

Matched Filter based Synchronization for OFDM Systems

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Abstract

An improved timing and carrier frequency synchronization scheme for orthogonal frequency division multiplexing (OFDM) systems based on matched filtering approach is discussed. This paper extends the work presented in [1] through incorporation of carrier frequency synchronization. The estimation variance is greatly minimized by utilizing matched filtering for training symbol based timing synchronization. Once it is established that timing synchronization achieves performance improvements in [1], the carrier frequency offset is estimated and compensated to achieve carrier frequency synchronization in addition to timing synchronization at the receiver. The overall performance of the scheme is satisfactory in dispersive channels as compared to other methods. The evaluation of bit error rate performance of the scheme for different channels, through computer simulations emphasizes the improvement obtained.

Keywords:

Timing synchronization; carrier frequency synchronization; OFDM; training symbol; matched filter

1. Introduction

Recent years have been marked as revolution in terms of advancement in mobile communication and wireless broadband internet services, which in effect, is based on development of communication systems that can cope with wireless channels effectively to provide better performance and high data rates. Usually the major liabilities of a wireless channel are the multipath effects and the fading that transmitted signal has to undergo. Thus, a reliable and efficient transmission scheme needs to be used to overcome these impairments. Due to the ability of OFDM systems to effectively counter multi-path fading channel effects, they are a popular choice for wireless communication. This advantage is ensured as the data is transmitted on orthogonal subcarriers as well as the use of cyclic prefix (CP). A loss of sub-carrier orthogonality consequently results in performance degradation caused by inter symbol interference (ISI) and inter carrier interference (ICI) due to lack of synchronization. Therefore, to avoid performance deterioration caused by synchronization errors, the problem is of considerable importance when discussing OFDM systems [2].

The timing and carrier frequency synchronization errors are more significant than clock offset errors, and

hence degrade the system performance [3]. OFDM transmission overcomes severe multipath channel conditions due to long symbol period and use of special guard interval known as cyclic prefix, having the ability to combat ISI and ICI [4].

OFDM synchronization has been an research area for some years now. Various authors have discussed timing and carrier frequency synchronization jointly as well as separately ([1]-[9]). The timing and frequency offsets can be estimated using correlation between cyclic prefix and received OFDM symbol ([5], [6]). The limitations observed in these techniques were of frequency ranges and/or the inability to perform timing synchronization correctly if channel affected the CP [3]. Schmidl did the foundation work addressing training symbol based timing and carrier frequency synchronization of OFDM systems [7]. He suggested the training symbol structure having two identical parts in time domain, to be transmitted before the OFDM data. In this way, timing metric was formed after correlation between received samples at receiver. The timing metric gave an indication of the timing position of the OFDM data or timing offset if any, consequently providing a solution to the timing synchronization problem. Further sharpening of this timing metric was proposed by Minn [8]. These methods provide an estimate of timing point expected to lie within guard interval samples of guard interval, and the possibility of finding the exact timing point is not guaranteed. A synchronization method was proposed by Puska in [3] using the training symbol of S&C, known to the receiver based on matched filtering. The performance of this method was satisfactory in dispersive tapped delay line modeled channels, but was found inadequate when the evaluation was carried out under severe channel conditions such as Stanford University Interim (SUI) channels. A scheme was proposed in [1] focusing on timing synchronization, considering perfect frequency synchronization, which ensured that the performance of proposed OFDM system is match- able with that of perfectly time synchronized scenario, even if the evaluation is carried out in drastic channel conditions. This paper is an extension to the work presented in [1] by withdrawing the perfect carrier synchronization assumption, and dealing the OFDM synchronization problem in presence of timing and carrier frequency offsets. The issues of timing offset estimation, channel estimation and carrier frequency offset estimation and compensation is addressed.

Regarding the organization of this paper, problem formulation is discussed in section II. In section III, the proposed method for OFDM synchronization is highlighted. The performance evaluation through computer simulations is discussed in section IV. Section V concludes the paper.

