is explained and then, the governing equations and boundary conditions of solution are explained in details.
To explain solution logics, consider eq.1; this equation shows the general form of momentum equation.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{1}{r^j}\frac{\partial}{\partial r}\left(r^j(\nu + \nu_t)\frac{\partial u}{\partial r}\right) \tag{1}$$

In this equation, if exponent j is considered to 1, momentum equation is solved in cylindrical system and if the exponent is equal to 0, the fluid momentum equation is solved for Cartesian coordinate system (channel flow). As the studied geometry is a channel, the exponent j will be considered to zero. On the other hand, the solution is implemented in a section of channel, at which the flow is fully developed. With such action, obstacle of changes in flow properties in x direction is removed. If φ refers to any turbulent or fluid properties, for any section of channel in fully developed region, eq.2 can be considered.

$$\frac{\partial \varphi}{\partial x} = 0 \tag{2}$$

As no dominant phenomenon is observed in angular direction in practice channel flows, hence:

$$\frac{\partial \varphi}{\partial \theta} = 0 \tag{3}$$

According to the above descriptions, it could be found that the basis of numerical solution is typically on one-dimensional solution. Although turbulent phenomenon has 3-D nature by itself and includes fluctuating velocities of u', v', w', with this explanation, one-dimensional solution can give satisfying results. As 3-D and even 2-D solution is time-consuming, using 1-D program is sufficient [14]. In other words, although turbulence phenomenon is a 3-D phenomenon, the results of 1-D solution can also confirm empirical data like time lag, mean flow velocity and different terms of TKE and there is no necessity to use 3-D analysis. That's means, by using $k-\varepsilon-v^2$ turbulence model, naturally 3-D phenomenon of turbulence can be modeled in form of a 1-D phenomenon. With this introduction, the governing equations will be presented as follow. Following equations are not-dimensional form of all parameters involved in equations.

$$u^+ = \frac{u}{u_\tau}\ , x^+ = \frac{u_\tau}{\nu}x\ , r^+ = \frac{u_\tau}{\nu}r\ , p^+ = \frac{p}{\rho u_\tau^2}\ , \quad t^+ = \frac{{u_\tau}^2}{\nu}t\ , \quad \nu^+ = \frac{\nu_t}{\nu} \tag{4}$$

Now, according to eq.4, total momentum equation (eq.1) becomes dimensionless.

$$\frac{\partial u^+}{\partial t^+} = -\frac{dp^+}{dx^+} + \frac{\partial}{\partial r^+}\left(\left(1 + {\nu_t}^+\right)\frac{\partial u^+}{\partial r^+}\right) \tag{5}$$

On the other hand, the wall shear stress according to the flow regime is as eq.6.

$$(j+1)\frac{\tau_w}{R} = -\frac{dp}{dx} \tag{6}$$

In eq.6, if j is considered to 1, equation is solved in cylindrical system and if considered to 0, the equation is solved for Cartesian coordinate. According to eq.4 the flowing equation can be considered:

$$-\frac{dp}{dx} = -\frac{d(\rho u_\tau^2 p^+)}{(x^+ \cdot \frac{\nu}{u_\tau})} = -\frac{\rho u_\tau^3}{\nu}\frac{dp^+}{dx^+} \tag{7}$$

By combining eq.6 and eq.7:

$$(j+1)\frac{\tau_w}{R} = \frac{\rho u_\tau^3}{\nu}\frac{dp^+}{dx^+} \tag{8}$$

According to the definition of friction velocity (eq.9), the friction Re (eq.10) can be extracted as follow:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \Rightarrow \tau_w = \rho u_\tau^2 \tag{9}$$

$$Re_\tau = \frac{u_\tau}{\nu}R \tag{10}$$

By substituting j=0 for Cartesian coordinate system (channel flow) and using eq.9 and eq.10; eq.8 will be as follow:

$$\frac{\tau_w}{R} = \frac{\rho u_\tau^3}{\nu}\frac{dp^+}{dx^+} \tag{11}$$

$$\frac{\rho u_\tau^2}{R} = \frac{\rho u_\tau^3}{\nu}\frac{dp^+}{dx^+} \tag{12}$$

$$\frac{R u_\tau}{\nu} = \frac{dp^+}{dx^+} \tag{13}$$

$$\frac{1}{Re_\tau} = \frac{dp^+}{dx^+} \tag{14}$$

Therefore:

$$\frac{\partial u^+}{\partial t^+} = \frac{1}{Re_\tau} + \frac{\partial}{\partial r^+}\left(\left(1 + {\nu_t}^+\right)\frac{\partial u^+}{\partial r^+}\right) \tag{15}$$

Eq.15 is the first equation which has to be discretized for numerical solution of the studied geometry. In this study a modification of $k-\varepsilon-v^2$ turbulence model is employed. $k-\varepsilon-v^2$ model is enhanced version of standard model $k-\varepsilon$. This model can enable systematic boundary conditions. No need to attenuation functions is one advantage of this model. Turbulent viscosity is as follows:

$$\nu_t^+ = C_\mu {\mathrm{v}^2}^+ T \tag{16}$$

$k-\varepsilon-v^2$ turbulence model is consisted of 3 main equations; k and ε and v^2 which are reported as follow in dimensionless form.

- k^+ is referred to turbulence kinetics energy which is as eq.17:

$$\frac{\partial k^+}{\partial t^+}=P^+-\varepsilon^++\frac{\partial}{\partial y^+}\left[\left(1+\frac{v_t^+}{\sigma_k}\right)\frac{\partial k^+}{\partial y^+}\right] \tag{17}$$

Where:

$$Production\ term: P^+=v_t^+\left(\frac{\partial u^+}{\partial y^+}\right)^2 \tag{18}$$

$$Diffusion\ term: \frac{\partial}{\partial y^+}\left[\left(1+\frac{v_t^+}{\sigma_k}\right)\frac{\partial k^+}{\partial y^+}\right] \tag{19}$$

- ε^+ is referred to dissipation term and is the second equation in $k-\varepsilon-v^2$ turbulence model , which is as follow:

$$\frac{\partial \varepsilon^+}{\partial t^+}=C_{\varepsilon 1}^{*+}\frac{P^+}{T^+}-C_{\varepsilon 2}\frac{\varepsilon^+}{T^+}+\frac{\partial}{\partial y^+}\left[\left(1+\frac{v_t^+}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon^+}{\partial y^+}\right] \tag{20}$$

- ${\mathrm{v}^2}^+$ is the third $k-\varepsilon-v^2$ turbulence model equation:

$$\frac{\partial {\mathrm{v}^2}^+}{\partial t^+}=k^++{f_{22}}^+-{\mathrm{v}^2}^+\frac{\varepsilon^+}{k^+}+\frac{\partial}{\partial y^+}[\left(1+\frac{{v_t}^+}{\sigma_k}\right)\frac{\partial {\mathrm{v}^2}^+}{\partial y^+}] \tag{21}$$

