Fluid Flow Modeling In Several Types of Industrial Electrowinning Cells

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Abstract
Hydrometallurgical method for copper production due to its benefits, including low cost, has many used in the world. The final step in this process is called, electrowinning (EW). In Ew due to Release bubble gas of oxygen in the reaction and bursting up at free surface caused acid mist. Due to the difficulty of the simulation of transport phenomena in the electrowinning process, a few work studied in electrowinning modeling in the world. In this article we modeling some of copper electrowinning cells in leaching and discussed and compared these models and results. This modeling be done with Two and three-dimensional modeling of fluid flow, stable and two phase, with Eulerian-Eulerian sight. 11 coupled partial differential equations, including equations of momentum, continuity, turbulent flow model, the transport equation of copper ions in the electrolyte solution to the problem have been solved. In this equation buoyancy, drag, turbulence and buoyancy concentration distribution have been considered.
Keywords:
Copper Electrowinning, Two-dimensional and three-dimensional hydrodynamic modeling, Multi-phase flow, Eulerian-Eulerian method, computational fluid dynamics

1. Introduction

Copper electrowinning is copper recovery process of the solid electrolyte on the cathode screen by passing an electric current through the electrolyte to absorb the cathode copper ions takes place. Electrowinning of copper in a cube-shaped geometry occurred with two electrodes facing each other. Flow between the electrodes move and copper ions on the cathode exhaust. Oxygen bubbles are produced on the anode.

Bubbles don't pass from the electrolyte solution and burst at the electrolyte free surface. Due to release of oxygen bubbles on the anode, the electrode should be neutral, to this end, the anode lead is used. It is known that oxygen bubbles create a large circular area and the rotation due to the nature of mixing has a large effect on the mass transfer of copper on the cathode. In the electrowinning process, reactions (1) and (2) are:

Anode reactions

(1) $$Cu^{2+} + 2e^- \rightarrow Cu \qquad E = 0.34V$$

Cathode reactions

(2) $$H_2O \rightarrow H^+ + OH^- \rightarrow \frac{1}{2}O_2 + 2H^+ + 2e^- \qquad E = -1.23V$$

Overall reaction of electrowinning total anodic and cathodic reactions in addition to sulfate ions

(3) $$Cu^{2+} + SO_4^{2-} + H_2O \rightarrow Cu + \frac{1}{2}O_2 + 2H^+ + SO_4^{2-}$$

Several studies on hydrodynamic modeling have been conducted in electrowinning cell. In this study, study some electrowinning cells that have more complete view than electrowinning cell modeling, modeling them.

Leahy and Schwarz [2], one of the newest articles about simulating fluid flow within the electrowinning cell considered. In this paper, a CFD model to simulate the process of electrowinning copper and to model the flow transfer and mass in the space between the electrodes, a pair of single (an anode and a cathode) with geometry and operating parameters of industrial EW was applied.

CFD models predict a separation zone. Two-phase Eulerian-Eulerian model flow gas was used. The electrolyte liquid continuous phase and bubble-gaseous discontinuous phase is oxygen. This article bubble diameter assumes about 150 microns. Turbulence model to solve in this paper is used; turbulence model is k, ω. The results of this study will be available include:

Calculate the velocity field, concentrations contours of copper, oxygen volume fraction contours, contours on viscosity of turbulent flow.

Filzwieser in the paper [4] electrowinning cell has specific laboratory to study the speed and boundary layer between two electrodes and by laser method (LDA) obtain accurate laboratory results.

Najmi et al in a paper [8] simulate the flow of electrolytic in copper refining cell with the help of computational fluid dynamics, and tried to electrolytic two-phase flow and bubble for cells of copper refining model.

Turbulence model to solve in this paper is used; turbulence model is k-ω. Najmi in article [] to model Sarcheshme copper electrowinning cell. In this paper, part of Sarcheshme copper cell is symmetric and modeling in

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three-dimensional aspect. Turbulence model to solve in this paper is used; turbulence model is k, ω.