2. Problem Formulation

A. OFDM System Architecture

OFDM system architecture is depicted in Fig. 1. Phase shift keying is employed for mapping the serial input data. The mapped symbols are arranged in parallel block for subsequent transmission on orthogonal subcarriers. This is implemented using N -point inverse fast fourier transform (IFFT) to form an OFDM symbol [4]. The cyclic prefix is formed by replicating the last N_g samples of OFDM symbol at the start of the symbol. An OFDM symbol containing N samples can be written mathematically as [3]

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} D(k) e^{j2\pi kn/N} \quad (1)$$

where k indicates subcarrier index, $D(k)$ represents modulated data and n denotes discrete time index [3]. The OFDM data is transmitted on N orthogonal subcarriers. The guard interval or cyclic prefix length N_g is chosen such that it is greater than the channel delay spread given by K . The signal comprises of multiple such blocks of OFDM symbols and is transmitted after digital to analog conversion and appropriate radio frequency modulation. The signal traverses the multipath wireless channel before reaching the receiver. The received signal is radio frequency down converted and analog to digital converted. Considering the transmission taking place in AWGN channel, the converted received symbol can be written as [3]

$$r(n) = s(n - \tau) e^{j\theta} + w(n) \quad (2)$$

Where τ denotes the propagation delay, $w(n)$ represents the additive white gaussian noise with variance σ^2 , and θ is phase introduced during the transmission [3]. Once the OFDM symbol is properly received, it undergoes fast fourier transform (FFT) and symbol de-mapping to generate original transmitted data as shown in Figure 1.

B. Effects of Synchronization Issues

As highlighted in [1], it is highly desirable that while estimating the timing point, the timing estimate should match the actual start point of useful symbol part (excluding CP), as in this way the sub carriers remain orthogonal and the OFDM symbol is recovered without any interference. The performance is similar to ideal timing synchronization case. If we talk about carrier frequency synchronization, the physical inherent nature of the oscillators induces an inevitable error between carrier frequencies at transmitter

and receiver [2]. The normalized carrier frequency offset epsilon is given by

$$\nu = f_{\text{offset}} / \Delta f \quad (3)$$

where Δf is the subcarrier spacing and f_{offset} is the carrier frequency offset. If the transmitted signal is denoted by $x[n]$ in time domain, and a carrier frequency offset of ν is introduced,

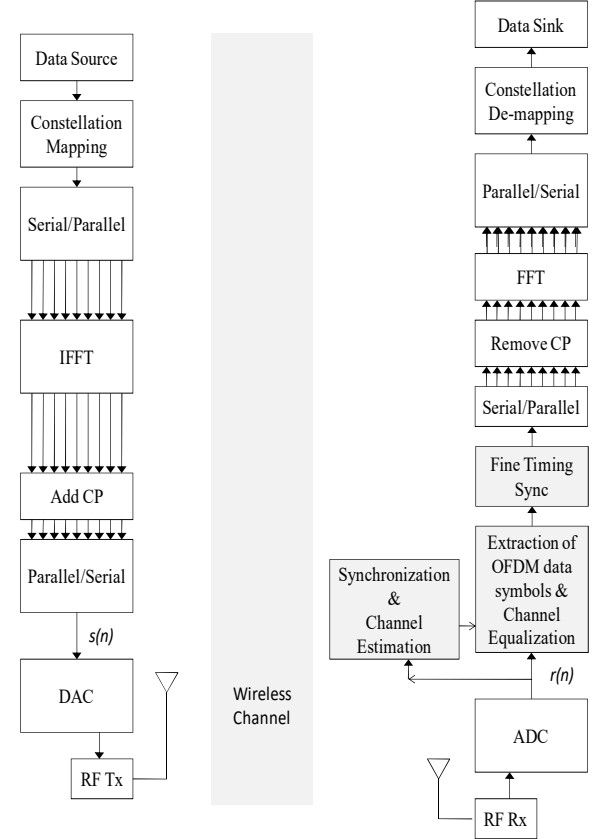


Figure 1. OFDM system architecture.

which eventually results in a phase offset of $2\pi n\nu$. As we can see, the phase offset is proportional to ν and time index n . Fig. 2 highlights that a CFO of ν induces a frequency shift of $-\nu$ in frequency domain signal $X[k]$, resulting in ICI [2]. This eventually results in corruption of different subcarriers by other subcarrier's influence.

