Large Deformation Effects on the Capacity Curve of the Structure

Mohammad Shirzadi[1*], Mohammad Hossein Vafaee[2]

[12] Faculty of Civil Engineering, Pooyesh Higher Education Institute, Qom, Iran

Abstract
The current paper considers the effect of large deformation on the capacity curve of the structure. In fact, by applying p-Δ effects, it seems that the energy dissipation by the building decrease in capacity curve, so the structural performance points are affected. The results show that given the large deformation effects, the structural performance points assign a greater amount of displacement.
Keywords:
Pushover analysis, capacity spectrum curve, response spectra curve, structural performance points

1. Introduction

Due to the increasing spread of technology and acceleration of computing, the long and complex computations are done by advanced computers. Accordingly, the engineers have achieved new principles and concepts regarding the behaviors of structures, along with the spread of science. The invention of a variety of new methods for analyzing and evaluating structure's performance to identify the forces, and displacement of the structure in different directions, are samples of current developments. To introduce the second order analysis, a few points are mentioned:

1. Types of structural analysis
2. Terms and conditions of the analysis

As we know, all analyzes are divided into linear and nonlinear groups. Linear analyses are those which examine the behavior of the structure in linear phase. These analyses come in two types:

A. linear static analysis
B. linear dynamic analysis

Linear static analysis is the most common types of structural analysis. It is used to study the structural behavior under gravity and lateral loads in the phase of linear behavior of structural components, and it can provide an excellent view of the behavior of the structure by simple analytical assumptions, but this analysis is not suitable for the structures that have a different behavior in nonlinear phase, and also the structures that vibrate under the lateral loads, which are different from the simplified single degree of freedom systems. In irregular structures, that their lateral force distribution is not linear due to the change in resistance after the yielding or the effects of different modes; dynamic analysis, which indicates the correct distribution of force, will be compulsory. Similarly, the analyses that examine the structural behavior in the nonlinear phase are divided into two categories:

C. Nonlinear static analysis
D. Nonlinear dynamic analysis

These analyses examine the behavior of the structure in nonlinear phase. As it is obvious, the gravity loading of the structure cause to deform the structure and change of stiffness of structural members, so by applying lateral loads, two results are obtained:

1. Displacement of force's point of action (linear-elastic effect): the point of action of gravity force is changed due to the lateral force, and creates a secondary gravity moment. These deformations may be exacerbated by changing stiffness.

2. Stiffness change (nonlinear – elastic effect): a column which has one degree of freedom and is under the influence of compressive gravity load, become softer; in other word, its geometric hardness changes. So the displacement caused by lateral load, which depends directly on its lateral hardness, will increase. As a result, this factor increases the displacement and forces of the structure and it will be different from the case that there is no hardness change in the structure. This effect is known as nonlinear second-order analysis.

Another aim of the study is to obtain performance point of capacity spectrum method that was provided by ATC-40. According to ATC-40, the intersection of demand and capacity curve spectra, in spectral acceleration-spectral displacement ordinates, is called performance point. In spectral acceleration-spectral displacement ordinates, the spectral displacement of the performance point of the structure is called Demand Displacement. Using the displacement, the amount of displacement of the control point is achieved and the performance level of the building is evaluated. In this method, based on a series of relationship, the capacity

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curve (pushover) and elastic response spectrum curve, with 5% damping, are converted to curves according to spectral acceleration (S_a) - spectral displacement (S_d) ordinates. Then the elastic response spectrum curve (demand spectrum) and the capacity curve (pushover) are converted to ADRS format. Both curves are drawn in a co-ordinate system, and a spectral displacement is supposed for structure and based on the spectral displacement, the effective damping of the structure is calculated and spectral reduction coefficients are achieved. Using the spectral reduction coefficients, the demand spectrum is reduced in ADRS format to intersect the capacity spectrum curve of the structure. What is actually obtained is demand displacement of the structure. The way for obtaining performance point of the structure or demand displacement in ATC-40 is as mentioned earlier. In this research, the structural models of Steel Moment Frame Buildings (SAC) are used. ETABS software is used in this study. Other parts of this research is as follows: the modeling technique in ETABS software, characterizing the structural properties and the details of studied structural models, analyzing the results of Static Pushover Curves (includes Resultant Base Shear vs Monitored Displacement curve and Capacity Spectrum curve), presenting the interpretation on the results of causes of errors and its value, and summarizing the results.

2. Methodology

As stated, there are two kinds of p-Δ : initial P-Δ and secondary P-δ. p- Δ effect is created by the eccentric gravity loads of the upper floors on the intended floor which called p- Δ force. If the lateral displacement of the intended floor is due to the lateral force of earthquake, a moment whose value is equal to the multiplication of p and Δ is added to the created moment on each floor. In order to achieve the significant results, many factors should be identified for nonlinear response of structure. One of these factors is p-effect. Δ

P-Δ effect:
The secondary effect of gravity loads on the framework, which is deformed laterally, is called p-Δ effect. This factor has a significant impact on the behavior of the building in a super elastic range. P- Δ Effect decreases the initial elastic stiffness slightly. Therefore, its effect on the structural seismic response in the elastic area is very low. While in super elastic and yielding phase, where the lateral resistance and stiffness of the structure is decreased, P- Δ Effect is very high. First it causes a yield step and negative slope of sheer curve in the face of lateral displacement, so that in relative lateral displacement (drift), 4% of lateral resistance is eliminated and the resistance base sheer is equal to zero. In the absence of p-Δ effect, the base sheer increases with a low slope. This deep difference in the super yielding behavior of the buildings, require taking account of the p- Δ effect as an important factor in response to seismic excitation. When p-Δ effect is taken into account, after a series of shifts in the initial vibration of the earthquake, the relative lateral deformation of the floor grows in one direction, without any return that indicates the dynamic instability. In contrast, in analysis without considering p- Δ effect, the response remains dynamic stable and fluctuating. So p-Δ effect should be considered in the seismic analysis of those buildings which enter the super elastic range mainly.