In this paper, the electrowinning cells modeling by Leahy, Filzwieser and Najmi have been studied. Choose this model because the three different types of cells in tem of geometry and number of dimensions and boundary conditions. They are as comprehensive in Ansys program packages in model and laboratory results or available CFD validated. Thus, a complete view of the complex multiphase simulations in different states will find it and unified modeling in Ansys CFX has modeling capabilities for many electrowinning cells.

2. The Main Equation

Solving method in this article is Eulerian-Eulerian model. The models solve momentum and continuity equations for each phase. Correlation of these equations through pressure and exchange coefficients takes place between fuzzy. This connection will depend on the type of phase. Multiphase flow is described as a continuum environment penetrated, expressing the concept of phase volume fractions that shown by α_q . Volume fraction represents the space occupied by each phase and the laws of conservation of mass and momentum are satisfied for each phase separately. The equation according to the definition of volume fractions for two-phase flow is as follows:

(1) $$\alpha_1 + \alpha_2 = 1$$

According to the project, while springs or wells and the production phase N is stable issue, equations of continuity as a whole will be defined by the following equation:

(5)
$$\frac{\partial\left(\rho_N \alpha_N u_{Ni}\right)}{\partial x_i} = S_N \quad \rightarrow \quad \nabla.\left(\alpha_N \rho_N \vec{U}_N\right) = S_N$$

N = 1,2, respectively is representative of electrolyte phase and oxygen gas. ρ_N is density phase, S_N in the above equation as copper wells in cathode or oxygen springs in anode that according to Faraday relations define in accordance with the boundary condition. Relationship momentum for each phase is as follows:

(6)
$$\frac{\partial}{\partial t}\left(\rho_N \alpha_N u_{Nk}\right) + \frac{\partial}{\partial x_i}\left(\rho_N \alpha_N u_N u_{Nk}\right) = \alpha_N \rho_N g_k + F_{Nk} - \delta_N \left\{\frac{\partial p}{\partial x_k} - \frac{\partial \sigma^D_{Gki}}{\partial x_i}\right\}$$

In the above equation $\delta_1 = 1, \delta_2 = 0$ and F_{Nk} reaction force on the phase of N and in direction k, ie we have:

(7) $$\sum_N F_{Nk} = 0$$

Using the unique continuity relationship of phase, momentum relationship for a unique phase in the other phases will be simplified as follows.

(8)
$$\rho_N \alpha_N \left\{\frac{\partial u_{Nk}}{\partial t} + u_N \frac{\partial u_{Nk}}{\partial x_i}\right\} = \alpha_N \rho_N g_k + F_{Nk} - \Upsilon_N u_{Nk} - \delta_N \left\{\frac{\partial p}{\partial x_k} - \frac{\partial \sigma^D_{Gki}}{\partial x_i}\right\}$$

Υ_N is rate of mass transfer to phase N and F_{Nk} reaction force on N phase. Of the four forces is given.

(9) $$F_{Nk} = F_{N,drag} + F_{N,turb} + F_{N,grav} + F_{N,conc}$$

To drag force the volume, bubbles in control volume that volume fraction α_D from that bubble is formed, by following simple relationship is obtained by Schiller-Newman as follows.

(10) $$F_{N,drag} = -\frac{3}{4}\frac{C_D}{d}\rho_C \alpha_D \left|W_k\right| W_k$$

The drag coefficient for the relationship Schiller-Newman is as follow:

(11) $$C_D = \frac{24}{\mathrm{Re}}\left(1 + 0.15\,\mathrm{Re}^{0.687}\right)$$

Concentration differences created in a continuous phase can cause differences in density and therefore differences in the weight of the fluid.

As gravitational force effect on weight differences that so-called concentration Ba-Yunus force of the continuous phase that the relationship is expressed as follows:

(12) $$F_{N,conc} = \delta_{N,1}\alpha_N\left[-\rho_N \vec{g}\,\beta\left(C - C_{ref}\right)\right]$$

In the equation of momentum transfer part of force is due to different concentrations, resulting in another relationship that must be solved and is coupled with the momentum equation, transport equation are associated with the concentrations.