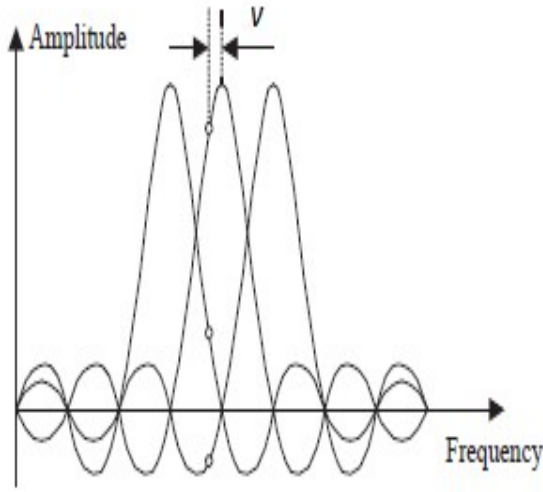


Figure 2. Inter-carrier Interference by CFO.

3. Proposed Scheme For Synchronization

C. Structure of Training Symbol

For synchronization purposes, a training symbol is usually appended with the transmitted signal. This effectively forms the start of every transmitted signal. As in [1], the received samples are correlated with known training symbol samples to get timing metric at the receiver. The known training symbol is composed of an N length pseudo noise preamble.

D. Timing Synchronization

Assuming burst mode transmission, the receiver keeps on receiving noise until the actual signal arrives; this is composed of training symbol followed by data symbols. The training symbol is searched at the receiver, and start position of the signal is estimated, eventually the training symbol is distinguished from noise and the receiver identifies the data symbol through timing synchronization. After estimating the start of each OFDM symbol, it is convenient to take N - point FFT of useful part of every symbol.

The scheme proposed in [1] assumes the training symbol having no repeated part, thus $L=1$ for timing synchronization purpose. If T is the training symbol part of length M , by considering $L=1$, the length of T will be N ($M=N$). Let R denote the received samples, then $P(d)$ is obtained by correlation of every M conjugated samples of R with T , where the discrete sample index d is incremented to shift the window of correlation. In order to normalize the timing metric let R_E represent the energy of T , then by

using (4) - (6), the timing metric M_T having a single peak as proposed in [1] is obtained.

$$P(d) = \sum_{k=0}^{M-1} R^*(d+k)T(k) \quad (4)$$

$$R_E = \sum_{k=0}^{M-1} |T(k)|^2 \quad (5)$$

$$M_T(d) = \frac{|P(d)|^2}{R_E} \quad (6)$$

E. Channel Estimation and CFO Estimation

Unlike the case in [1], the assumption of perfect carrier frequency synchronization is withdrawn in this paper, so there will be a carrier frequency offset induced in the received data symbols causing a phase offset in the received symbol. Hence it is necessary to estimate the offset, and the estimate is used to compensate the CFO effects at the receiver during the channel estimation process [9]. For such cases, a training symbol with $L=2$ is transmitted after our primary training symbol in order to estimate the CFO. If the received samples, after the peak location t_{coarse} are given by $\{y(k); k=0,1,\dots,N-1\}$ then the normalized frequency offset v can be estimated as [9]

$$\hat{v} = (L/2\pi) \sum_{m=1}^H w(m) \phi(m) \quad (7)$$

$$w(m) = \frac{3(L-m)(L-m+1) - H(L-H)}{H(4H^2 - 6LH + 3L^2 - 1)} \quad (8)$$

$$\phi(m) = [\arg\{R_y(m)\} - \arg\{R_y(m-1)\}]_{2\pi} \quad (9)$$

where $1 \leq m \leq H$

$$R_y(m) = \frac{1}{N-mM} \sum_{k=mM}^{N-1} y^*(k-mM)y(k) \quad (10)$$

where $0 \leq m \leq H$

Channel estimation is carried out utilizing the same training symbol which has been used for timing synchronization. For transmission of one block of q OFDM symbols, the channel response is assumed to remain constant for the interval. The channel taps are denoted as $h_0, h_1, \dots, h_{K-1}$. The following matrices are defined to model the system [9]:

$$\mathbf{r}(0) \square [r(0) \ r(1) \ \dots \ r(N-1)]^T \quad (11)$$

$$\mathbf{h} \square [h_0 \ h_1 \ \dots \ h_{K-1}]^T \quad (12)$$

$$\mathbf{w} = [w(0) \ w(1) \ \dots \ w(N-1)]^T \quad (13)$$

$$\mathbf{S} = \begin{bmatrix} s(0) & s(-1) & \dots & s(-K+1) \\ s(1) & s(0) & \dots & s(-K+2) \\ \vdots & \vdots & \ddots & \vdots \\ s(N-1) & s(N-2) & \dots & s(N-K) \end{bmatrix} \quad (14)$$

The signal $s(n)$ contains the $N+N_g$ samples of training symbol and corresponding CP and can be written as

$$\{s(n) : n = -N_g, -N_g + 1, \dots, N-1\}$$

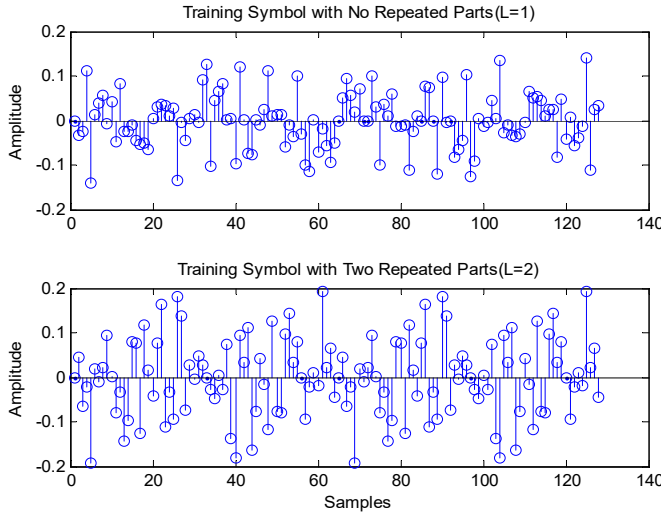


Figure 3. Training symbol structure for L=1 and L=2.

Furthermore, the N length received samples are represented as

$$\{r(n) : n = 0, \dots, N-1\}$$

Finally, the noise samples are written as

$$\{w(n) : n = 0, \dots, N-1\}$$

$\mathbf{S}$ is assumed to be full rank which is justified due to random nature of raining symbol data. Using this model, we can write the received training symbol as [9]

$$\mathbf{r}(0) = e^{j\theta} \mathbf{W}(\nu) \mathbf{S} \mathbf{h} + \mathbf{w} \quad (15)$$

where θ is phase introduced by the channel. When assuming, perfect carrier frequency synchronization $\nu=0$, so the matrix $\mathbf{W}(\nu)$ becomes an identity matrix as in [1]. In this paper, the CFO is non-zero so using matrix hermitian transpose, the

maximum likelihood channel response can be estimated as [9]

$$\hat{\mathbf{h}} = [\mathbf{S}^H \mathbf{S}]^{-1} \mathbf{S}^H \mathbf{W}^H(\nu) \mathbf{r}(0) \quad (16)$$

where $\mathbf{r}(0)$ is taken as N samples after the t_{coarse} timing estimate. Therefore using the known training symbol shaped into $\mathbf{S}$ with the received $\mathbf{r}(0)$, we can easily estimate the channel impulse response. In presence of CFO, the offset effects are easily compensated using estimate of normalized frequency offset ν in (16).

F. Channel Equalization and Fine Timing Synchronization

The estimated channel $\hat{\mathbf{h}}$ represent the multipath wireless channel effects and carrier frequency offsets. This channel information can be used to nullify the channel effects through equalization of the received data stream. Let the received symbols be represented in frequency domain as R_X , the estimated channel frequency response be denoted by H_e . This can lead to design of a filter G_e which is the inverse of H_e . The equalized data R_{EQ} in frequency domain can be obtained by multiplying G_e with R_X as outlined in Figure 4.

$$R_{EQ} = R_X \cdot G_e \quad (17)$$

where $G_e = 1/H_e$ is the inverse filter to nullify the channel effects. If needed, the equalized data can be processed further to undergo fine timing synchronization stage. Since the equalization operation nullifies the multipath channel effects and compensates the carrier frequency offset errors on the training symbol, the peak location can be estimated again more accurately during this stage. Since the stream is already equalized, the performance of fine timing synchronization is much better than the cases of coarse timing synchronization (without equalization) and perfect timing synchronization (without equalization) [1]. In case of highly dispersive channels, or transmission at lower SNR, coarse timing may not provide accurate timing estimation and CFO compensation. In such a case, fine timing synchronization improves the results.