Where:

$$T^+=max\left[\frac{k^+}{\varepsilon^+},6\left(\frac{1}{\varepsilon^+}\right)^{1/2}\right] \tag{22}$$

$$L^{+2}\frac{\partial^2 f_{22}^+}{\partial y^{+2}}-\partial^2 f_{22}^+=(1-C_1)\frac{\left(\frac{2}{3}-\frac{v^{+2}}{K^+}\right)}{T^+}-C_2\frac{P^+}{K^+} \tag{23}$$

Table 1: constants used in $k-\varepsilon-v^2$ model

C_1	C_2	C_L	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_μ	C_η	σ_k	σ_ε	a_1
1.3	0.3	0.2	1.44	1.9	0.3	90	0.9	1.3	0.1

$$c_{\varepsilon 1}^{*+}=c_{\varepsilon 1}(1+a_1\frac{p^+}{\varepsilon^+}) \tag{24}$$

$$L^+=C_L\max[\frac{k^{+\frac{3}{2}}}{\varepsilon^+},C_\eta\left(\frac{1}{\varepsilon^+}\right)^{\frac{1}{4}}]$$
(25)

$$f_{22}^+=\frac{v}{u_\tau^2}f_{22} \tag{26}$$

$$\mathrm{v}^{+2}=\frac{\mathrm{v}^2}{u_\tau^2} \tag{27}$$

Table 1 presents constants in this model. Eq.17, eq.20 and eq.21 are three next equations which have to discretized for numerical solution. Therefore, the numerical solution basis is based on discretizing and solving 4 main equations;

- Momentum equation
- k^+ equation
- ε^+ equation
- ${\mathrm{v}^2}^+$ equation

Khaleghi has tried to make some changes in $\mathrm{k}-\varepsilon-\mathrm{v}^2$ model in order to obtain a modification of this model. He put the modification strategy basis on recognizing the effect of changing parameters weighted coefficients and comparing the difference between calculated values and lab results with difference between values in standard model and lab results. The parameters existed in equations include k ، ε ، v^2 and f_{22}, which can be used for modification of model. Through changing each parameter and comparing their results with empirical results, it could be found that making changes in parameter ${\mathrm{v}^+}^2$ can shows more appropriate and close to lab results for

TKE. Hence, it seems that the first option to modify $\mathrm{k}-\varepsilon-\mathrm{v}^2$ model is making changes in parameter ${\mathrm{v}^+}^2$. Hence, through making some changes in the said parameter, a modified version of this model could be obtained [18]. Therefore, the relevant model was modified as follows:

$$\frac{\partial \mathrm{k}^+}{\partial \mathrm{t}^+}=\mathrm{p}^+-\varepsilon^++\frac{\partial}{\partial \mathrm{y}^+}\left[\left(1+\frac{\upsilon_\mathrm{t}^+}{\sigma_\mathrm{k}}\right)\frac{\partial \mathrm{k}^+}{\partial \mathrm{y}^+}\right]-\mathrm{C_K}\frac{\partial \mathrm{v}^{2+}}{\partial \mathrm{t}^+} \tag{28}$$

Where $\mathrm{C_K}$ is a constant which had been obtained from comparing the empirical results with the modified turbulence model results, by testing different values for $\mathrm{C_K}$. According to Khaleghi's investigations the best value for the mentioned constant in order to best fit to empirical results is $\mathrm{C_K}=0.6$ [18] In order to solve partial derivatives using Finite Difference Method, Crank - Nicholson Method is applied. This method has high second-degree accuracy for time and place. Crank-Nicholson method is used to solve trigonometric equations obtained from three diagonal matrix approach (TDMA). As Non-uniform grid (250 points along Height with coefficient of expansion to 1.02) is used, h direction gridding is used for one-dimensional condition. Boundary conditions are as follows:

$$\mathrm{h}=0 \qquad \emptyset=0 \tag{29}$$

$$\mathrm{h}={}^{\mathrm{H}}/_{2} \qquad \frac{\partial\emptyset}{\partial \mathrm{y}}=0 \tag{30}$$

Equations 29 and 30 are defined for $\emptyset=\mathrm{u},\mathrm{v},\mathrm{u}',\mathrm{v}',\mathrm{u}'\mathrm{v}',\ldots$. Input velocity to studied section is obtained as follow:

$$u(x,r,t)=(u_f-u_i)\times{}^{t}/_{T}+u_i \tag{31}$$

In eq.31, " T " refers to acceleration time to the flow and u_i refers to initial velocity to 0.138m/sec and u_f refers to ultimate velocity to 0.891m/sec. As it was mentioned, the less the acceleration time is, it means that fluid velocity has reached its ultimate velocity in shorter time. Hence, the momentum applied on flow has been higher. Accordingly, the more the acceleration time is, the less applied momentum on flow would be. As the solution of turbulence model used here is based on $k-\varepsilon-v^2$ model, boundary conditions for this model are also reported as follows:

$$y^+ = 0 \quad k^+ = 0, \quad \varepsilon^+ = 2\frac{k^+}{y^{+2}}, \quad v^{+2} = 0, \quad f_{22}^+ = 20\,\frac{v^{+2}}{\varepsilon^+ y^{+4}} \qquad (32)$$

$$y^+ = H/2 \qquad \frac{\partial k^+}{\partial y^+} = \frac{\partial \varepsilon^+}{\partial y^+} = \frac{\partial v^{+2}}{\partial y^+} = \frac{\partial f_{22}^+}{\partial y^+} = 0 \qquad (33)$$

3. Validation

First, accuracy of the data obtained from solution for total TKE in accelerated mode in pipe is validated and is tested using the data obtained from He and Jackson's examinations [1]. In order to be able to compare the obtained results, the studied geometry should be in consistence with desired geometry by this study for validation purpose. Hence, flow velocity in the pipe and channel is increased from 0.139 to 0.891m/sec. During the acceleration, different ranges of momentum could be applied on flow. Accordingly, during the acceleration period of 5sec, the most momentum is applied and in acceleration period of 100sec, the lowest level of momentum is applied on flow. Calculating TKE terms is done in equations 17, 20 and 21 in section of governing equations. Figure 2, 3 and 4 illustrate changes in total TKE for acceleration time of 5sec in 3 different radiuses in the pipe. In each diagram, the relevant results of empirical data and the data obtained from $k-\varepsilon-v^2$ turbulence model and the enhanced $k-\varepsilon-v^2$ model are presented.