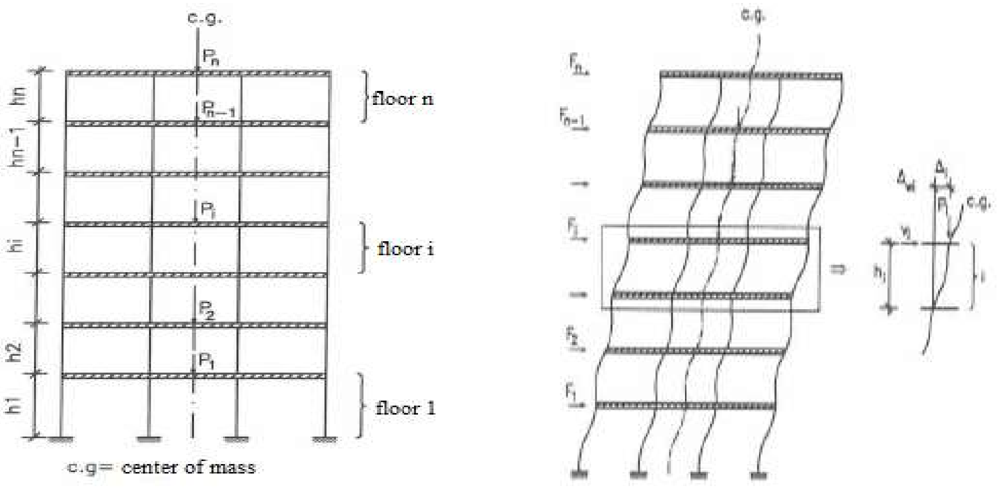
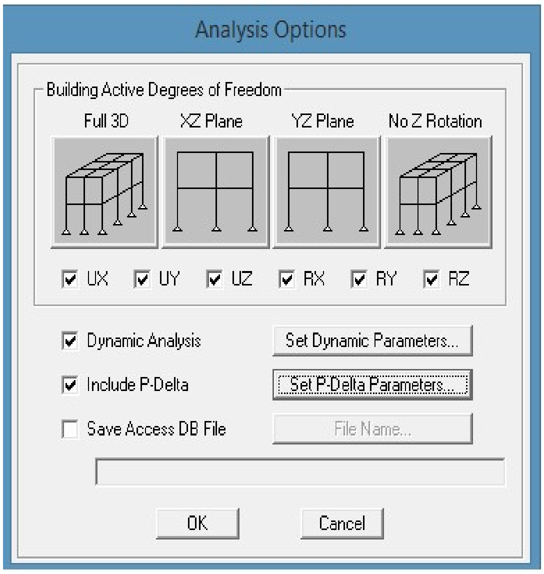


Figure 1: the effect of p-Δ on the structure [1].

According to what was said, applying the effect of large deformation on the structure is essential, so we will illustrate the way of applying these effects in ETABS software pictorially. Based on the follow instruction, the initial p- Δ is applied in ETABS software:



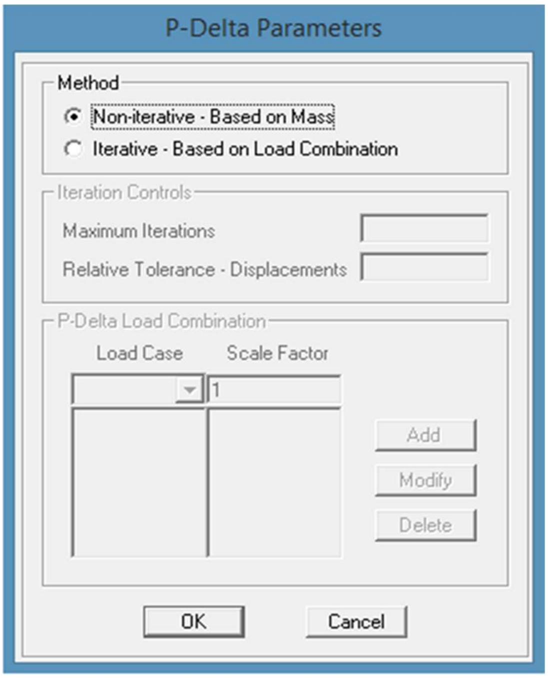


Figure 2: the way of applying p- Δ effect in ETABS software

The way to do these changes is as follows:

Analyze→ Set Analysis options→ Include P-Delta→ Non-iterative – Based on Mass

The effects of second-p- δ

To consider the second order effects, it is better to use design codes in the software. To consider these effects, moment's amplification factor should be placed against each other around both strong and weak axes. One way of applying p- δ effect on dynamic analysis, is considering axial forces obtained during first order dynamic analysis, combining them with static loading, using existing equations in this field, and estimating the value of secondary effects.