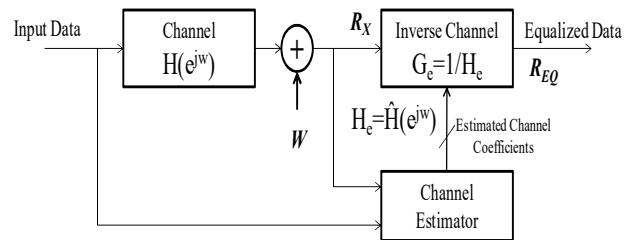


Figure 4. Equalization process for OFDM data.

4. Computer Simulation

G. Simulation Parameters

MATLAB has been used to carry out the computer simulations. BPSK is used to map the input data available in binary form, and choice of number of sub carriers is selected as 128, $N_g=128$. The length of guard interval samples is chosen as $N_g=8$. For block transmission, q OFDM symbols are transmitted per block, $q=10$. The number of repeated parts $L=1$ for timing synchronization followed by a training symbol with $L=2$ for estimation and compensation of CFO. The channels available for terrain type C, SUI-1 and SUI-2 are utilized to carry out performance evaluation in SUI channel scenario. The parameters of SUI channels are listed in Table-I. The number of simulation runs is 10,000 times, and signal to noise ratio (SNR) values range from 1 to 25 for different channel realizations. The performance evaluation and comparison is highlighted using bit error rate (BER) plots of the OFDM system in different channel environments.

H. Performance Evaluation

In this work, problem is addressed when there is need of carrier frequency synchronization in addition to timing synchronization unlike the situation in [1], where performance evaluation has been done focusing on timing synchronization, considering perfect frequency synchronization. Once it is established that the proposed timing synchronization scheme matches the performance of ideally time synchronized OFDM system even in drastic channel conditions, the performance is now tested for the cases when there is a need to perform carrier frequency synchronization as well due to presence of carrier frequency offsets.

TABLE 1 SUI CHANNEL PARAMETERS

Type	Channels Terrain Type C								
	Delay Spread (micro seconds)			Power Attenuation(dB)			Doppler (Hertz)		
	Taps			Taps			Taps		
	1	2	3	1	2	3	1	2	3
SUI-1	0	0.4	0.9	0	-15	-20	0.4	0.3	0.5
SUI-2	0	0.4	1.1	0	-12	-15	0.2	0.15	0.25

Figure 5 highlights the performance of matched filter based synchronization as it compares the performance of proposed method with an ideal synchronization scenario in SUI-1 channel. It is evident that matched filter based timing synchronization performance (without equalization having

zero carrier frequency offset) almost provides the same BER as in the case of perfect timing synchronization without equalization. Estimated channel coefficients are utilized to equalize the channel distortion, which results in overall BER improvement as shown by the dashed curve in black.

It is also highlighted in Figure 5 that introduction of CFO degrades the performance. However, even in presence of normalized CFO, $\nu=0.1$, the system performs timing synchronization (without equalization) to exhibit performance closely matched with the performance of perfect timing synchronization case. Further performance improvement is possible through equalization process, when the CFO effect is compensated using the estimated CFO. This scenario is depicted in the figure using blue curves.

Figure 6 highlights the performance of matched filter based synchronization in SUI-2 channel as it compares the performance of proposed method with a perfect synchronization scenario. The performance of matched filter based timing synchronization scheme (without equalization having zero carrier frequency offset) is slightly degraded as compared to perfect timing synchronization without equalization. It is also highlighted that introduction of CFO degrades the performance. However, even in presence of normalized CFO, $\nu=0.1$, the system performs timing synchronization (without equalization) to exhibit performance having a slight difference than the performance of perfect timing synchronization case. Further performance improvement is possible through equalization process, when the CFO effect is compensated using the estimated CFO. This scenario is depicted in the figure using blue curves.