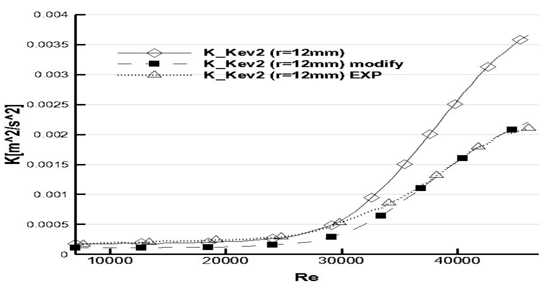

Figure 2: comparing TKE level of 3 methods for 12mm radius for acceleration time of 5sec in pipe

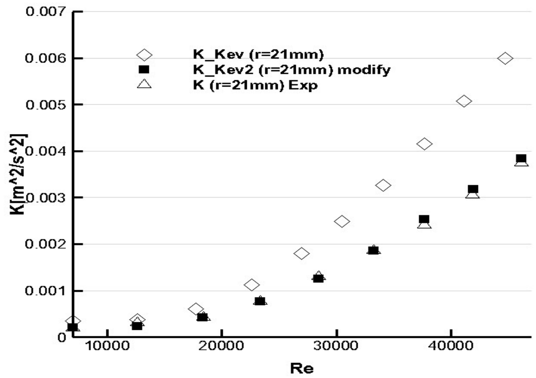

Figure 3: comparing TKE level of 3 methods for 21mm radius for acceleration time of 5sec in pipe

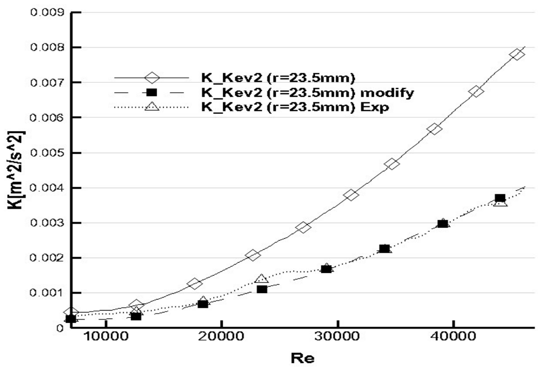

Figure 4: comparing TKE level of 3 methods for 23.5mm radius for acceleration time of 5sec in pipe

The first point found from these figures is that there is good consistency between the results obtained from model solution using modified $k-\varepsilon-v^2$ model with the results obtained from He and Jackson experiments on in-pipe flow. The insignificant difference between them is also because the pipe in real 3-D state in vitro. It means that in addition to motion in desired radius, the particle can experience any kind of rounded or other mode of motion allowed by real 3-D pipe. Same issue is true for channel. The particle in a real and 3-D channel can have any mode of motion in rectangular section of channel. For example, for the pipe, particles can just move along the radius and for channel, these particles can just move along Height of channel section. The difference in nature of real flow and simulated flow can cause a little difference in numerical answers or in vitro results. This issue is also assessed in section of results

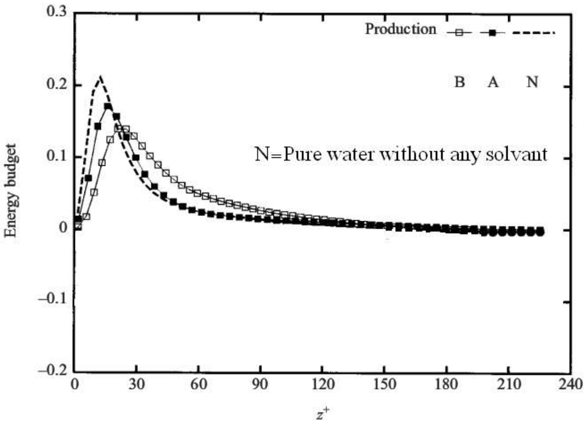

Figure 5: portion of production term of TKE in channel in 5500 Reynolds value. The mode N is for pure water fluid with no soluble and modes A and B are for water fluid containing soluble with different concentrations [15].

In diagram in figure 5, portion of production term of TKE in channel with longer length than channel section dimensions in a turbulent flow is illustrated. Petasinsky et al [15] have investigated effect of existence of polymer soluble in the flow in channel using direct numerical solution (DNS) and using FENE-P model and have also assessed portion of production term of TKE in a flow free from any kind of solvent in channel. The diagrams are named as A, B and C. The diagram of mode N is related to conditions that water fluid is free from any kind of impurity and A and B modes for water fluid contain soluble matter with different concentrations. Hence, the basis of comparison and validation is on diagram N.

Figure 6 illustrates portion of production term of TKE in fluid flow in 9500 Reynolds value in steady flow, which is based on the program arranged based on enhanced model. Comparing figure 5 with figure 6 shows consistency of the two diagrams for production term as well.

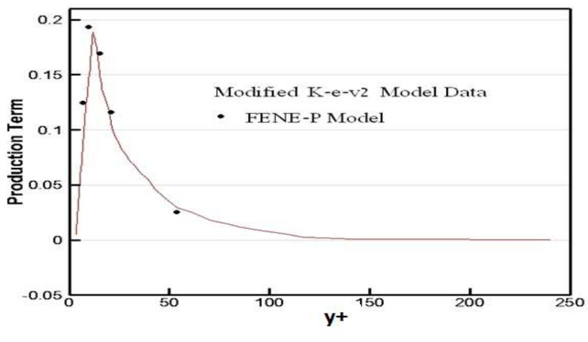

Figure 6: portion of production terms of TKE in channel in 9500 Reynolds value using solution with enhanced model $k-\varepsilon-v^2$

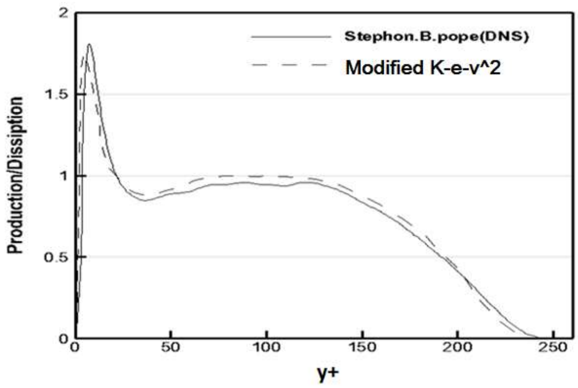

Figure 7: comparing diagram of production term to dissipation term ratio in Reynolds value of 13600

In the diagram in figure 7, the comparison of production term to dissipation term ratio in Reynolds value of 13600 is illustrated. The first diagram is presented by Stephen Pop [19] in his book "Turbulent Flows", which has calculated production to dissipation term ratio in a 3-D channel using DNS in steady state and with Reynolds to 13600 [19]. In second diagram, modified $k-\varepsilon-v^2$ model is used to obtain production to dissipation term ratio in same Reynolds level. Good consistency is observed between two diagrams. Therefore, these results for unsteady state in pipe, report total TKE accurately. On the other hand, in steady state for channel, production term and production to dissipation term ratio are also predicted truly. Hence, it is expected that portion of different terms of TKE in unsteady state in channel can be also predicted appropriately.

4. Results

Here, the results obtained from this study are reported. In the following diagrams, portion of different terms of TKE in channel under different acceleration conditions and in different distances from channel center line are reported. In this state, h refers to lateral distance from channel central symmetry line.

Portion of different terms of TKE in channel for distance to center of 5.08mm and different ranges of momentum:

As it is illustrated in diagram in figure 8, TKE production term in distance of 5.08mm from channel center line is significantly affected by the momentum applied on the flow, so that the time lag phenomenon in TKE production term under effect of acceleration can be easily observed in this figure. In other words, the more momentums are applied on channel flow, the more resistance would be against production of TKE. Under such condition, very high acceleration of flow (acceleration time of 5sec) has been able to control production of TKE to

Reynolds about 38000; although the lower the acceleration is, the lower the resistance would be, so that through applying lowest momentum (acceleration of 100sec), production of TKE is begun from Reynolds about 15000.