Applying the second order static hardness matrix instead of the first order matrix in the first order analysis of the dynamic analysis does not reflect the effects properly, because the axial forces caused by dynamic loads is different from the general form of the static loading. It can be said that although the horizontal displacement is more in taller structures, but due to the phase difference, it seems that the second order hardness change causes attenuation, or more correctly, energy absorption by the structures. The difference in rotational displacements of the lower floors eventually leads to a large moment in lower nodes, which of course is not apparent in first order dynamic analysis. As a result, the difference in transferring and rotational deformations of the nodes of the upper floors is more regular, but at the nodes of the lower floors, because of large phase difference, it is more turbulent. On the first floor beams of the buildings, axial force resulting from the of the second order analysis almost does not exist, while the upper floors beams have a relatively large axial forces which are ignored in the analysis of the first order.

Since the software calculates displacements only at the nodes, in order to consider the effects, deformations between the two end nodes (in the column) must be known. To do this, the column must, using the command division, be divided internally. Divided internally prevents problems related to column length and free length buckling coefficients. In this section, the main focus will be on the column.

When we consider the effect of p- Δ , structural capacity is severely reduced, so it is possible structures under earthquake without the p- Δ stay in the linear region; but in another earthquake and taking into account p- Δ effects, it may lay on the nonlinear area with plastic hinges. The p- Δ effect is not negligible on construction and other work.

Under study buildings

In this study SAC group structures are used. 3, 9 and 20 stories buildings of SAC group are designed in order to express the behavior of short and tall steel buildings with flexural frame system under gravity and seismic loads.

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These structures were designed for office applications and the hard soil was selected for this application. In this design no evidence of distinguished disorder in plans and in height was considered. The steel flexural frame systems were considered for Peripheral parts of the buildings and the interior systems were including strong gravity load resisting elements. In designing all three buildings seismic loads were considered as dominant loads of flexural frames. In designing the structure of SAC gravity loads considered for inside the structures and lateral loads considered for peripheral parts of the structures. As gravity loads are not related to the subject or this article, we do not regard them and we focus on lateral loads instead. In this research the N-S direction frames are selected for analyzing. The building plans and the under analysis frame façade, beams and columns of the buildings are shown in figures 3 to 6. The effective seismic load of the buildings is equally distributed between two flexural frames in each direction. Figure 6 shows a 20 story building. As in each story equal sections were used, each figure shows one sample of interior and exterior beams and columns.

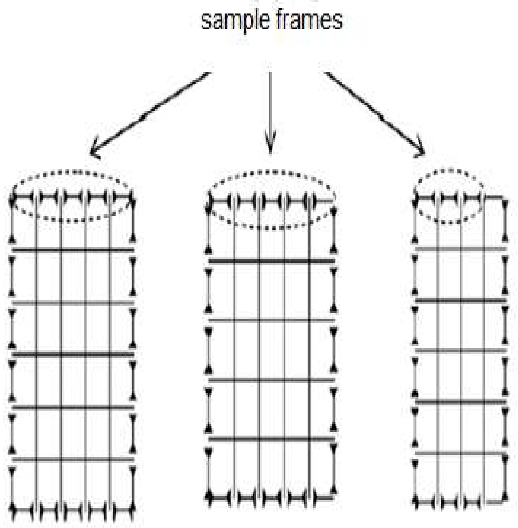

Figure3: sample frames (3, 9, 20 floors, left to right)