(13) $$\nabla\cdot\left(\alpha_1 C_{Cu}\vec{U}_1\right) = \nabla\cdot\left(\alpha_1\left(\rho_1 D_1 + \frac{\mu_{T,1}}{Sc_T}\right)\nabla\left(\frac{C_{Cu}}{\rho_{Cu}}\right)\right) + S_{Cu}$$

This equation shows how the copper ion due to concentration differences in the fluid field moved, in fact, the mass transferred. D, Sc_T and Sc_u is copper ion diffusion coefficient, turbulent Schmidt number, spring semester mass flux for example in the Andes and mass transfer flux well at the cathode. To term due to buoyancy force we have the following density change in equation:

(14) $$F_{N,grav} = \alpha_N\left(\rho_N - \rho_{ref}\right)\vec{g}$$

ρ_N is phase density N, ρ_{ref} reference density or input of the electrolyte and $\vec{g}$ the acceleration of gravity. The term comes from the Turbulenceforce include:

(15) $$F_{N,turb} = \left(-1\right)^N C_{td}\,\rho_N\, k\,\nabla\alpha_N$$

Two-phase flow can be streamed each phase of type calm or chaotic flow in the electrowinning cell since the Reynolds number for gas and liquid flow may seem small, is quiet but because of big natural flow caused by the concentration difference.

Drag the bubble motion has Rayleigh number that is very large thus disturbing the flow of the electrolyte phase and gas phase also disturbed and in some cases, because of the small size of the bubbles assume quiet. There are different ways to analyze the turmoil. One of the best ways to solve the problem because of solving electrowinning issue due to walls and the predominant natural flow is $k-\omega$ two-equation method. Single-phase equation is as follows:

(16)

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \rho P - \beta'\rho\omega k + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{1}{\sigma_k}\rho\frac{k}{\omega}\right)\frac{\partial(k)}{\partial x_j}\right]$$

$$\frac{\partial(\rho\omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = \frac{\alpha\omega}{k}P - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{1}{\sigma_\omega}\rho\frac{k}{\omega}\right)\frac{\partial(\omega)}{\partial x_j}\right] + \frac{\rho\sigma_d}{\omega}\frac{\partial(k)}{\partial x_j}\frac{\partial(\omega)}{\partial x_j}$$

$\mu_t = \rho\dfrac{k}{\omega}$ and constants are defined as follows:

$$\beta' = 0.09$$
$$\alpha = 5/9$$
$$\beta = 0.075$$
$$\sigma_k = 2$$
(17) $$\sigma_\omega = 2$$

Turbulent viscosity in inner viscosity used in the Navier-Stokes equations is as follows:

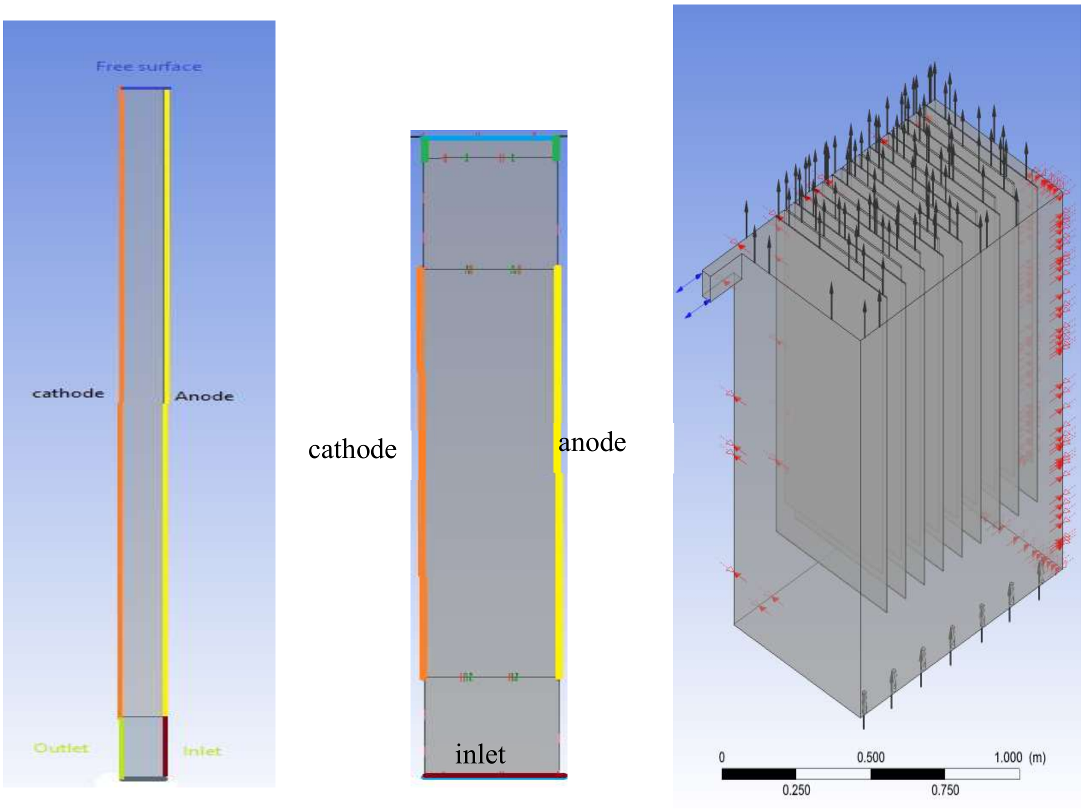
(18) $$\mu = \mu_{electrolyte} + \mu_t$$

3. Research Methodology

Modeling done in this paper is based on Eulerian-Eulerian, two-dimensional and three-dimensional, stable, no heat transfer, mass transfer, and two-phase turbulent done. Flow in the gas phase due to low Riley and Reynolds number is quiet and for liquid electrolyte phase due to high turbulent Rayleigh number is intended. Approach turbulence model is $k-\omega$.

3.1 Geometrical and physical characteristics of cells

In this paper models simulate that are two-dimensional electrowinning cell with input line with anode and cathode plates (Leahy model), two-dimensional electrowinning cells with perpendicular input to the anode and cathode plates (Filzwieser model) and symmetric and simplified three-dimensional electrowinning cell (Najmi model).



a b c

Figure 1 shows the geometry of the cells using Ansys model

Figure 1 electrowinning cell geometry, respectively, from left to right a) Leahy, b) Filzwieser and c) Najmi

In Table 1 the characteristics of these cells is displayed. Electrolyte temperature inside the cell is 50 ° C and molecular mass of copper $M_{Cu} = 63.546 \frac{gr}{mol}$ is considered.

Table 1 features three different cells

Parameter	Leahy's values	Filzwieser value	Najmi value
Current density i	**300 A/m2**	**200,A/m2**	**191 A/m2**

fluid input speed	$2.5\times10^{-2}\frac{m}{s}$	$2.61\times10^{-5}\frac{m}{s}$	$8\frac{m^3}{hr}$
density of the electrolyte	$\rho_1=1200\frac{kg}{m^3}$	$\rho_1=1200\frac{kg}{m^3}$	$\rho_1=1220\frac{kg}{m^3}$
density of oxygen	$\rho_2=1.2\frac{kg}{m^3}$	$\rho_2=1.2\frac{kg}{m^3}$	$\rho_2=1.2\frac{kg}{m^3}$
viscosity of the electrolyte	$\mu_1=1.18\times10^{-3}\frac{kg}{m.s}$	$\mu_1=0.835\times10^{-3}\frac{kg}{m.s}$	$\nu_1=1.34\times10^{-6}\frac{m^2}{s}$
viscosity of oxygen	$\mu_2=2.18\times10^{-5}\frac{kg}{m.s}$	$\mu_2=2.18\times10^{-5}\frac{kg}{m.s}$	$\mu_2=2.18\times10^{-5}\frac{kg}{m.s}$
diffusion coefficient D	$8.62\times10^{-10}\frac{m^2}{s}$	$8.62\times10^{-10}\frac{m^2}{s}$	$8.62\times10^{-10}\frac{m^2}{s}$
Number transfer t_+	0.0849	0.0849	0.0849
Coefficient of expansion β	$0.0019\frac{m^3}{kg}$	$0.0019\frac{m^3}{kg}$	$0.0019\frac{m^3}{kg}$
input concentration and reference C_{ref}	$40\frac{kg}{m^3}$	$50\frac{kg}{m^3}$	$50\frac{kg}{m^3}$
Acid concentration	$180\frac{kg}{m^3}$	$165\frac{kg}{m^3}$	$165\frac{kg}{m^3}$
Cell dimensions	$40mm\times1000mm$	$291mm\times30mm\times110mm$	$1015mm\times3.2mm\times1052mm$ $943mm\times6mm\times920mm$