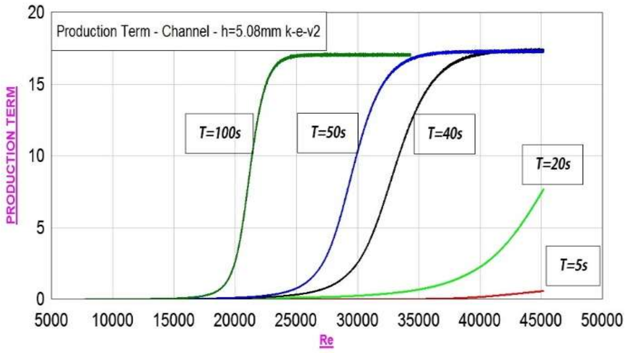

Figure 8: TKE production term under different accelerations for lateral distance from channel center (h=5.08mm)

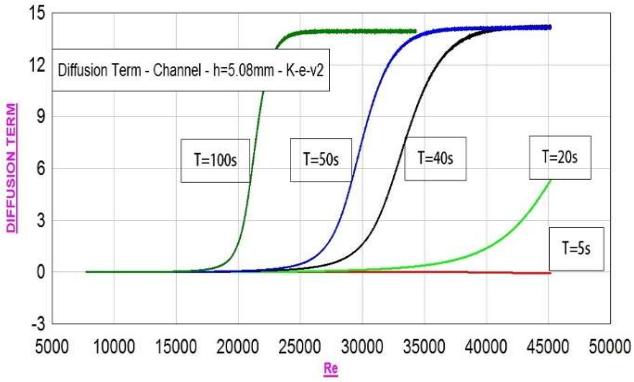

Figure 9: TKE diffusion term under different accelerations for lateral distance from channel center (h=5.08mm)

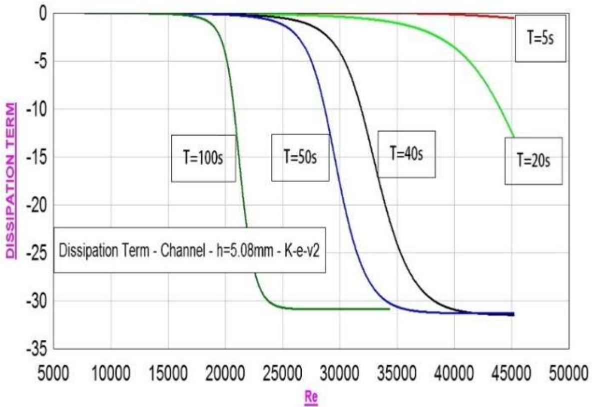

Figure 10: TKE dissipation term under different accelerations for lateral distance of channel center (h=5.08mm)

For dissipation and diffusion terms of turbulence kinetic energy in diagrams of figures 9 and 10, again sensitivity of flow in channel to momentum is evident. This time, again the higher the momentum applied on flow is, the more time lag in growth of dissipation and diffusion terms of TKE would be. It is observed that for diffusion term of TKE in distance of 5.08mm from channel center with the acceleration time of 5sec, growth of this term is almost controlled absolutely. For TKE dissipation term, this growth has been controlled to desired level through applying 5-sec acceleration.

Portion of different terms of TKE in channel for the distance to center of 12.273mm and different momentums:

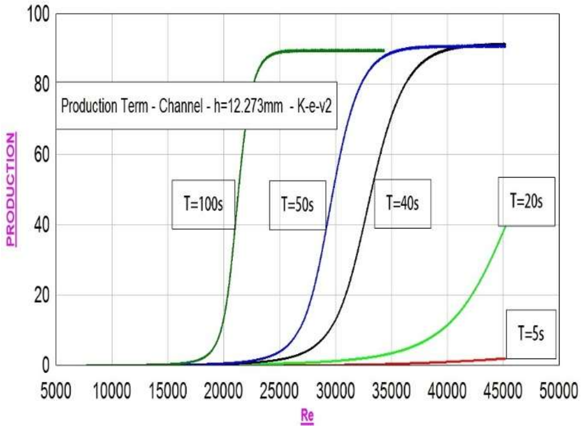

Figure 11: TKE production term under different accelerations for lateral distance from channel center (h=12.273mm)

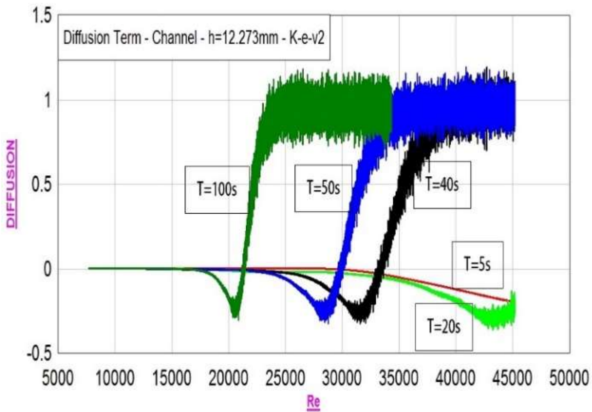

Figure 12: TKE diffusion term under different accelerations for lateral distance to channel center (h=12.273mm)

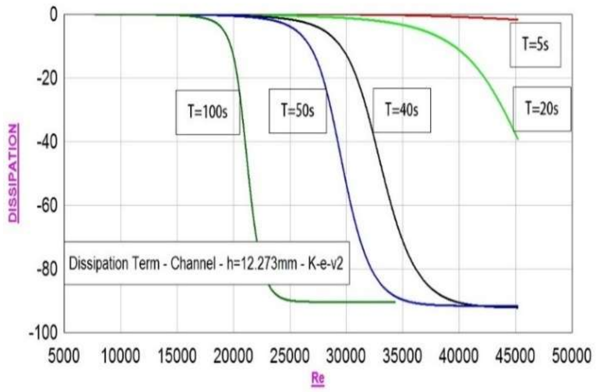

Figure 13: TKE dissipation term under different accelerations for lateral distance of channel center (h=12.273mm)

According to figures 11, 12 and 13, it could be found that in 12.27mm distance to channel wall, sensitivity of flow to applied momentum on the flow is still existed. It could be observed that the more going close to wall, the more resistance of production and dissipation terms has become and they have been able to achieve higher numerical values compared with distances close to channel center line; although the momentum applied on flow has been able to control the growth of production and dissipation terms same as 5.08mm distance. In other words, it could be mentioned that values of momentums applied on flow for distance of 12.273mm from center have been able to control growth of production and dissipation of TKE same as 5.08mm distance; although after that the flow resistance was disrupted, the portion of these two terms for 12.273mm distance, was increased with high slope and achieved higher values compared to 5.08mm distance from channel center. Also, it could be observed that for diffusion term, the results have become fluctuating to some extent and the most probable cause can be numerical problems in implementing the program. However, through considering mid-level of values, it could be observed easily that through going close to wall, portion of TKE diffusion term is decreased significantly. This time, again the applied momentum has been able to control portion of this term.