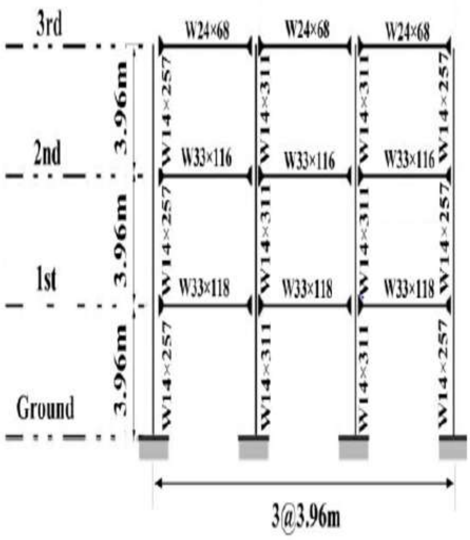

Figure 4: the frame used in the 3-floor building

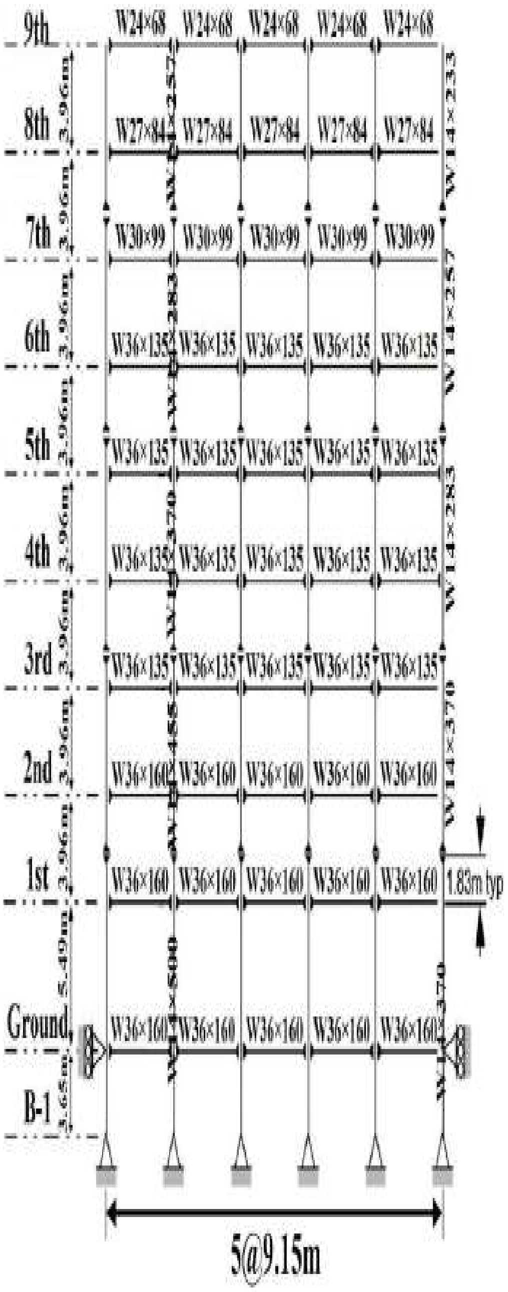

Figure 5: the frame for 9-floor building

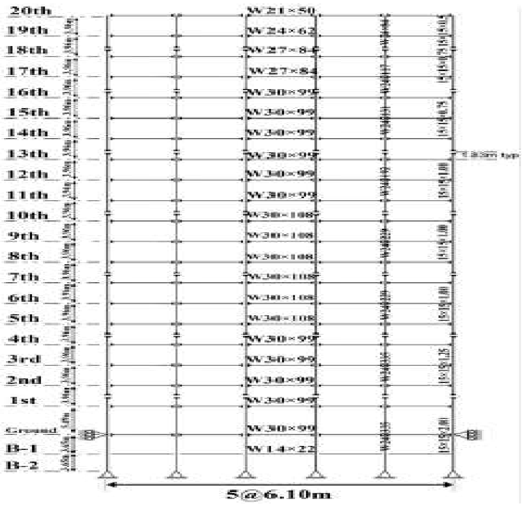

Figure 6: the frame for 20-floor building

Figure 7 is a non-linear element for modeling the beam-columns in steel and concrete buildings. This element consists of elastic beam geometry, two concentrated plastic hinges at the end of the beam and the rigid ends.

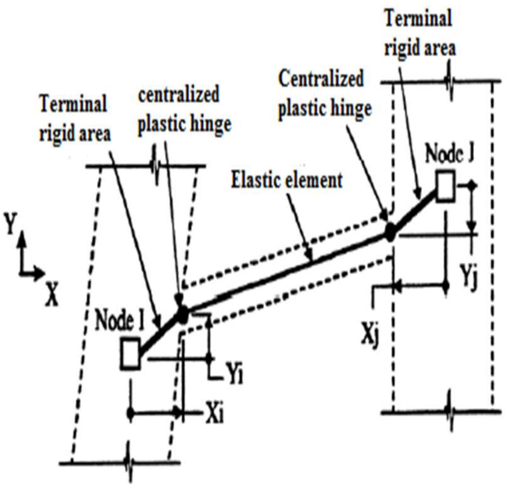

Figure 7: Plastic behavior modeling of structure elements

The arbitrary put of the beam-column elements in the X-Y sheet is possible. These elements have flexural and axial stiffness. The yielding is just possible in the plastic joints and yielding moments of the joints are determined in two ends of the element for positive and negative directions. Although it is assumed in these elements that yielding occurs only in bending and without any axial deformation, the direction of the moments and the effect of axial force on the bending strength regarding the effect of p-Δ are obvious in figure 8.

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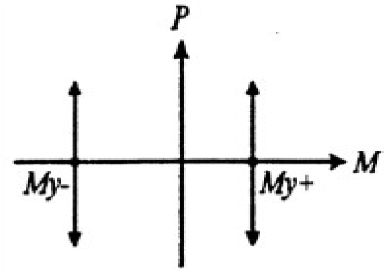


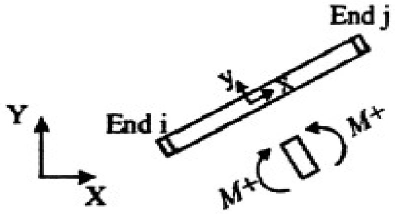
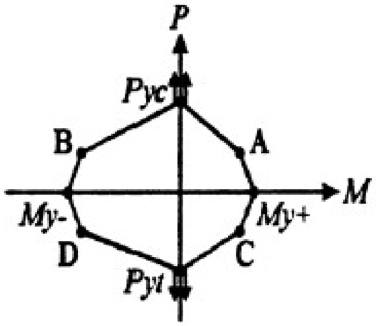


Figure 8 (a) the directions of positive moment Figure 8 (b) the model of beam under the effect of bending moment



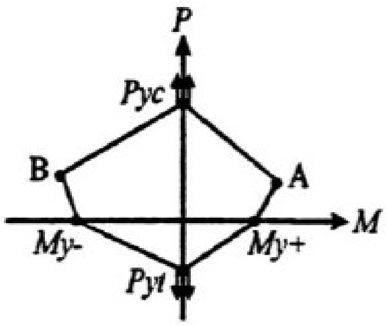


Figure 8 (c) model of concrete columns Figure 8 (d) model of steel column

Static loads exerted on the element length can be replaced with the equivalent amounts of the fixed end forces. Each element of beam-column like **Figure 9** has three general moods of deformation including axial length change and the rotating bending at the first and end of the element. These deformation moods are related to the initial and terminal change components of beam-column element in the modeling.