4. Boundary Conditions

Wall boundary condition for the restriction can be divided into two types of non-slip condition and free-slip condition, non-slip condition $U_{Wall}=0$ and free slip condition which is $\begin{cases}U_{n\,Wall}=0\\ \tau_w=0\end{cases}$. All the walls of the cell and the anode and cathode have a wall non-slip boundary condition to electrolyte phase and free sliding wall for gaseous oxygen phase. High free surface of cell has boundary condition of degassing and these conditions for the electrolytic or the continuous phase condition of the wall plays a role with a free bet slip and to disperse phase as the output with speed of gas reaching to free surface performed. As a result, oxygen gas reaches the surface as fast as leaves it. Anode with non-slip boundary condition to electrolyte phase and slip-free wall for gaseous oxygen phase and since oxygen production reaction also occurs; it is spring production of oxygen as well.

Quantity of material as oxidation is carried that according to Faraday's law related to cell electric current. The total volume of oxygen produced at the anode to the electric current is linearly related. According to Faraday's law to apparent Dbi of produced gas are as follows.

(19)

$$Flux_{OX} = \frac{IM_{O_2}}{4F} = \frac{32\frac{gr}{mol}}{4\times 96485\frac{A\cdot s}{mol}} I = 8.291\times 10^{-8} I \left[\frac{kg}{s}\right]$$

I is total current on anode and cathode surfaces. For the cathode boundary condition we have:

(20)

$$Flux_{Cu} = \eta\frac{IM_{Cu}}{zF} = \frac{63.45}{2\times 96485}\eta I = 32.881\times 10^{-8}\eta I \left[\frac{kg}{s}\right]$$

η is cathode current efficiency is usually about 88 to 92 percent, which is linked to the working conditions of the cells. It is important to note that the electrolyte as single-phase mixtures of continuous variable considered which contains two copper ions fluid and diluted sulfuric acid. Since the variable is intended that the mass fraction of copper indicated.

For input boundary condition, constant dbi is considered that include only electrolyte with real copper concentration.

For example, Najmi geometry input electrolyte to cell is $3\frac{kg}{s}$. This input by diagonal holes in the bottom of the tube is removed. In the simple geometry, half that amount entered into the cell. Matter fuzzy conditions specified input is the only electrolyte and enters with the copper concentrations $36\frac{kg}{m^3}$.

Outflow boundary condition based on faster convergence experience at the beginning for solving the boundary condition for opening is selected. Fuzzy terms in output, copper concentration $32\frac{kg}{m^3}$ and electrolyte is considered. The opening boundary conditions where the application will flow in and out of the border have the capability.

Because our border is output in real terms at the end of solving the general boundary condition will be opening into the outlet. This condition will only answer quickly reaches higher. There are also physical symmetry in the boundary plane of symmetry and geometric symmetry, asymmetric or symmetric boundary conditions in accordance with $U_n = 0$, $\frac{\partial\phi}{\partial n} = 0$ relationships established which

ϕ contains all scalar parameters such as pressure, density and volume fraction, and so on.