Portion of different terms of TKE for distance to center of 20.733 and different accelerations:

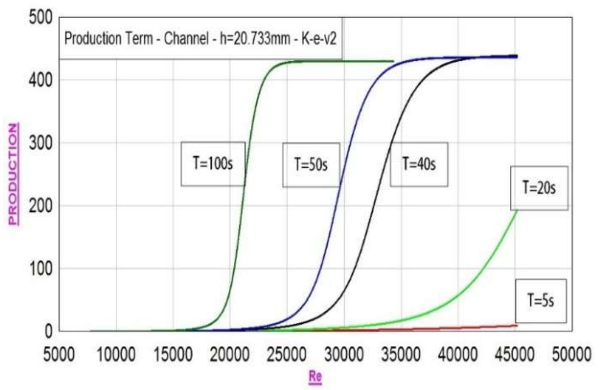

Figure 14: TKE production term under different accelerations for lateral distance of channel center (h= 20.733mm)

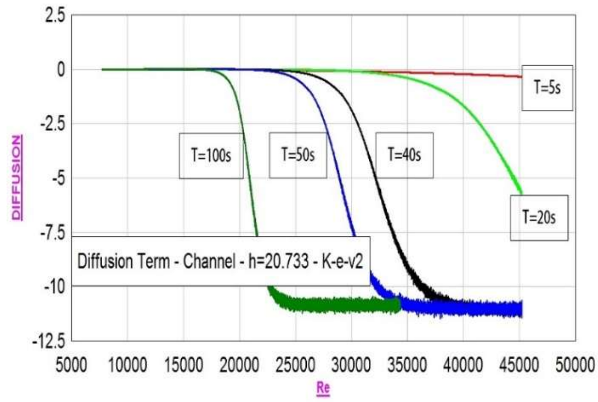

Figure 15: TKE diffusion term under different accelerations for lateral distance from channel center (h= 20.733mm)

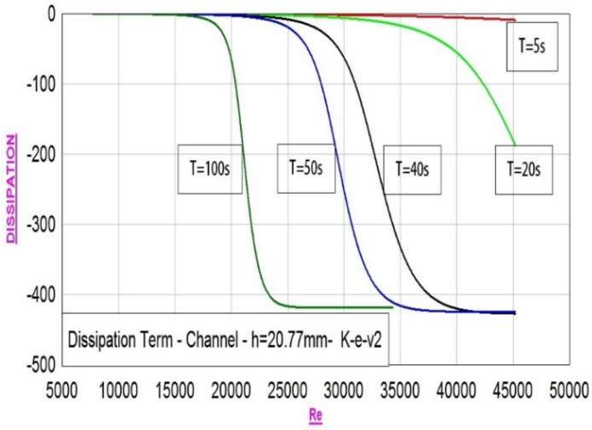

Figure 16: TKE dissipation term under different accelerations for lateral distance from channel center (h= 20.733mm)

In figures 14, 15 and 16, portion of different terms of TKE in channel and with distance of 20.733mm from channel center are illustrated. It should be noted that in this state, numerical solution is done for a channel with depth of 25.4mm (1inch). Hence, the distance of 20.733mm of channel center means very close distance to wall. Similar to the for distance of 5.08mm and of 12,273mm from channel center, this time again close to wall, the portion of TKE production term is increased significantly. This issue is evident in diagram in figure 14. In this diagram, it could be observed that applying intense momentum (acceleration time of 5sec) has been able to control growth of TKE production term to Reynolds about 34000. Numerical values achieved by TKE production term after disrupting resistance to accelerated flow averagely is about 400. Diagram in figure 15 illustrates portion of TKE diffusion term in distance of 20.733mm of channel center. Through applying acceleration time of 5sec, the flow has been able to control negative growth of TKE diffusion term to Reynolds about 30000. This value is in range of Reynolds about 20000 for distance of 12.273mm from channel center. This shows that with going close to wall can increase ability of flow to control TKE diffusion term. In diagram in figure 16, portion of TKE dissipation term under different accelerations is observed. The highest level of acceleration (5sec) has been able to stop growth of portion of this term to Reynolds about 34000. This time, again obtained values from distance of 20.733mm from channel center are higher than the values obtained for distance of 5.08mm and 12.273mm from channel center.

Comparing portion of different terms of TKE in channel for distance to centers and different accelerations:

In order to observe that which term of TKE is more sensitive to applied momentum on the flow, the diagrams in following figures should be observed. In these figures, comparison of different terms of TKE is presented in distances of 5.08, 12.273 and 20.733mm and in response to acceleration times of 5sec and 50sec and 20sec. According to diagrams in figures 17, 18 and 19, more exact and interesting results could be obtained. In diagram in figure 17, response of flow to acceleration times of 5 and 50sec in distance of 5.08mm from channel center is observed against portion of different terms of TKE. In this diagram, it is observed that increased applied momentum to 10 times (reduced acceleration period 50 to 5sec) has been able to control growth of portion in different terms of TKE.

The most interesting result obtained from figure 17 is that TKE dissipation term in distance of 5.08mm from wall is weaker than other terms, since 10 times increase in momentum has been able to control growth of this term up to Reynolds about 45200 and has been also able to decrease numerical value of this term more than other terms. Hence, TKE dissipation in areas close to channel center has low power and shows more intense response to the momentum applied on flow. In next position, TKE production term is placed. Although growth in this term is controlled to the Reynolds about 45200 same as dissipation term, numerical value of portion of this term is decreased less than dissipation term. Therefore, production of TKE in distance of 5.08mm from channel center has more power compared to dissipation term. However, the diffusion term shows the lowest reaction to applied momentum on flow. With a similar analysis of figures 18 and 19, it could be observed that for the distance of 12.273mm and 20.733mm of channel center, the most sensitivity is shown by production and dissipation terms and the changes in diffusion term are insignificant compared to the two other terms. Through analyzing the recent diagrams, it seems that in turbulent flows in channel, the main role is played by two terms of production and dissipation of TKE. For exact analysis, pay attention to figures 20, 21 and 22. In these diagrams, portion of different terms of TKE is illustrated in acceleration times of 50 and 20sec.