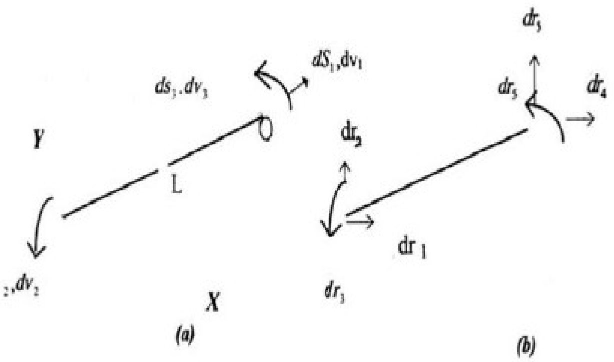


Figure 9: the forces and the types of deformation moods of beam-column element

Specifications of the selected models

As mentioned above, the constructions of SAC group include two parts of gravity bearing inside the structure and lateral bearing around the structure. Regarding that the topic discussed in this research does not relate to the gravity loads; the structure bearing the gravity is ignored and it is focused on their lateral bearing. Seismic effective weight classes and specific sections of buildings Group SAC are presented in Tables 1-3.

Table 1: Seismic effective weight of floors in the building 3 in SAC group

Floors	All floors except the roof	Roof
Effective Weight (In Kgf-m)	12278.2354	13412.5025

Table 2: Seismic effective weight of floors in the building 9 in SAC group

Floors	All floors except the roof	Roof
Effective Weight (In Kgf-m)	10708.5776	11643.8096

Table 3: Seismic effective weight of floors in the building 20 in SAC group

Floors	All floors except the roof	Roof
Effective Weight (In Kgf-m)	7560.2705	976.2528

Dynamic Characteristics of Structures of Group SAC: The dynamic characteristics of structures have been determined with 5% damping (ξ = 5%) and the effective mass of floors in accordance with the expressed values. These characteristics have been proposed in follow:

Table 4: Period of Construction 3 in the building 3 in SAC group

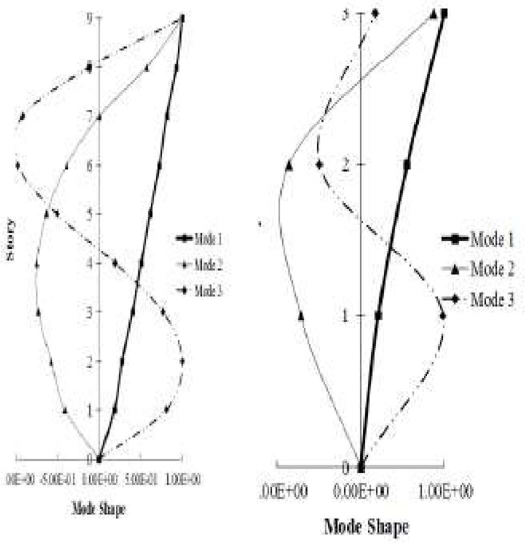
Modes / Characteristics	Mod 1	Mod 2	Mod 3
Period In seconds (Tn)	1.0174	0.3292	0.1782

Table 5: Period of Construction 9 in the building 3 in SAC group

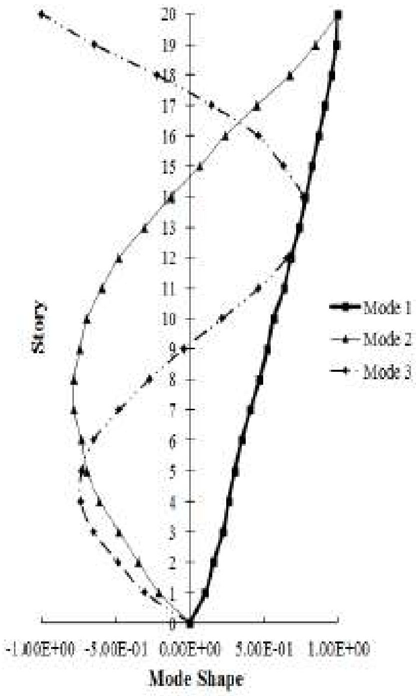
Modes / Characteristics	Mod 1	Mod 2	Mod 3
Period In seconds (Tn)	2.2497	0.8446	0.4909

Table 6: Period of Construction 20 in the building 3 in SAC group

Modes / Characteristics	Mod 1	Mod 2	Mod 3
Period In seconds (Tn)	3.7807	1.2926	0.7440



10-1 (floor 3) 10-2 (floor 9)



10-3 (floor 20)

Figure 10: The three mods of structure 3, 9, and 20 in SAC group

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3. Presentation of Results

The pushover analysis is an analysis of nonlinear static which affected with increasing lateral loads. The aim of increasing nonlinear static analysis is estimating the expected behavior of a system to help estimate the requirements of structural and deformation resistance, by performing an analysis taking into account the seismic design and then compares the required quantities with capacities in behavioral or functional level. This estimation is based on the identification of important parameters including lateral displacement behavior; change the relative figures members and connections, and so on. Analytical methods that are considered in the design basis seismic performance and improvement mainly are based on the nonlinear static analysis. The priority of using this type of analysis is its high speed in terms of performance, simplicity for interprets the results, and acceptable accuracy.