5. Results

This section reviews the results and compares them. In Figure 2 the linear vertical speed diagram between anode and cathode is shown in the middle Filzwieser cells. Compare modeled by this study and tests show very low error.

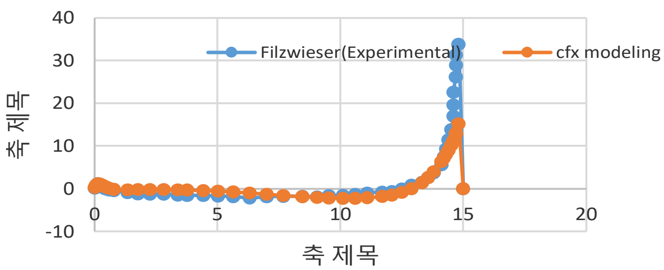


Figure 2 compares results of vertical speed tests between the anode and the cathode in the Filzwieser test cell with Ansys CFX modeling results

Figures 3, 4 and 5 compare the numerical results of Leahy electrowinning cells modeling with the results obtained in this study are discussed. Compare the figures show the validity of the modeling done in this paper for the cell has high precision.

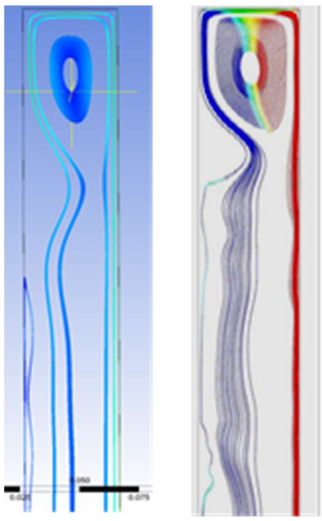


Figure 3 compares the current contour of left and right Leahy modeling

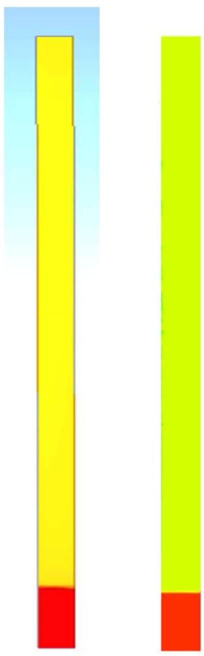


Figure 4 compares the color concentration analysis left and right Leahy modeling

Figure 5 compares the two-phase flow vertical speed between the anode and the cathode in the Najmi 3-dimensional electrowinning cell modeling has been done by this study. A small error can be seen in the results due to the creation of eddy currents induced in the 3-dimensional turbulence is assumed.

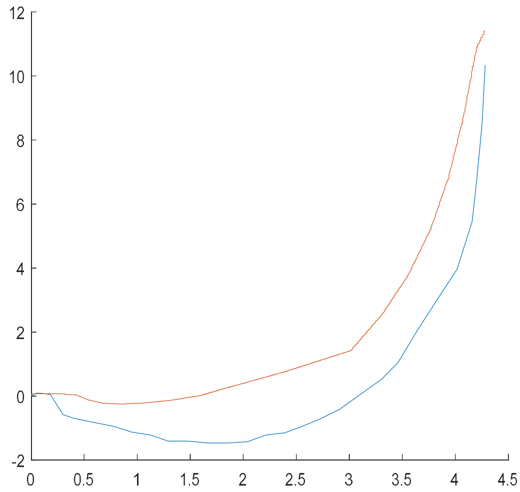


Figure 5 shows a red line analysis by blue modeling and Najmi Nouri modeling

6. Conclusion

In this article, copper electrowinning cells using 3 modeling have been investigated. These three types consisted of Leahy two-dimensional cell with perpendicular input flow to the electrodes, Filzwieser two-dimensional cell with input flow parallel to electrodes and Sarcheshme copper three-dimensional cell (Najmi). After modeling for validation with experimental results were compared each of the cells, good agreement between them indicating the accuracy of the modeling of two-phase electrowinning cells. Current obtained in three models represent the upward motion of flow close anode due to drag force, upward motion at a slower pace due to the depletion of copper cathode near the cathode and electrolyte style and speed falling in the middle of the stream is due to rotate. The main advantages of this study are ready to create a general model for the majority of two-dimensional and three-dimensional electrowinning cells and recognizing the hydrometallurgical properties of cells.