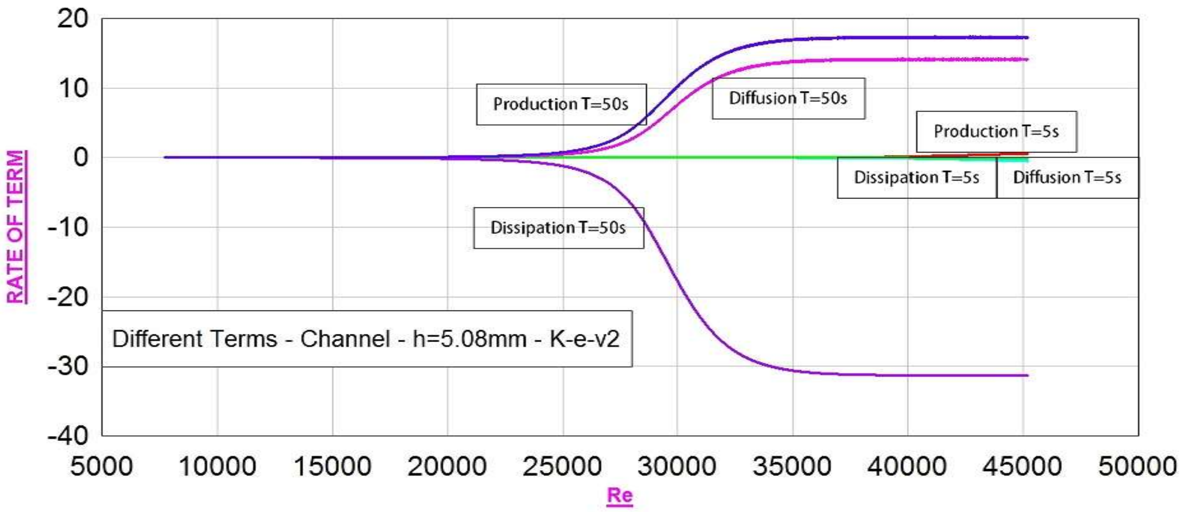

Figure 17: comparing different terms of TKE in distance of h= 5.08mm from channel center and in response to acceleration times of 5 and 50sec

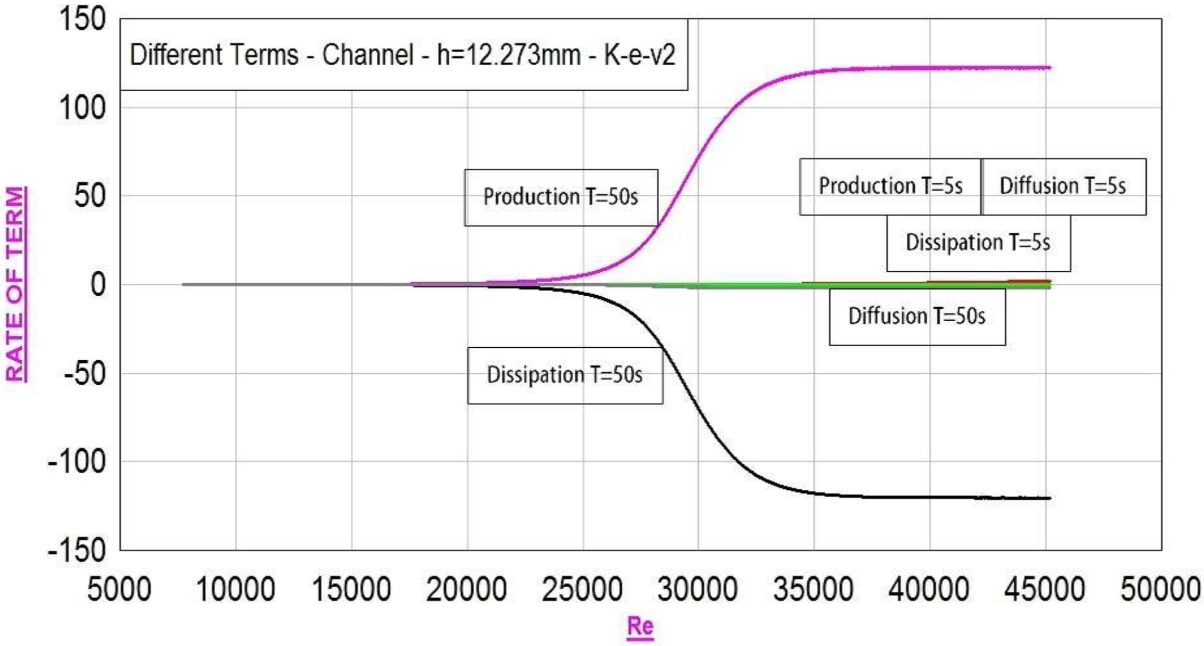

Figure 18: comparing different terms of TKE in distance of h= 12.273mm from channel center and in response to acceleration times of 5 and 50sec

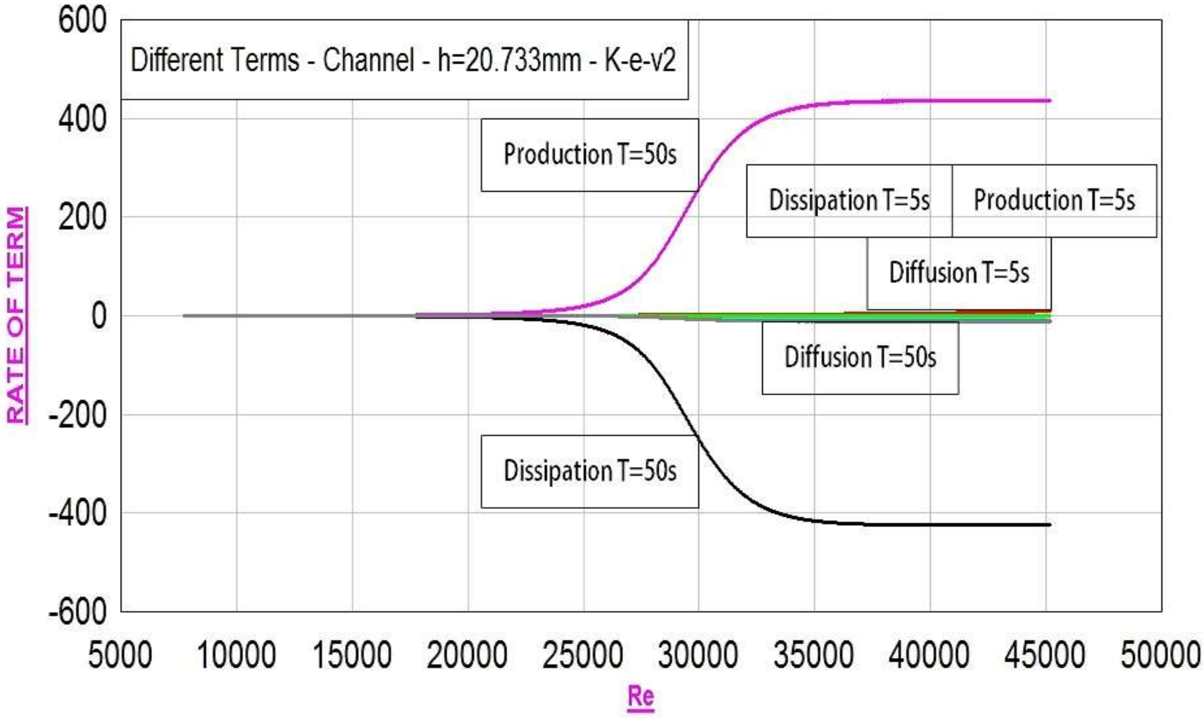

Figure 19: comparing different terms of TKE in distance of h= 20.733mm from channel center and in response to acceleration times of 5 and 50sec

In diagrams in figures 20, 21 and 22, the results could be obtained easily. The diagram in figure 20 showing the results related to response of portion of different terms to acceleration time of 20 and 50sec in distance of 5.08mm of channel center, it could be observed that the highest numerical difference caused by decreased acceleration time from 50 to 20sec for dissipation term is about 30 units and is about 17 units for production term and 12.5units for diffusion term. As it was mentioned before, the most response to the momentum applied on flow in distance of 5.08mm of center is related to dissipation term and then production and diffusion terms in last positions.