Load combinations used in nonlinear static analysis: In the nonlinear static analysis, especially in irregular structures, the applying of lateral load can lead to a different result in positive and negative directions. Therefore, two modes of analysis should be considered for each model of lateral load. The differences for these analyses are related only to the line of loading. Two modes of analysis will be introduced for each pattern of lateral load. In the first mode the value for Scale Factor id positive and in the second mode, it will be introduced to the program in a negative value. In the nonlinear static analysis, each category has a separate calculation.

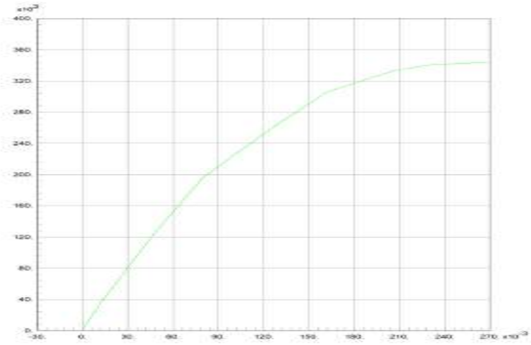
The lateral load distribution caused by the earthquake is a function of dynamic characteristics of structures and its non-linear behavior. This will be altered during the earthquake.

Lateral load distribution caused by the earthquake is a function of dynamic characteristics of the structure and its nonlinear behavior. Lateral load distribution cause change in the force distribution and shape of structures. Since each structure has several possible failure modes, the lateral load distribution should be selected in a way of the most critical failure mode. We used the lateral load distribution based on Triangular Pattern.

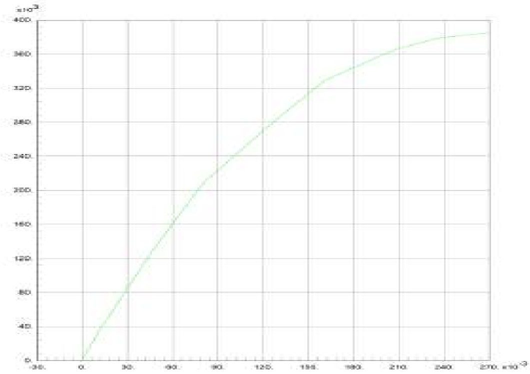
The Pushover Curve

First, we draw the pushover curves. According to the research will be different by using or not using of p- Δ . It means when we apply p- Δ we see the curve comes down and therefore its energy is absorbed less. The following figures indicated this situation.

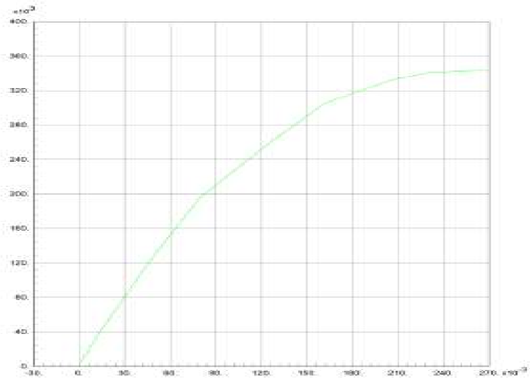
Curves of 3 floors of Group SAC:



(a) The pushover curve by applying p- Δ

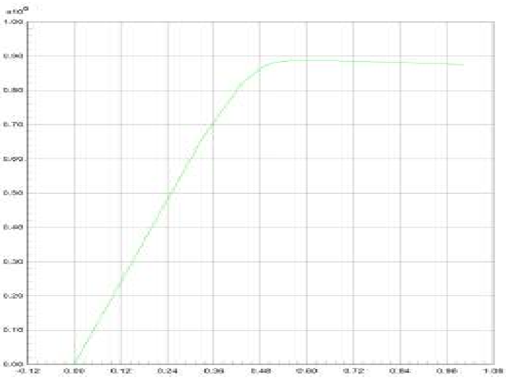


(b) The pushover curve by applying p- Δ

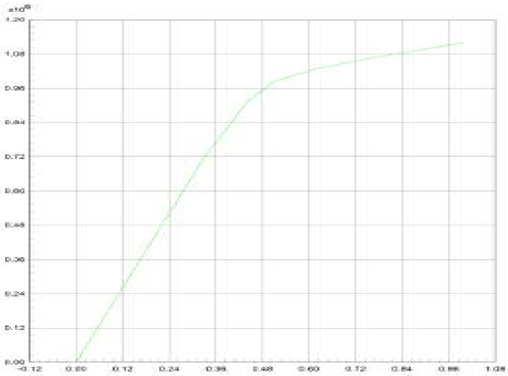


(c) The pushover curve by applying p- Δ and p- δ

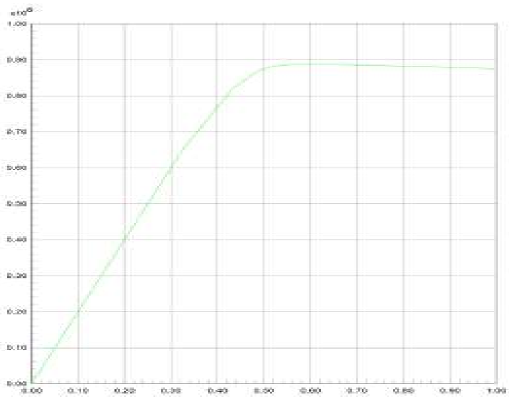
Figure 11: Curve capacity 3 floors structure with and without considering the effects of large deformations