References

[1] C Geankoplis, "Transport Processes and Separation Process Principles", *Book*, 2003.
[2] M.J. Leahy, M. P. Schwarz, "Flow and Mass Transfer Modelling for Copper Electrowinning: Development of Instabilities along Electrodes", *Hydrometallurgy* 147–158,2014.
[3] K.R. Kim, S.Y. Choi, S. Paek, J.Y. Park, I.S. Hwang , Y.Jung ,"Electrochemical Hydrodynamics Modeling Approach for a Copper Electrowinning Cell" , *International Journal Of Electrochemical Science* 12333-12347, 2013.
[4] A. Filzwieser, K. Hein, G. Mori,"Current Density Limitation and Diffusion Boundary Layer Calculation Using CFD Method", *Research Summary of Copper Electrodeposition*, 2002.
[5] K.R. Kim , S.Y. Choi , S.H. Kim ,J.B. Shim , S.Paek , I.T. Kim, "Computational electrochemo-fluid dynamics modeling in a uranium electrowinning cell" , *Journal Radioanal Nucl Chem* 165–170, 2014.
[6] A.S. Filippov , I.A. Korotkin , A.A. Kanaev, V.V. Kondakov I O. Urazov , O.A. Ushakova , P.G. Yakovlev, "Thermoelectrical Regime of a Vertical Electrolyzer. Part 2:Numerical Modeling of Two-Phase Electrolyte Convection" , *Journal of Engineering Thermophysics*, 320–327, 2008.
[7] R. Wedin , L. Davoust, A. Cartellier a, P. Byrne, "Experiments and modelling on electrochemically generated bubbly flows" , *Experimental Thermal and Fluid Science*, 685–69, 2003.
[8] H. Hemmati , A. Mohebbi , A. Soltani , S. Daneshpajouh, "CFD modeling of the electrolyte flow in the copper electrorefining cell of Sarcheshmeh copper complex", *Hydrometallurgy*, 54–63, 2013.
[9] L. Abdelouahed, R.Hreiz, S Poncin, G Valentin, F. Lapicque, "Hydrodynamics of gas bubbles in the gap of lantern blade electrodes without forced flow of electrolyte: Experiments and CFD modelling", *Chemical Engineering Science*, 255-265, 2014.
[10] S. Kawai, T. Miyazawa, "CFD modelling and simulation of industrial-scale copper electrorefining process", *Minerals Engineering*, 81-90, 2014.
[11] M.P. Schwarz,"Improving zinc processing using computational fluid dynamics modelling –Successes and opportunities", *Minerals Engineering*, 12-18, 2012.
[12] L. Abdelouahed, G.Valentin, S. Poncin, F.¸Lapicque, "Current density distribution and gas volume fraction in the gap of lantern blade electrodes", *Chemical Engineering Research And Design*, 559–570, 2014.
[13] R.AlShakarji, "Mechanisms of acid mist formation in electrowinning", *PhD thesis, James Cook University*, 2012.
[14] S.h. Hashemabadi, "CFD simulation of multiphase flows with Fluent software" book, 2011
[15] T.A.Muhlare, D.R.Groot, "The effect of electrolyte additives on cathode surface quality during copper electrorefining", *The Journal of The Southern African Institute of Mining and Metallurgy*, 371-378, 2011
[16] M. Rahimi and A. Parvareh, "CFD study on mixing by coupled jet-impeller mixers in a large crude oil storage tank" *Computers & Chemical Engineering*, **Vol. 31**, Issue 7, 737-744 (2007).
[17] M. Najmi Nouri, A Nouri; "concentration boundary layer analysis in the paper" presented at the Seventh International Conference on Application of CFD in chemical and petroleum industries, Kerman Shahid Bahonar University, May 1395.