The diagram in figure 21 is related to the results of response of portion of different terms of TKE to acceleration time of 20 and 50sec in distance of 12.273mm of channel center. In this diagram, the highest numerical difference created by reduction of acceleration time from 50 to 20sec, for dissipation term is about 86 units; 87 units for production term and is about 2 units for diffusion term. As it was mentioned, in the measurement of portion of different terms of TKE for channel, it seems that the main roles are played by production and dissipation terms and these are the most important terms and the diffusion term, except for distances close to channel central line with partial role, plays no significant role in other distances of wall. Diagram in figure 22 is related to the response of portion of different terms of TKE to acceleration time of 20 and 50sec in distance of 20.733mm from channel center. In this diagram, the highest numerical difference created under effect of reduced acceleration term from 50 to 20sec is about 400 units for dissipation term, about 408 units for production term and is about 7 units for diffusion term. Through analyzing the diagram in figure 22, it could be found that portion of production and dissipation terms of TKE is more than diffusion term in channel and as it was mentioned, although diffusion term has partial portion in far distances from channel wall, this partial portion is decreased with going close to wall. In other word, in close distances to wall, portion of diffusion term is “too” insignificant.

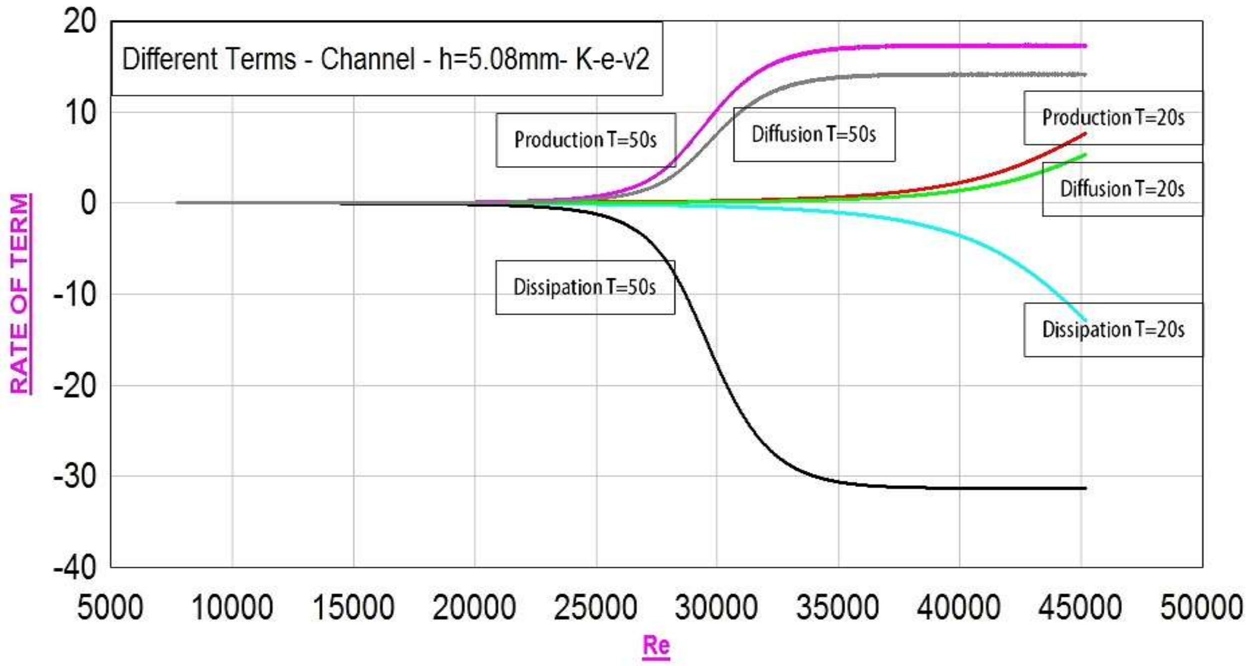

Figure 20: comparing different terms of TKE in distance h= 5.08mm of channel center in response to acceleration times of 20 and 50sec

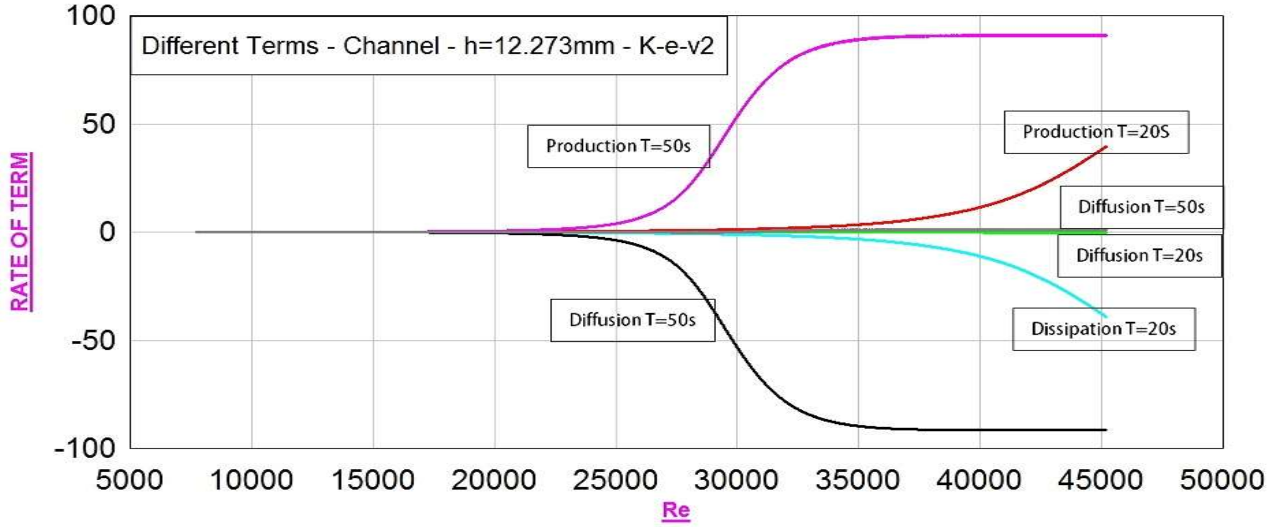

Figure 21: comparing different terms of TKE in distance h= 12.273mm of channel center in response to acceleration times of 20 and 50sec