(a) The pushover curve by applying p- Δ

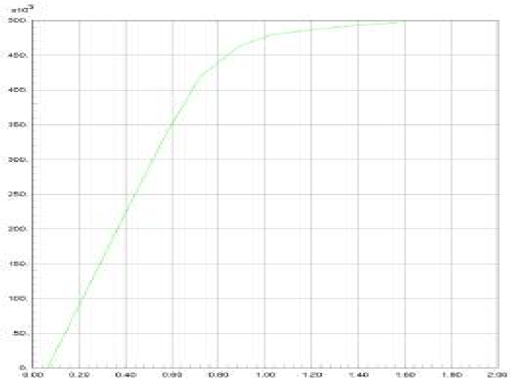


(b) The pushover curve by applying p- Δ

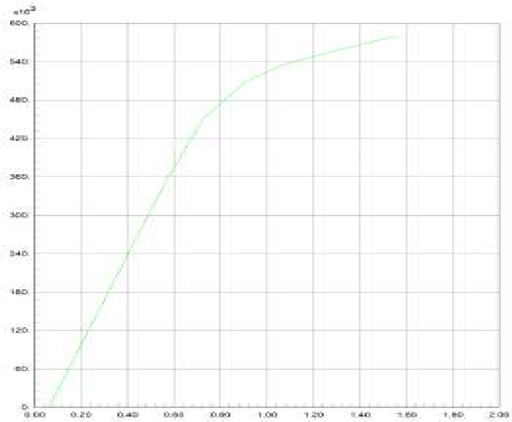


(c) The pushover curve by applying p- Δ and p- δ

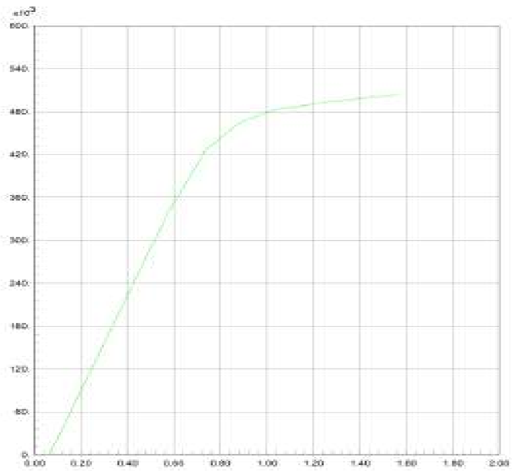
Figure 12: Curve capacity 9 floors structure with and without considering the effects of large deformations



(a) The pushover curve by applying p- Δ



(b) The pushover curve



(c) The pushover curve by applying p- Δ and p- δ

Figure 13: Curve capacity 20 floors structure with and without considering the effects of large deformations

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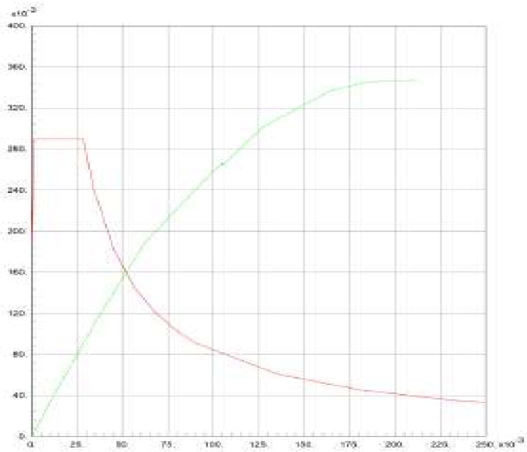
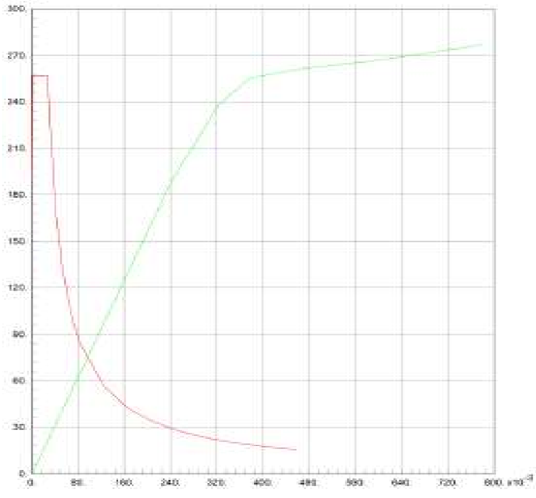
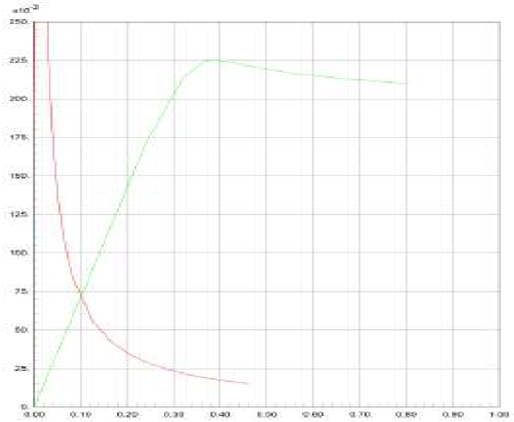
Determination of Demand Displacement in ACT-40: The method for obtaining the yield point in ACT-40 is based on CSM (Capacity Spectrum Method). In ACT-40 the point where the capacity spectrum and demand spectrum meets each other is called Performance Point. Here, the displacement spectrum of performance point's structural shift in demand is called Demand Displacement. Using this displacement, we can assess the building performance levels. In this method, the capacity curve (pushover) and the response elastic curve spectra with 5% damping turn to curves with spectrum displacement (S_d) and spectral acceleration (S_a). ADRS (Acceleration Displacement Response Spectrum) is the displacement acceleration response spectrum. Also, we can draw "Curve Spectrum capacity" and "Curve Spectrum different damping demand" due to the displacement acceleration spectrum (ADRS format). After drawing the two curves, due to the displacement spectrum, the effective damping is calculated. According to the effective damping, the reduced spectrum coefficients will be calculated. The spectrum displacement will be the same with the structural demand displacement if the percentage difference between the displacement spectrum point of intersection obtained and the spectrum displacement are in the acceptable range. This spectrum displacement is the structural demand displacement. In means that the new obtained structural demand displacement is based on the reduced spectrum coefficients. The new obtained structural demand displacement is not the real structural demand, and therefore the obtained effective damping is not the real effective damping of the structure. As you can see, we should repeat and repeat the process of calculation to achieve the required result in ACT-40. The following Figures indicated the three mods of structure 3, 9, and 20 in group SAC:



(a) Performance point of structure 3 through applying p-Δ and p-δ



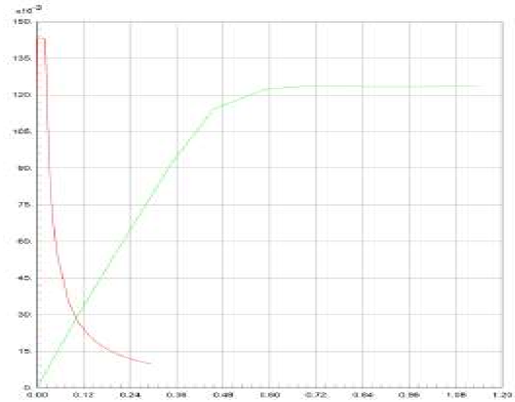
(b) Performance point of 9 floors structure through applying p-Δ



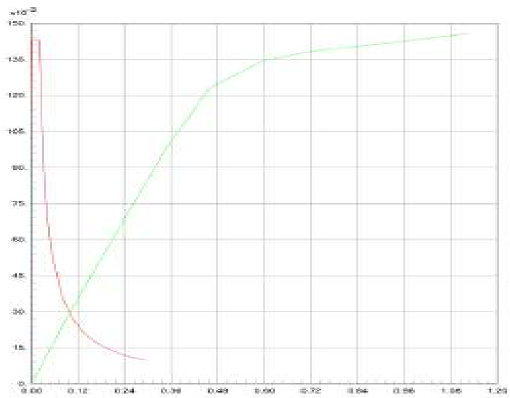
(c) Performance point of 9 floors structure through applying p-Δ and p-δ

Figure 15: Control point of SAC 9 floors structure to the method of capacity spectrum with and without application of p-Δ

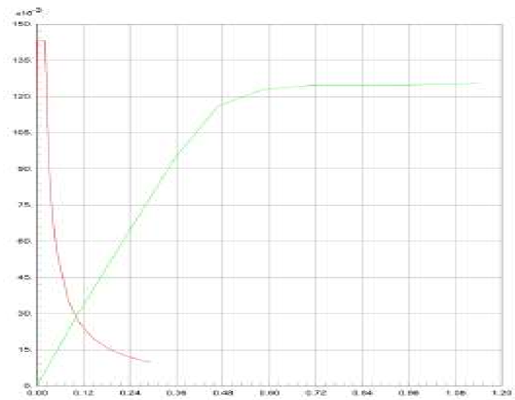
The most amount of effectiveness in considering change effects of massive shapes is usually observed in the structures with average height due to the limit of their periodicity time. As it is observed based on figure 15, movement of structure performance point to the right hand is well observable through consideration of the effect of p-Δ in the average height structure such as SAC 9 floor structure. Performance point of 20 floors structure of SAC group.



(a) Performance point of 20 floors structure through applying p-Δ



(b) Performance point of 20 floors structure through applying p-Δ



(c) Performance point of 20 floors structure through applying p-Δ and p-δ

Figure 16: Control point of SAC 20 floors structure to the method of capacity spectrum with and without application of p-Δ

Regarding 20 floor structures, also according to the drop of absorbance capacity of structure energy by considering the effect of p-Δ, performance point of structure has slightly moved the right hand. Generally, based on doubled effect of p-Δ in imposing additional forces to the structural elements, we can assume that specifically, the usage of plasticity capacity of structure in post-delivery area requires the non-linear static methods to need displacement increase especially in control point of structure. According to the comparison of drawn curve, it is inferred that following the application of p-Δ and p-δ, since capacity curve (pushover) descends, performance point of the structure will be transferred to the right hand of the curve and in fact it advances more.

4. Conclusion

In this paper, we consider massive displacements in first and second type in the calculation of performance point of structure. Initially, we calculate pushover curve related to these three buildings without p-Δ and then with p-Δ and p-δ and compare them together. According to the obtained evaluations and comparisons, we concluded that following the application of p-Δ and p-δ, structure capacity curve would drop-off and as a result, it would cause less energy loss. Then according to ATC-40 regulation, capacity spectrum and response spectrum of the mentioned structures have been drawn in order to earn performance point of structures which is intersection between capacity spectrum and response spectrum. In the investigated structures, it was observed that when p-Δ and p-δ are applied, since capacity spectrum curve descends, performance point is directed toward the right hand namely toward more front part. Of course, p-Δ is considered to be larger than p-δ.

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