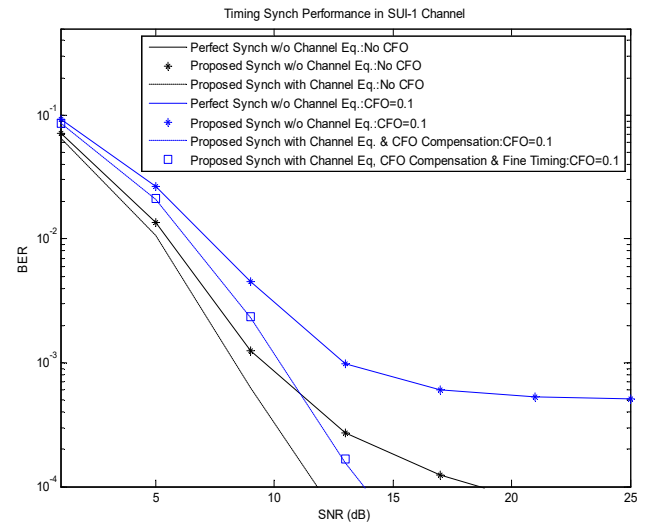


Figure 5. Performance of Proposed Method in SUI-1 channel in presence of CFO=0.1

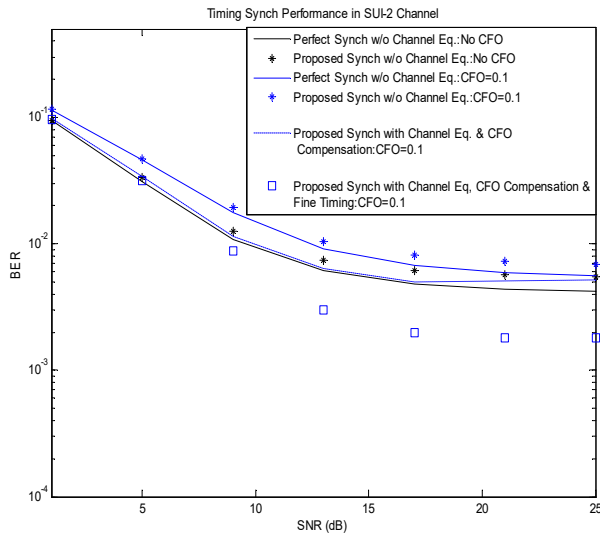


Figure 6. Performance of Proposed Method in SUI-2 channel in presence of CFO=0.1

The significance of fine timing synchronization is evident in the case of SUI-2 channel which is more dispersive than SUI-1, and hence fine timing synchronization plays its role in performance improvement as shown by blue boxes. This validates the claim that fine timing synchronization is needed in case of high channel degradation.

5. Conclusion

Using matched filtering; a scheme for synchronization of OFDM systems is addressed in this work. The scheme proposed in [1] is extended by incorporating carrier frequency synchronization in addition to timing synchronization. The structure of training symbol for timing synchronization used is unique, as used in [1] followed by the Schmidl training symbol for carrier frequency synchronization. To achieve improvement in performance, matched filtering is utilized to perform timing synchronization, maximum likelihood approach is used for channel estimation and Minn's method is considered for CFO estimation. Benefitting from the transmission of a training symbol which is known to the receiver, significant performance improvement is achieved as compared to other methods. Further performance improvement is possible utilizing the channel estimation and CFO compensation

process leading to channel equalization and fine timing synchronization at receiver. The performance of the matched filter based timing synchronization method is compared in presence of CFO, ensuring that significance of synchronization and equalization method is highlighted. The BER plots obtained through computer simulations depict that the presented method offers almost perfect results without overhead of additional training symbol for channel estimation, as the receiver already has knowledge of training symbol which is used to determine the channel estimates. The presented method effectively extends the scheme for timing synchronization proposed in [1], and highlights its robustness in SUI channels even in presence of CFO. The extension of this work to support MIMO OFDM systems is planned to appear in future publications.

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