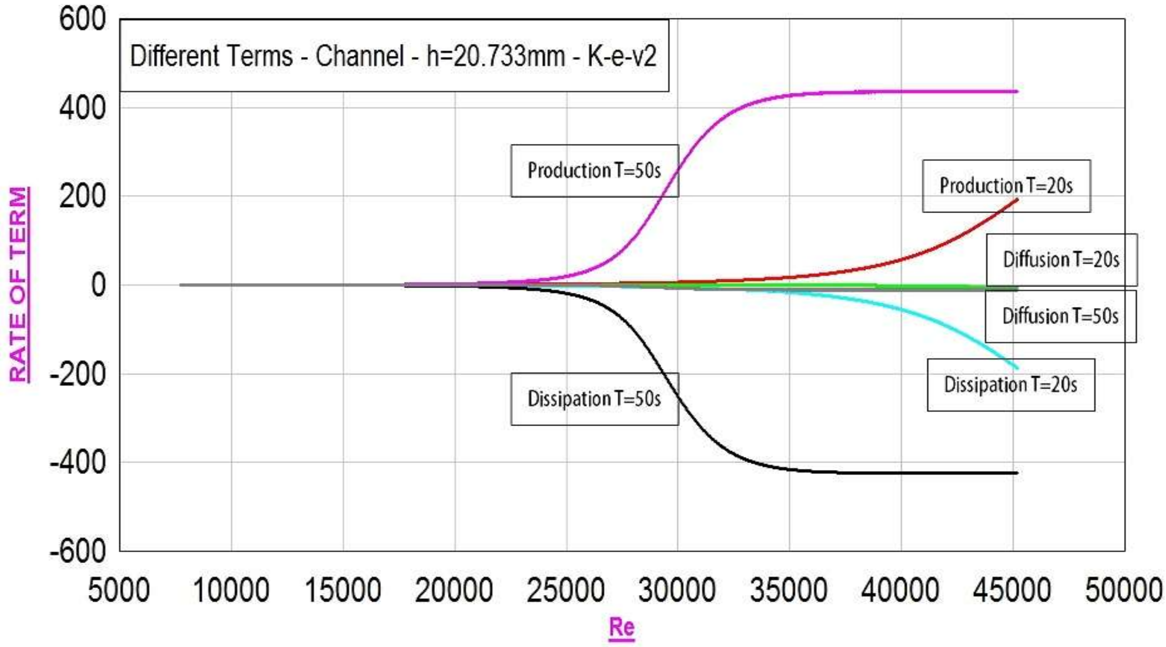

Figure 22: comparing different terms of TKE in distance h= 20733mm of channel center in response to acceleration times of 20 and 50sec

5. Conclusion

According to the investigations of diagrams related to portion of different terms of TKE for channel accelerated flow, it could be found that in general, production and dissipation terms play the key role in forming turbulent flow in channel; although the portion of diffusion term is generally insignificant. Through moving towards channel wall, the power of two terms of production and dissipation is increased and on the other hand, the power of diffusion term is decreased. Changes in applied acceleration on channel flow, have the most effect on production and dissipation terms. “Time lag” is evident in growth of portion of different terms of TKE for channel. In channel, after failure of flow resistance against the tendency of different terms of TKE for growth, these terms (especially production and dissipation terms) go toward high numerical value with sharp slope. Through considering distance from central line that each term has lowest power, in which the accelerated flow has been able to control the growth TKE terms more than other distances, the most sensitive term to applied momentum could be found. For production term in distance of 5.08mm from center and acceleration time of 5sec, the flow has been able to postpone beginning of production of TKE from Reynolds 13000 to 37500 This means a kind of time lag which is respectively equal to 24500 Reynolds units. This value in same distance is equal to 20850 for dissipation term and is uncertain for diffusion term. If changes and responses of diffusion term are ignored in channel because of being insignificant in forming TKE, it could be mentioned that in channel, after diffusion term, the most sensitive term to applied momentum on flow is production of TKE and the most resistant term against the momentum is dissipation term.

Table 2: the time lag (tlag) in growth of each term of TKE as a result of applying momentum (values in percent form are related to tlag ratio for the beginning of relevant term growth to total duration of acceleration)

Diffusion			**Dissipation**			**Production**			**TKE term**	
T=50s	**T=20s**	**T=5s**	**T=50s**	**T=20s**	**T=5s**	**T=50s**	**T=20s**	**T=5s**	**Acceleration period**	
15.1% Tlag= 15.1s	60% tlag= 12s	82% tlag= 4.1s	15% tlag= 15.0s	65.5% tlag= 13.1s	69.6% tlag= 3.48s	15.7% tlag= 15.7s	60% tlag= 12.0s	79.8% tlag= 3.99s	**5.08 mm**	**Distance from channel center line**
37.1% tlag= 37.1s	66.5% tlag= 13.3s	>100% tlag >5s	14.6% tlag= 14.6s	34% tlag= 6.8s	57.4% tlag= 2.87s	14.5% tlag= 14.5s	47.1% tlag= 9.42s	60.2% tlag= 3.01s	**12.27 mm**	
>100% tlag >100s	>100% tlag >20s	>100% tlag >5s	13.6% tlag= 13.6s	31.4% tlag= 6.28s	43.8% tlag= 2.19s	10.6% tlag= 10.6s	30.1% tlag= 6.02s	34% tlag= 1.70s	**20.73 mm**	

In table 2, it has been tried to obtain time lag in turbulent and accelerated flow per 3 acceleration times T=5s (intense momentum); T=20s (medium momentum) and T=50s (slow momentum) in 3 different distances from center. It should be noted that in this study, time lag is defined as the postpone time of growth of portion of relevant term. The values given in table2 are obtained by

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putting the difference value between the Reynolds number which the relevant term begins to growth in steady state and the Reynolds number which the relevant term begins to growth in accelerated flow, in Re equation and the main kinetic momentum equation. In other words, the difference of Reynolds value of growth of relevant term in steady state compared to accelerated state is a value which replaced instead of parameter Re in main Reynolds equation. The Reynolds value can be extracted easily from diagrams presented in the study. For example, for acceleration time of 5s in distance of 5.08 from channel center for production term, the time lag of 3.99s is obtained. This time lag is interpreted in this way that flow in this state, out of total acceleration time of 5s, has shown unconsciousness about 3.99s for the beginning of production term and has postponed its TKE production for 3.99s.

This time lag is about 79.8% of total duration of acceleration time. The interpretation can be also presented for other terms in different distances and accelerations. According to table 2, the more the momentum applied on flow is, the more the lag of terms of TKE would be. In other words, the more the momentum applied on flow is, the more time the flow is in unconsciousness out of total acceleration period. According to table 2, it could be observed that the more going close to wall in channel, the more consciousness of flow to production and dissipation terms would become. On the other hand, the less the applied acceleration is, the more consciousness to all TKE terms would become. On the other hand, production and dissipation terms in areas close to wall have more impact and as a result, the accelerated flow in these areas shows more consciousness to these two terms. However, the reverse issue is true for diffusion term and accelerated flow in central areas has more consciousness to this term against the applied momentum.

In this study, only effect of positive momentums (increasing velocity) on portion of different terms of TKE is investigated. As a further study, effect of decreasing momentum on portion of different terms of TKE, whether in channel or pipe, could be investigated.

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